

ALGEBRA RELATED TO PARTICLES  
OF SPIN  $3/2$

ONE of us (B.S.M.) has considered [(1942), referred to here as I], the question of deriving commutation rules for the matrices  $\beta$  appearing in the relativistic wave-equation of a particle of arbitrary spin in the form

$$\partial_\mu \beta_\mu \psi + \kappa \psi = 0 \quad (1)$$

of the famous Dirac equation for a particle of spin  $1/2$ . It was there shown that this problem could be solved by making the new assumption that the spin operator  $t_{\mu\nu} = i s_{\mu\nu}$  satisfies the condition

$$t_{\mu\nu} = (\beta_\mu, \beta_\nu) \equiv \beta_\mu \beta_\nu - \beta_\nu \beta_\mu \quad (2)$$

for all spins.

The general commutation valid for all spins can then be written as

$$(\beta_\mu, t_{\nu\rho}) = (\beta_\mu, (\beta_\nu, \beta_\rho)) = \delta_{\mu\nu} \beta_\rho - \delta_{\mu\rho} \beta_\nu \quad (3)$$

The special cases of spins  $3/2$  and 2 were considered in I, and the restricted forms of (3) on the further assumption that the eigenvalues of  $s_{\mu\nu}$  for a particle of spin  $s$  are  $s, s-1, \dots, -s+1, -s$ , were also given there [I, (26) and (34)] for these cases.

The imposition of condition (2), while it solves this particular problem of deriving the commutation rules, has also very far-reaching consequences in that it makes wave-equation (1) itself of fundamental importance in obtaining properties of the elementary particles. Bhabha (1945, a, b, c) has recently considered the full implications of the assumption (2) and shown that the problem of finding all irreducible equations of form (1) can be connected with that of finding all irreducible representations of the Lorentz group in five dimensions. He has further shown that, on this theory, a particle of maximum spin  $n$  must appear with  $n$  different values of the rest-mass if  $n$  is an integer, and  $n + 1/2$  values if  $n$  is half an odd integer, the higher values of the rest-mass being simple rational multiples of the lowest value.

On the basis of his new theory Bhabha has considered (1945, c) in particular, the two possible equations of the form (1) for a particle of maximum spin  $3/2$ , and indicated, by consideration of the non-relativistic approximation, that the equation given by one of the representations denoted by  $R_\pi(3/2, 1/2)$  may possibly describe the behaviour of the proton. The degree of this representation, *viz.*, 16, and



