Exploring cultural diversity of the people of India

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The People of India project of the Anthropological Survey of India has assigned the entire Indian population to 2753 communities distributed over 32 states/union territories. This article explores: (a) number of communities in relation to population size, (b) number of and similarity in the traits exhibited by different sets of communities, (c) clustering of the traits in relation to geography, food habits, subsistence strategy and (d) impact of modernization on the division of labour within the Indian society.

THE Indian subcontinent, with its tremendous range of environmental regimes, supports a large human population that is a fascinating mosaic of varied cultural traits. This cultural diversity is organized in the form of tens of thousands of endogamous caste groups that function as largely closed breeding units. Furthermore, in rural India, social interactions and hence transmission of cultural traits overwhelmingly involve members of an endogamous caste group, with inter-group interactions primarily focusing on economic transactions. Members of an endogamous caste group therefore tend to share cultural traits, so that between-group variation in cultural traits is significantly larger than the within-group variation. India's cultural diversity is therefore best investigated in terms of such endogamous caste groups. Presently, however, this is an impossible task given the very large numbers of groups involved. These endogamous caste groups, however, form a smaller number of culturally homogeneous clusters. These correspond to the 'communities' investigated by the People of India project[1,2]. We then have at our disposal extensive information on 2753 such communities making up 4635 elements when a community population in each state/union territory is counted as a separate element. A yes/no response to 776 individual items of information ranging over identity, ecology, food habits, occupation, kinship patterns, marriage rules, art and music, as well as educational status and impact of development programmes is now available for these 4635 community elements. Undoubtedly this represents information of varying degrees of reliability and value from an ecological, anthropological and developmental perspective. Detailed investigation of this material must therefore be based on careful sifting. But before that begins it is worthwhile enquiring into the broad patterns emerging from this rich set of data; this is what we propose to undertake here.

How many communities?

A good way to begin is by asking how many different communities occur in a given geographical area. The POI data furnishes this number for the 25 states and 7 union territories with population ranging from 40,000 in Lakshadweep to over 110 million in Uttar Pradesh[3]. In addition, the total number of communities for India as a whole is 2753 (Table 1). The relationship between the number of communities and human population for the states and union territories is described well by the equation (see Figure 1):

Table 1. States with their populations and number of traits

State/UT	Population (millions)	No. of communities	Mean no. of traits	Mean dissimilarity	Total no. of traits	Abbrev.**
Andhra Pradesh	53.550	386	215.8	0.394	723	AP
Assam	19.897	115	220.9	0.393	684	ASM
Bihar	69.915	261	223.2	0.335	677	BHR
Gujarat	34.086	289	175.8	0.437	682	GUJ
Haryana	12.923	82	231.7	0.336	636	HAR
Himachal Pradesh	4.281	116	229.2	0.348	698	HP
Jammu and Kashmir	5.987	111	201.1	0.454	691	J&K
Karnataka	37.136	300	178.4	0.399	684	KAR
Kerala	25.454	225	195.2	0.363	676	KER
Madhya Pradesh	52.179	342	182.4	0.430	695	MP
Maharashtra	62.784	305	204.7	0.433	735	MHR
Manipur	1.421	29	225.0	0.382	564	MNP
Meghalaya	1.336	27	235.4	0.434	636	MGH
Nagaland	0.775	25	285.2	0.320	629	NGL
Orissa	26.370	279	224.3	0.360	708	ORS
Punjab	16.789	95	192.7	0.400	644	PUN
Rajasthan	34.262	228	171.6	0.417	649	RAJ
Sikkim	0.316	25	208.7	0.379	539	SIK
Tamil Nadu	48.408	364	213.7	0.374	733	TN
Tripura	2.053	78	218.3	0.312	566	TRP
Uttar Pradesh	110.862	307	206.4	0.413	721	UP
West Bengal	54.581	203	210.4	0.376	701	WB
Andaman and Nicobar	0.189	20	179.6	0.482	571	A&N
Arunachal Pradesh	0.632	66	218.3	0.358	572	ARU
Chandigarh	0.452	42	192.0	0.388	600	CHN
Dadara	0.104	14	180.9	0.308	409	DDR
Delhi	6.220	147	214.4	0.330	681	DEL
Goa	1.008	35	177.6	0.401	525	GOA
Lakshadweep	0.040	7	215.3	0.123	287	LAX
Mizoram	0.494	17	256.2	0.197	460	MIZ
Pondicherry	0.604	67	213.2	0.346	629	PON
Daman	0.079	28	188.2	0.314	493	DMN
Total (All India)	685.185	2753*	204.1	0.449	776	ALL

*If a community occurring in more than one states is counted separately for each state, there are 4635 community elements.
**These abbreviations are employed in Figures 1, 3, 4, 5, 6 and 7.

$$C = 0.074\ P^{0.46}, \quad (1)$$

where C is the number of communities in a state/UT, P the population of that state/UT. (For the log–log least square fit, $r = 0.94$, the 95% confidence limits for r are 0.88 and 0.97).

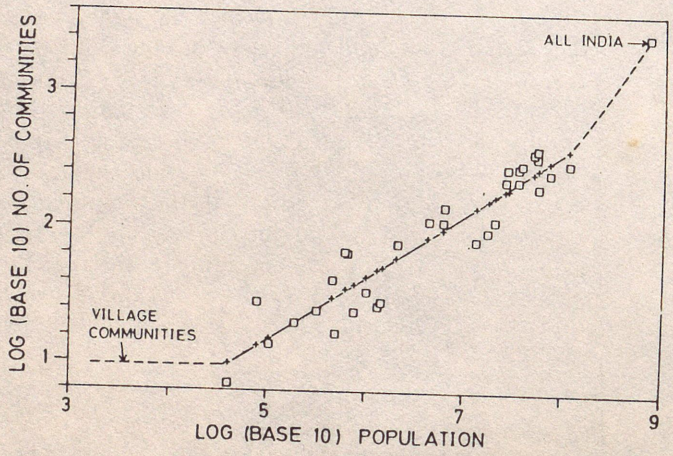

Figure 1. The number of communities occurring within a state/union territory as a function of its population in 1981. The dotted lines represent the number of communities co-occurring at the village level and at the level of the country as a whole.

Union territories are reasonably homogeneous; states are somewhat more heterogeneous, in particular, large ones like Uttar Pradesh that at once embrace the thickly populated Gangetic plains and the more thinly populated hill areas. For units of this level of homogeneity with populations ranging between 1 million and 100 million, the total population size effectively predicts the number of communities. However, the total number of communities predicted for the country as a whole on the basis of 1981 population (685 million) using equation (1) is 900, which is far smaller than the actual number of 2753. This implies that new communities are added *across* states/UTs at a much higher rate than they are *within* the culturally more homogeneous units like the states/UTs (see Figure 1).

Extrapolating equation (1) at the lower end of population size suggests that a village settlement, typically with 500–1000 inhabitants, should support either 1 or 2 communities. Extensive data are available on the composition of village communities throughout India, the best source being the 188 monographs accompanying the 1961 census[4]. Only a very small number of such villages located in tribal tracts support just 1 or 2 communities. More commonly villages with populations ranging from 500 to 1000 support 8–16 communities. This much larger number of communities at the lower end of population size reflects the traditional division of labour in rural India, so that while 1 or 2 cultivator (or more rarely, fishing or pastoral) communities make up the bulk of the population, several traditional service/artisan communities occur in smaller numbers (see Figure 1).

How many traits per community: α-diversity

One may explore the diversity of individual communities and groups of communities in terms of measures of diversity used in community ecology[5–7]. If an individual human community is taken to be analogous to a biological community, the number of traits reported by it corresponds to the number of species of a biological community. This is species richness, one of the measures of α-diversity. When one compares several biological communities along an environmental gradient, the number of new species added and old ones deleted is termed species turnover, a measure of β-diversity. The number of traits which are not shared in common by two human communities, or trait dissimilarity would then be an analogous measure of diversity for the POI data. Finally the total number of species reported by a whole set of biological communities of any region is termed γ-diversity. In the case of POI data this would correspond to the total number of traits reported by a group of human communities. This may be termed as trait accumulation.

The currently available computerized POI data base is a list of those attributes (out of a possible total of 776) for which any given community has a positive response. Figure 2 depicts the frequency distribution of

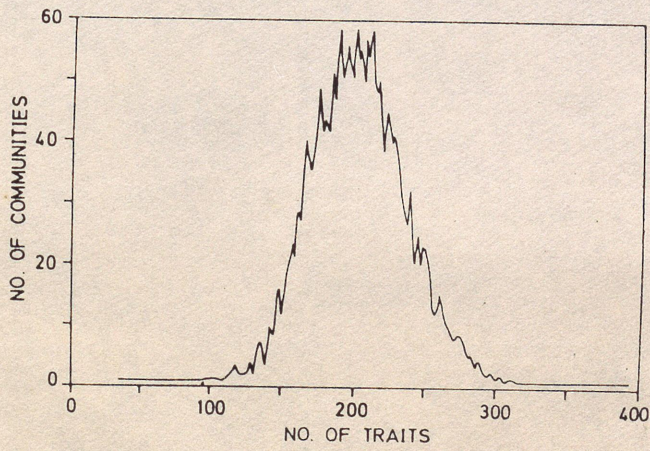

Figure 2. Frequency distribution of the number of traits (out of a possible maximum of 776) reported by the 4635 community elements.

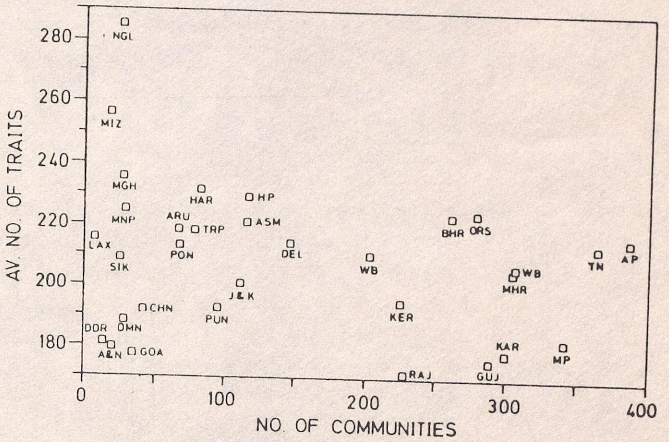

Figure 3. Mean number of traits reported by a community as a function of the number of communities in a state/union territory.

the number of traits which a community reports: the minimum being 32, the maximum 406 with a median of 202 and a mean of 204 with an s.d. of 35.4. The 32 different states and UTs show noteworthy differences in the number of traits reported on an average by communities belonging to that territory (Figure 3). North-eastern states of Nagaland (285) and Mizoram (256) report much higher and the north-western states of Gujarat (176) and Rajasthan (172) report a much lower number of traits than the bulk of other territories (Table 1). This corresponds to the fact that the north-eastern hill states are not only more heterogeneous topographically, but harbour tribal populations which have few food taboos, rather fluid cultural practices and minimal occupational specialization. Indeed most communities reporting large number of traits from other states are also tribals or groups dependent much more directly on natural resources such as Khas of Himachal Pradesh or Gonds of Maharashtra. The rather arid territories of Rajasthan and Gujarat are topographically more homogeneous, and harbour caste populations with extensive food taboos, more rigid cultural practices and considerable caste-based occupational specialization. Groups reporting very small number of traits from other states are either little-studied tribals of Andaman and Nicobar islands such as Sentinelese, Jarwas and Shompens or communities constituted out of normal endogamous groups through some special process like Faqirs (ascetics) of Maharashtra or Hijras (sexually abnormally developed individuals such as gynandromorphs) of Uttar Pradesh.

Differences between communities: β-diversity

The second component of diversity would then be trait dissimilarity, or the proportion of unshared traits amongst two communities (see Appendix). Of the 2753 POI communities, 791 occur in two or more states:

These communities may be expected to exhibit lower levels of trait dissimilarity in comparison with two randomly picked communities. Although the data do exhibit such a tendency with Mann–Whitney test resulting in significant differences at $p < 0.01$, the tendency is weak in that the means of the two distributions are not significantly different. Evidently geographical proximity is an important component governing the distribution of the POI traits.

In addition to territory, the 2753 POI communities can be classified on a variety of other bases, such as religion, scheduled caste or tribe, current occupation, rural or urban, nomadic or settled. Amongst such categories, the clearest trend with respect to dissimilarity obtains in case of religious categories. Followers of the ancient Jain religion overwhelmingly follow business/trade as an occupation and are concentrated in a contiguous geographical tract encompassing Rajasthan, Gujarat, Maharashtra and Madhya Pradesh. They exhibit the lowest levels of trait dissimilarity, 0.31 within a state and 0.36 across states. Followers of Christianity have converted much more recently, over a wide geographical tract and from very different cultural backgrounds. They follow an exceedingly wide range of traditional occupations. Trait dissimilarity is highest amongst them for any religious category, 0.38 within a state and 0.46 across states. Sikhs, Hindus, Muslims and Buddhists exhibit, in that order, increasing levels of trait dissimilarity. The trends amongst occupational categories are less clear; nomadic pastorals exhibit the highest levels of trait dissimilarity and agriculturists the lowest with business/trade communities occupying an intermediate position.

Figure 4 depicts the levels of trait dissimilarity as a function of number of communities for the 32 states/UTs. Lakshadweep harbours only 7 communities, all followers of Islam, all Malayalam speakers and all subsisting on fishing and coconut cultivation. They exhibit a very low level of trait dissimilarity of 0.12 (Table 1). So does the predominantly tribal north-eastern state of Mizoram, whose 17 communities exhibit a dissimilarity of 0.2. On the other hand, the Andaman and Nicobar islands have complemented the 6 original tribal communities with 14 others derived from widely different cultural background, so that these have a high dissimilarity level of 0.48. Amongst the more populous mainland states with over 250 communities, Bihar (261 communities) has the lowest dissimilarity level of 0.34 and Maharashtra (305 communities) the highest dissimilarity level of 0.44. Bihar was the seat of the 2500-year-old agricultural civilization of Magadha, and appears to have acquired over centuries a high level of cultural homogeneity. Maharashtra achieved widespread settled cultivation much later and is known as a state transitional between the northern and southern cultural zones. This seems to be reflected in the trait dissimilarity level. We expect low trait richness to be correlated with high levels of trait dissimilarity. As Figure 5 shows, this is borne out by the statistically significant negative correlation ($r = -0.438$, the 95% confidence limits are -0.68 and -0.11) between these two variables, represented by the equation, $\beta = 0.627 - 0.0012\alpha$, where β is the average proportion of unshared traits in a pair of communities, and α is the average number of traits per community in the state/UT.

How many traits in a state: γ-diversity

Trait accumulation, analogous to γ-diversity, is a third measure of diversity that may be examined on the basis of the POI data. Trait accumulation is the total set of traits represented in a group of communities, such as

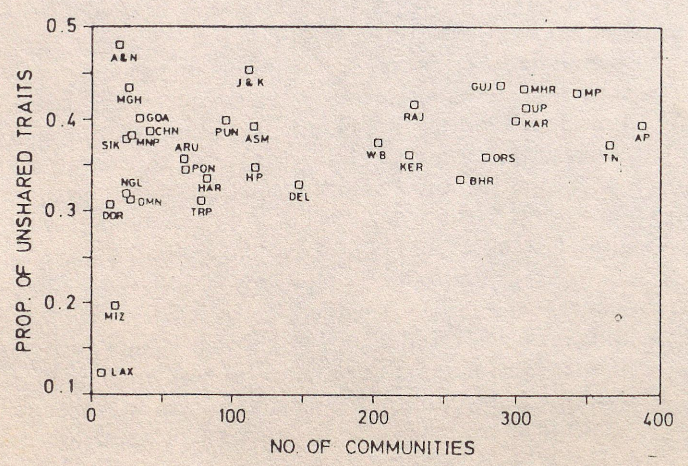

Figure 4. Mean value of dissimilarity (proportion of traits not shared) between communities within a state/union territory as a function of the number of communities in the state/union territory.

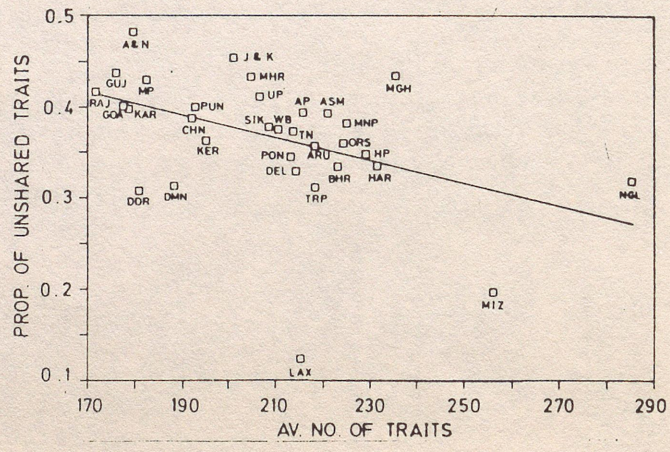

Figure 5. Mean value of dissimilarity between communities as a function of the mean number of traits reported by communities in the 32 states/union territories.

those co-occurring in a state/UT. Obviously trait accumulation is a function of trait richness and dissimilarity. In addition, it depends on how the traits are distributed across communities. If there is a wide variation in the distribution of traits (i.e. some traits being very common, some very rare, etc.) an equation of the form

$$T = An/(B+n) \tag{2}$$

should provide a reasonable fit[8], where A and B are functions of trait richness and dissimilarity, and n is the number of communities. Empirically, the POI territory level data on trait accumulation (Figure 6) are well-fitted by the equation:

$$T = 730\,n/(10+n) \tag{3}$$

(least square fit for $1/T$ vs $1/n$, $r = 0.95$, the 95% confidence limits for r are 0.90 and 0.97).

As the number of communities becomes large (say > 100), T asymptotically approaches a value of 730. In the POI data Maharashtra with 305 communities has the highest T value of 735, with Tamil Nadu with 364 communities close behind at 733 (Table 1). The very high accumulated trait set of Maharashtra seems to reflect its intermediate position in the north–south cultural gradient, with communities representing features typical of north as well as south. Lakshadweep with only 7 rather specialized and homogeneous set of communities has the lowest accumulated trait set of 287, well below the maximum of 406 for a single community, that of Khas in the Himachal Pradesh.

Correlation analysis: searching for clusters of traits

Beyond measuring diversity, one may look for patterns in the distribution of diversity in terms of geography as well as social and economic stratification. The simplest geographical hypothesis may be that the set of traits held by groups of communities diverges with distance between them. Thus while neighbouring Bihar and Orissa share 63% of traits with each other both share only 54% of traits with the distant Gujarat (see Appendix). But further exploration reveals this hypothesis to be overly simplistic. Thus while Himachal Pradesh shares 55% of traits with neighbouring Jammu and Kashmir, it shares 56% with distant Assam, and Haryana shares 58% of traits not only with neighbouring Punjab, but distant Tripura as well. However, while distance is not a simple predictor of divergence between two groups of communities as in states/UTs, a multivariate analysis reveals these territorial elements to cluster in an interpretable fashion. Figure 7 is a representation of the similarities between different territorial elements through multi-dimensional scaling. Geographical proximity evidently does explain cultural similarity to a fair extent.

Indian society continues to be predominantly an agricultural society. It is therefore of considerable interest to enquire whether there are clusters of cultural traits associated with the use of specific food plants (see Appendix). An exploration of the People of India data reveals several such clusters; six of these, associated with wheat/moong/masur/mustard, jowar/ragi/tur/groundnut, barley/maize/mustard, coconut, peas/beans and ragi merit further discussion. The first of these, associated with wheat/moong/masur/mustard spans over the Indus–Ganga–Brahmaputra valleys and includes a set of linked cultural traits involving *gotra*-based exogamy, *kanyadan* during marriage, use of vermilion

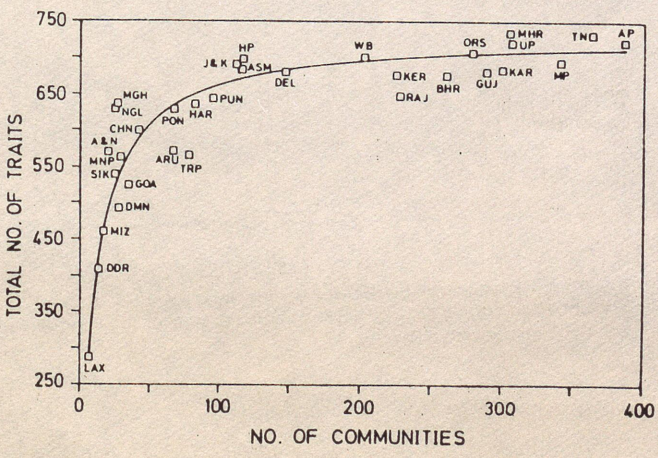

Figure 6. Total number of traits reported as a function of the total number of communities for the 32 states/union territories.

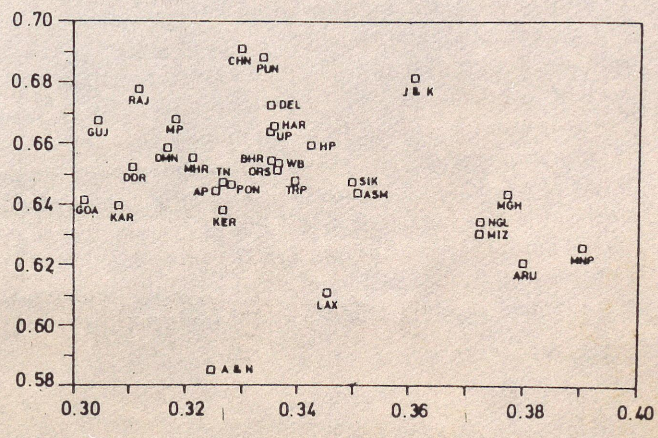

Figure 7. Extent of similarity between states and union territories for the traits reported by the constituent communities. This depiction is based on a multi-dimensional scaling analysis.

and bangles as symbols of a woman's married status, cremation of dead, joking relationship between sister-in-law and younger brother-in-law, avoidance between daughter-in-law and father-in-law as well as between sister-in-law and elder brother-in-law, and acceptance of water from members of other communities. The complex of jowar/ragi/tur/groundnut characterizes the south Indian plateau and Tamil Nadu plains and is associated with a set of linked cultural traits including surname-based exogamy, practice of uncle–niece and cross-cousin marriages and the use of *tali* and toerings as symbols of married status of women. Coconut, universally used on the west coast is linked to use of betel leaf and snuff, cross-cousin marriages, performance of marriage ceremony in bride's residence, use of *tali* as a symbol of married status and observance of puberty and pre-delivery rituals. The complex of barley/maize/mustard characterizes the hilly areas of Western Himalayas and is linked to terrace cultivation, smoking of loose tobacco, acceptance of levirate form of marriage, participation of caste council in resolution of disputes over land, practices of exorcism and divination and attribution of putative kinship to members of other communities. We are, of course, not stating anything novel in identifying the geographical spread of these traits, since diffusion of cultural traits is well known to have taken place within these four broad geographical tracts. However, the identification of such expected patterns from our data strengthens our faith in other, less obvious patterns.

Hilly, still partially forested regions of central India and northeast harbour many tribal communities which share several traits, not so much because they have been in communication, but because these traits adapt them to their more primitive mode of resource use. There is indeed a large cluster of traits associated with shifting cultivation, primarily an activity of tribals. This cluster includes habitation in forest habitat and extensive use of forest produce, use of streams as a water source, hunting, fishing and gathering activities, terrace cultivation, maintenance of chicken and pigs, basket weaving, women's active participation in fishing, agriculture and fuel collection, role of traditional village council in punishing offences like adultery and theft, animistic beliefs, practice of exorcism and participation of both sexes in singing folk songs and in dancing.

Less expected, and therefore more noteworthy is the linkage of a number of traits related to occupation with specific food habits. The consumption of peas and beans is strongly linked to a community's participation in modern service, trade and industrial sectors, high levels of education for both boys and girls, use of piped drinking water, watching television and saving through banks and other institutions. On the other hand, use of ragi is strongly linked to unskilled agricultural labour, employment of child labour, use of fuelwood, women's involvement in collecting fuelwood, fetching water and participation in agricultural and animal husbandry activities, as well as wage earning, community's involvement in barter economy and weekly markets and worship of village deities.

A significant component of Indian population is nomadic, and POI records 276 non-pastoral and 37 pastoral nomads amongst its 4635 community elements. One expects pastoral nomads, i.e. shepherds, cowherds, etc. who move over large distances with their livestock primarily to occur in habitats where grazing resources are highly seasonal. This would be the case in regions of scanty rainfall or seasonal snowfall. Indeed while only 26 out of 3954 (0.65%) communities in regions of high or medium rainfall practice nomadic pastoralism, as many as 9 out of 646 (1.5%) of communities in tracts of low rainfall, 4 out of 103 (4%) in tracts of scanty rainfall and 15/196 (7.5%) in tracts of snowfall do so. Part of cultural diversity of the Indian subcontinent can thus be neatly explained by the ecological setting.

Occupational diversity: traditional and contemporary patterns

Finally, cultural traits of Indian communities are in no way stagnant; they are undergoing rapid change. The POI data set has substantial information on this aspect as well, information pertaining to changing patterns of religious practices, marriage relationships, family planning practices, health care, food habits, education, occupation and so on. We illustrate the emerging patterns here by looking at occupation, since transmission of occupation from one generation to the next was a notable feature of the Indian caste society. The POI data classify occupations in 42 categories and record traditional as well as current practices. As expected, traditional occupations show a strong mode at 1 (Figure 8), the average (computed by excluding

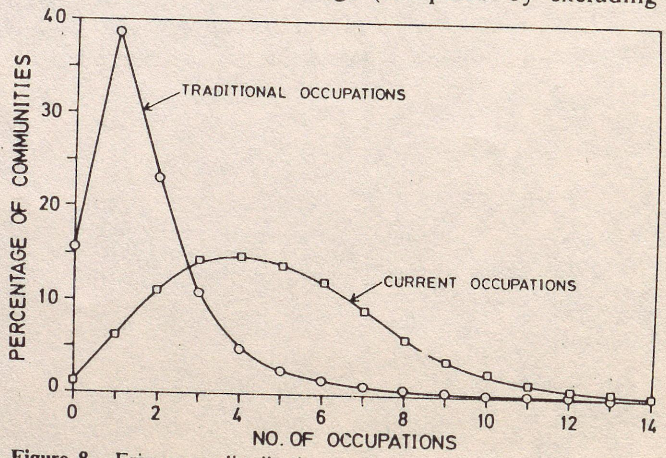

Figure 8. Frequency distribution of the number of traditional and current occupations (out of a possible maximum of 42) reported by the 4635 community elements.

communities for which no information exists) being 2.8 with a s.d. of 1.8. The maximum is 17. It is notable that all communities reporting large numbers of traditional occupations are tribal communities, not within the ambit of the traditional caste-based division of labour. In contrast, the mode of current occupation has shifted to 4, with a mean of 5.1, an s.d. of 2.2 and a maximum of 25. Indian communities are clearly widening their base of occupation. It is also noteworthy that while the five commonest traditional occupations, in decreasing order, were settled cultivation, unskilled labour, animal husbandry, trade and business and hunting and gathering; the five commonest current occupations (in terms of involving at least some members of a given community), again in the same decreasing order are Government service, settled cultivation, unskilled labour, self-employment and private service. People of India are evidently part of a dynamic society. However it is significant that fully 528 out of the 4635 community elements have apparently failed to enter even marginally the most valued occupational categories of Government and private service.

Appendix

Similarity between communities

For a pair of communities, this is defined to be the probability that a randomly picked trait reported by one of the communities is also reported by the other.

If T_1, T_2 are the number of traits reported by the two communities, and $T_{1\,2}$ is the number of traits seen in both the communities, then the similarity will be: $S_{1\,2} = 2 \cdot T_{1\,2}/(T_1 + T_2)$, and the dissimilarity is given by $D_{1\,2} = 1 - S_{1\,2}$.

The similarity and dissimilarity are thus the proportions of shared and unshared traits, respectively, for a pair of communities.

Similarity within a group of communities

Similarity within a group is the expected similarity between two communities, randomly chosen from the same group. If a group (e.g. a state) contains N communities there can be $N(N-1)/2$ comparisons of pairs of communities. The *mean similarity*, computed over the $N(N-1)/2$ similarities, has been termed as the similarity between communities within a group. Thus the statement that 'the dissimilarity within Mizoram is 0.2' implies that the similarity between two randomly picked communities from Mizoram is expected to be 0.8.

Similarity between groups of communities

Similarity between two groups is the expected similarity between a pair of randomly picked communities, one from each of the two groups. If the two groups contain N_1 and N_2 communities, there can be $N_1 \times N_2$ such pairs, and the mean similarity computed over all these pairs is termed as the similarity between the two groups. Thus, the statement that 'Bihar and Orissa share 63% of traits' implies that the similarity between two randomly picked communities, one each from Bihar and Orissa, is expected to be 0.63.

Correlation between traits

For two traits X and Y, let

a = Number of communities showing both X and Y
b = Number of communities showing X but not Y
c = Number of communities showing Y but not X, and
d = Number of communities showing neither X nor Y.

For the present data set, for any pair of traits,

$$a + b + c + d = 4635.$$

The Pearson product moment correlation coefficient between the two traits is given by

$$r = \frac{ad - bc}{[(a+b)(a+c)(b+d)(c+d)]^{1/2}}.$$

Testing the significance of correlations

When the number of observations (N) is large, the standard error of the correlation coefficient (r) is $1/(N-3)^{1/2}$. For the correlation to be significantly different from zero at 1% level, the absolute value of r should be higher than 2.58 times the standard error. We have computed correlations between all pairs of 711 traits (such that each trait was reported by at least 30 communities). However, since r between each trait with all the 710 others is tested for significance, about 7 traits are expected to turn out to be significant at 1% level, by chance alone. For such a situation of multiple comparisons, the Bonferroni confidence intervals have been used by carrying out the testing of significance at 1/710 per cent, i.e. at 0.0014% level.

Clusters of traits

A cluster of traits is a group of traits such that all the traits in the group are significantly and positively correlated with each other. To obtain such a cluster centered around trait X, say, we initially identify all the traits significantly positively correlated with X. Suppose there are M such traits. We next examine the correlation of each of the M traits with the remaining $M-1$ traits. In case not all pairs are significantly correlated with each other, that trait which is correlated to the least number of other traits is dropped from the set, to leave a set of $M-1$ traits. This process is iterated till a set of M' traits is left, which are all significantly positively correlated to each other. This is the desired cluster. The analysis presented in the text is based on an examination of a large number of such clusters, obtained using many different traits as starting points.

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Tiger census: Role of quantification

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Current system of censusing tiger populations in India, based on comparisons of tiger pug mark tracings, is controversial. Our attempts to make this method more objective, quantitative and reproducible, by employing modern statistical techniques, are reported.

A key question in the field of wildlife management concerns how many individuals of a species there are in a given population. Various estimation techniques such as water-hole counts, line transect sampling, capture–recapture, etc. are practised depending on their suitability in each case. In the case of tigers, the currently practised method is based on tracings of pug marks. The method is described briefly as follows: During a predetermined period, tracings of fresh pug marks are collected from the study area after an exhaustive survey of the area. Two pug marks produced on the same day are assumed to originate from two distinct animals if their locations are far apart compared to normal distances covered by a tiger in a day. Pug marks found in close vicinity of each other are compared visually in a qualitative manner, to determine whether they could be of the same animal or not. Visual inspection of the tracing is also used to decide the sex of the animal.

Census figures resulting from this method are often contested by ecologists and wildlife experts. Arjan Sing[1] has argued that unless a pug has some deformity or abnormality, it is not generally possible to distinguish between different animals of the same sex and size. Karanth[2] criticised these census figures on the grounds that they have not been validated on a known population anywhere. It is therefore necessary to devise a procedure for discrimination which will be objective and based on sound statistical principles.

Quantifying shape and size of the pug mark

A typical pug mark tracing (see Figure 1) consists of impression of four toes and pad. Typically, foresters (e.g. Panwar[3]) take into consideration features like size, inclination of the top edge of the pad, shape of the lobes at the base, etc. in a qualitative way for discrimination. Instead we have identified a string of variables that can be measured manually or with the help of a digitiser unambiguously. These include areas of the toes and pad, distances and angles between the lines joining centres of these figures, maximum widths, maximum lengths, etc. These measurements can be easily automated using simple programs (see Gogate *et al.*[4]).

Let us assume that these measurements extract all the information about the size and shape of a given tracing.

To be useful in discrimination they must have two

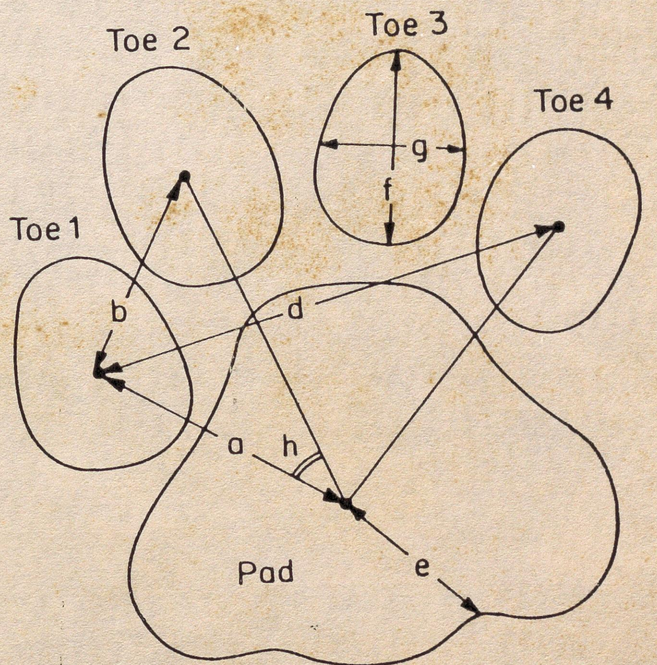

Figure 1. A typical tiger pug mark tracing indicating possible measurements. a, Distance between pad centre and centre of toe 1; b, distance between centres of toe 1 and toe 2; d, distance between centres of toe 1 and toe 4; f, maximum toe length; g, maximum toe width; h, angle; g, distance between pad centre and indent 1.

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more properties. They must be sensitive to variation between animals and must be insensitive to other extraneous factors like substrate, operator, etc. Specifically we must ask the following series of questions.

1. To what extent do the measurements vary if (a) the same pug mark is traced by different individuals, (b) the same person traces different pug marks of the same animal in a trail, (c) the same animal produces pug marks in different substrates?

2. Do the pug marks of males differ systematically from those of females?

3. To what extent do the pug marks of animals in one reserve differ from those in another?

Partial answers to these questions based on pilot experiments are now available.

Variation due to operators, substrates and replication

In one experiment six foresters were asked to trace the same three pug marks from a trail (i.e. the same animal), in Project Tiger, Melghat, Maharashtra. Measurements of these tracings were subjected to analysis of variance for each variable separately. In 28 out of 36 variables examined, neither the operator-to-operator variation nor the pug mark-to-pug mark variation was statistically significant. These variables therefore can be judged provisionally to be stable. Of the eight remaining variables, six showed significant operator effect while two showed significant replication effect. These latter variables therefore should either not be used for discrimination between individual animals or if used, suitable extra allowance should be made to account for this sensitivity.

A similar pilot experiment conducted also at Melghat involved three substrates (fine soil, sand, wet mud) and two animals. These prints were also subjected to analysis of variance. In this analysis 32 variables turned out to be unaffected by the substrate while four showed a significant effect.

The combined list contains 10 variables that exhibit some effect of the factors considered. Thus it appears that 26 out of 36 variables considered are insensitive to extraneous factors and hence potentially useful in discrimination.

Sex discrimination

Foresters generally believe that sex of a tiger can be guessed from the pug mark. Sommerville[5] suggested that pug marks of a male tiger are larger and toes are square while in the female prints these are more rounded and slender. Sankhla[6] claimed that at three months of age, the area of the male's pad is double that of a female, a difference that is maintained throughout life. Panwar[3] suggested that hind paws (not just toes) of the male fit in a prominently squarish frame whereas those of a female fit in a relatively rectangular frame. These ideas need to be made precise and validated with known cases.

Sagar and Singh[7] have recently attempted this. They have come up with what may be called the '1.5 cm rule'. They fit a pug mark within a rectangle and use the difference between length and breadth to decide the sex. If the difference is smaller than 1.5 cm, the animal is classified as male otherwise as female. Paranjpe *et al.*[8] have offered two alternatives to this. The intuitive alternative involves visual inspection of histograms of lengths, breadths and differences. The histogram which exhibits the most prominent pair of modes with a sharp depression in between is used for classification. The x value corresponding to the depression is used as cut-off. The other alternative offered by them is based on the graphical technique due to Bhattacharya[9]. In this approach it is assumed that a suitably chosen variate, say breadth, follows a normal distribution for one sex, say male. It follows a normal distribution in case of females also, but with different parameters. The technique leads to estimation of parameters of both the populations as well as the proportion of each sex in the given set of prints. A rule for classification of individual pug marks can easily be based on these estimates. Stride, the distance from the top end of a left hind pug to the top end of the next impression of the same pug, has been proposed[10] as a measure to distinguish tracks of a leopard and a tiger cub. The above methods can be applied to measurements on stride as well.

These techniques have one common weakness, viz. they are not based on known cases. Hence a small pilot experiment was conducted on two males and three females. These animals were induced to walk on specially prepared ground and around 100 prints were traced. The technique of logistic regression analysis (see Shanubhogue and Gore[11]) was used to develop a sex identification rule. Most of the variables measured on the pug mark tracing turned out to be of marginal use in sex identification. Three variables selected for use were (a) distances between pad centre and centres of first toe (X_1) and second toe (X_2), (b) distance between centres of first and fourth toe (X_3). Intuitively, the first two variables indicate the length of the print and the third indicates the breadth.

If p is the probability that a given pug mark represents a female, then the logistic regression analysis of these three variables yields the formula.

$$\ln \frac{p}{1-p} = 56.06 - 2.76X_1 - 3.56X_2 - 1.1X_3.$$

Given the values of X_1, X_2 and X_3 for a pug mark, this formula yields the value of p. The sex identification rule, proposed here is as follows. If p is below 0.25

classify as male, if p is above 0.75 classify as female, if p is between 0.25 and 0.75 the case is left unclassified. This rule applied to the 97 prints (known sex) yielded the results given in Table 1.

Variations between two reserves

Ninety pug mark tracings from Project Tiger, Kanha, MP and the same number of pug mark tracings from Melghat, Maharashtra were taken for this analysis. Half of each set was used for calibration and the remaining half was used for testing. The two calibrating sets were used to construct a discriminant function based on 10 variables measuring distances between various centres (see Anderson[12]). We denote by $\bar{X}_k$ the set of means of all variables for Kanha and by $\bar{X}_m$ the same for Melghat. Let S_k and S_m be the corresponding sample variance–covariance matrices. The classification rules are slightly different if covariance matrices are unequal.

Take the vector of measurements X of a print to be classified. Compute likelihood assuming X belongs to Kanha. Let it be L_k. Compute likelihood assuming X belongs to Melghat denoted by L_m. Compute $T=L_k/L_m$. If T is >1 the print is classified as belonging to Kanha and to Melghat otherwise. The extent of misclassification is indicated in Table 2.

It seems reasonable to conclude that reserve-to-reserve variation in pug marks is substantial enough to allow correct classification to the extent of about 90%.

An algorithm for counting tigers from pug marks

We now propose in outline an algorithm for determining the number of tigers on the basis of the pug marks collected by foresters from a specified area.

To state the problem formally, we have n independent vectors, $X_1, X_2, \ldots, X_n$, each with p co-ordinates. X_i follows a p-variate normal distribution with mean vector $\vec{\mu}_i$ and variance–covariance matrix Σ (assumed to be known), which is the same for every i. The aim is to find k, the number of *distinct* mean vectors $\vec{\mu}_i$ among the n mean vectors.

At one extreme we may have $\vec{\mu}_1=\vec{\mu}_2=\ldots=\vec{\mu}_n$ in which case $k=1$ and all the pug marks are attributable to one tiger. At the other extreme, $\vec{\mu}_i \neq \vec{\mu}_j$ whenever $i \neq j$ and the collection of pug marks represents n distinct individual tigers.

The algorithm involves first testing the hypothesis $H_0: k=n-1$; if that is accepted, then testing $H_0: k=n-2$ etc. We continue testing until $H_0: k=g$ is rejected. Then we conclude that there are $g+1$ tigers.

How do we test the hypothesis $H_0: k=g$? To fix ideas consider the case $g=1$. If all pug marks belong to the same tiger then an overall measure of variation among n pug marks must be small. In particular

$$\sum_{i=1}^{n} (X_i-\bar{X})' \Sigma^{-1} (X_i-\bar{X}),$$

where $\bar{X}$ is the mean of n vectors, has to be less than the upper, say, 95% value of the chi-square distribution with $(n-1)p$ d.f. If not, the notion that all vectors are from the same distribution is deemed untenable.

For a general value of g, the test is conducted as follows: Divide the n vectors into g groups such that every group has at least one member. For each group, compute a measure of variation between vectors in that group. Pool these measures for all g groups and compare with upper 95% value of the chi-square distribution with $(n-g)p$ degrees of freedom. If the pooled measure of within groups variation is too large reject the hypothesis.

Notice that the number of ways of dividing n individually identifiable vectors into g groups can be astronomically large. If we encounter one of these ways for which the pooled measure of within-groups variation is small, the null hypothesis can be accepted without having to examine all other ways. However, for terminating the algorithm we need to assure ourselves that even for the best way of grouping, the within-groups variation is unacceptably large. If enumeration of all ways is not feasible due to an excessive volume of computations, a large enough sample of these ways can be used instead.

Mathematical justification for the various aspects of the algorithm will be given in detail elsewhere.

Conclusions

1. Size and shape of a tiger pug mark can be quantified and measured objectively.

2. These measurements have some potential use in deciding the sex of an animal. The variables that seem to play a major role in sex discrimination are (a)

Table 1. Sex identification using logistic regression

Decision based on p	Known sex M	F	Total
M ($p<0.25$)	23	7	30
F ($p>0.75$)	1	59	60
Amibiguous ($0.25 \leq p \leq 0.75$)	6	1	7
Total	30	67	97

Table 2. Reserve identification using discriminant function true classification

Decision based on T	Kanha	Melghat	Total
Kanha	40	4	44
Melghat	5	41	46
Total	45	45	90

distances between pad centre and centres of first two toes and (b) distance between first toe centre and fourth toe centre.

3. Most of these measurements are robust in the sense that effect of extraneous factors on these measurements may not be significant.

4. There appear to be significant variations between prints from two reserves namely Kanha and Melghat. This may be a new dimension revealed by the present analysis.

5. The findings presented in this article seem sufficiently promising to warrant a study on a much larger scale to arrive at definitive recommendations for use in the field. In particular, it is imperative to generate a substantial data base of prints of known animals (see Watve *et al.*[13]) to set up numerical decision rules and to test their validity.

6. The possibility of using these and similar techniques for identification of individual animals needs to be explored.

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Book Reviews

Teaching of mathematical modelling and applications by M. Niss, W. Blum and I. Huntley, Prentice–Hall, 66, Wood Lane End, Hemel Hempstead, Hertfordshire, HP2 4RG, England, 1991, pp. 427, $97.

Mathematical modelling has been the 'in' thing for quite some time. It is not at all uncommon to hear the eager and hopeful question "Can we do some modelling with this data that I collected over oh so many years?". While the process of using the mathematics to describe real-world phenomena began centuries ago, the fancy term 'mathematical modelling' seems to be of a more recent origin. In the good old days, people used to write good old differential equations to describe how different variables change with time; now they set up mathematical models! (It is the same case everywhere; recent batches of students – technobrats – no longer write computer programs, they 'develop software'). Despite knowing all this, I was still unprepared to anticipate the extent to which these activities have proliferated—apparently, there have been numerous international conferences devoted to the teaching of mathematical modelling (and 'applications' thrown in for good measure).

The book under review is a result of one of such conferences, in fact, the fourth one – ICTMA-4, held at Roskilde University, Denmark, in July 1989. There are 44 articles by 56 authors from 13 countries, with UK contributing almost 50% share. India, the proud possessor of the second (third?) largest pool of scientific and technical manpower (rather, in deference to the recent fair and just nomenclature, 'person-power') is conspicuous by absence; but then so are USSR (Russia?), China, Japan and France, just to name a few superpowers. However, what the book lacks in representativeness from this angle is more than made up for in the coverage of mathematical topics, and especially in the very systematic organization of the sections and chapters.

The first section – surveys – is possibly the best part of the book, in terms of scope, information as well as entertainment. In the deceptively serious title 'Applied mathematics as a social instrument', P.J. Davis gives a clear, forthright and delightful description of what applied mathematics is all about, with a balanced mix of the philosophical thoughts, abstract language and down-to-earth examples laced with pithy prose. A couple of illustrations: ". . . the three goals are *description, prediction and prescription*; that is, tell me what is, tell me what will be, tell me what to do about it." or "Men have been sent to jail for practicing applied commercial mathematics which employs the wrong kind of axiom system" and the most quotable of all "Is modelling important in the federal decision making process? You'd better believe it. In 1979, half a BILLION dollars were spent on development, use and maintenance of (mathematical) models".

In contrast, the ponderous sounding title of the next chapter by W. Blum is not deceptive at all. Terms such as modelling, application and even real-world problems are *explicitly* defined. On the one hand, one marvels at the orderliness of the mind (parallelling those who relish the exquisite beauty of the highly rigorous rules of Sanskrit grammar). On the other hand, at a cursory glance, the whole exercise looks so pointless and pompous—just an act of belaboring the obvious. This impression is strengthened by the various tables, which indicate the importance of the different topics by decorating them with one, two or three stars! A closer look at the chapter, however, brings out the important contribution made by the author by clearly indicating the very fundamental goals—not just of mathematical modelling, or even of mathematics, but of education in general. While everyone is familiar with these in a somewhat intuitive and nebulous way, the explicitness and clarity of the writing makes this a very readable and useful chapter. The thoroughness of the treatment can be judged by the fact that the discussion includes detailed arguments both for as well as *against* teaching of mathematical modelling. (Halfway through the chapter, I felt the exposition to be far too systematic to be of non-German origin, and a quick look at the author's institutional affiliation confirmed the diagnosis). However, this is no ivory tower sermon—practical aspects such as reluctance of students to take on anything which will make examinations more complex and demanding, reluctance of teachers to acquire more non-mathematical knowledge, etc., are described with a matter-of-fact frankness.

The next chapter on 'Building mathematics curricula' also retains such contact with the real-world situations, but describes them in a refreshingly lucid language. The diverse attitudes adopted by the teachers towards teaching mathematical modelling (some think that it can be taught only to the brightest; others, only to the poorest, students), the unimaginative nature of the many examples described in the routine courses (and the even less-imaginative ways of solving them) and the like have been commented upon with a wry sense of humour. The case for bringing a logical structure to the field of modelling (which otherwise is a maze of bewildering complexity of applications and techniques) has been very well presented. What is most useful, however, is the author's insight into the crucial factors which contribute to the difficulties of the students when it comes to grasping mathematical modelling—it is their unfamiliarity with the use of arithmetic! They have never been taught to 'play' with numbers, nor have they been familiar with the use of even the most basic mathematical operations in a way that connects them with real-life problems. Also instructive is the demonstration of the multiple meanings of even the most ordinary sounding words—number, for example, has six different connotations: count, measure, location, ratio comparison, code and a derived constant. The author's step-by-step approach towards defining the philosophy behind the designing of better curricula can be very profitably read by all instructors of mathematical modelling. Also fascinating are the snippets from history, for example, algorithms for multiplication and division were newly invented in the 15th century, and were therefore taught only to the students of university age!

The last of the surveys deals with computing mathematics, and lists some of the things that can be done with computers (apart from the trivial applications to numerical calculations) *e.g.*, interactive video, algebraic manipulations, special-purpose languages such as Hope, ML, Miranda which deal with functions, predicates, clauses, etc. In short, some very esoteric stuff. A few interesting problems and some unusual applications (bar codes on books) have been thrown in to give a flavour of what can be done in a class-room. A less than cogent and coherent treatment, compared to the previous three articles.

The next section deals with the theoretical aspects of mathematical modelling. This is a thoroughly mixed lot, and contains articles that are:

* Sublimely philosophical—"Platonism . . . is characterized by the belief that mathematics is the highest form of knowledge, and therefore the form of knowledge God must have used when he created the universe. . . . Unfortunately . . . the credibility of Platonism has suffered a series of devastating blows . . . discovery of the contradictions of set theory, Godel's incompleteness theorem and quantum theory, to the popular rejection of its historicist and elitist social assumptions".

* Passionately emotional—"Against ill-founded, irresponsible modelling", . . . "Blind and powerful: ruthless, model-based, digitalized control" . . . "the vicious acceptance of the technological risks and the irrationalities of growth economy" . . . (and so on).

* Amusingly historico–social—"The two distinguishing trends in the international debate (on applications and modelling in mathematics instruction) are: the *pragmatic trends* from the English language area and the *scientific–humanist trend* from the Romance language area". ". . . appropriate understanding of mathematics . . . adequate attitude towards mathematics is needed for **becoming a human being**" (emphasis mine). "The German-speaking debate . . . shows three trends . . . the emancipatory, the science oriented and the integrating".

* Mix of optimistic, pedestrian and esoteric—"Future trends indicate a shift from *problem-doing* to *problem-solving*"; "Importance of teaching mathematical modelling in Bangladesh . . . as an efficient tool to assist in resolving complex problems such as population growth, floods, epidemics. . ."; "Role of the specification language Z, developed by the Programming Research group at Oxford, as taught at Sheffield City Polytechnique.

* Almost farcial—the question–answer sessions from a class, recorded as a part of a micro-ethnographical study: "Teacher: Mrs. Sweet wants to get some strawberries . . . . Is price the only thing. . . .? Student: If it's a long way away, and I come back with the strawberries and the road is rough or something, and the strawberries could go bad or something. [Laughter]"

The next three sections discuss in detail the experiences of teaching mathematical modelling at the Lower Secondary (ages 12–16), Higher Secondary (ages 16–19) and Tertiary (B.Sc./M.Sc.) levels. In the Lower Secondary section, the article by P. Abrantes from Portugal presents a very well thought-out curriculum, with many interesting examples. Laced with dry humour by the author ("this is a critical school level in Portugal because many students drop out due to successive failures, and mathematics strongly contributes to this situation") and the students (their responses to the course – ". . . I liked Geometry because of the *mentalities* which we had to use"), the article puts forward many thought-provoking points of view, *e.g.*, the role of the teacher as one who organizes activities and gives suggestions instead of teaching how to do, and checking if answers are correct. Other equally enjoyable articles demonstrate, through the use of activity-based syllabi (designing board games, planning trips, making pop-up cards, drawing topological trees from results of volleyball tournament, running a petrol bunk), how the learning of mathematical modelling can be made very attractive as well as useful.

The chapters dealing with Higher Secondary level, in contrast, look more like a matter-of-fact account of what is being done in different countries. Again, the major innovation is the activity-based approach. The projects at this level are rather more academic—traveling salesman, properties of the number 1729, applications of regression analysis to calibrate manometers, etc. A couple of articles continue to discuss (yaaaawn!) organizational and implementational aspects; while others are straight-forward single-theme tutorials—conic sections, automatic discrimination of written language, Weibull distribution, fossil fuels, consumption pattern analysis, the inevitable inclined plane for Newtonian mechanics and groundwater modelling. The only breath of fresh air in this section is the account of a very imaginative exercise in physical modelling by A. Kitchen from Manchester, UK. The students were taken to an amusement park, given joy-rides in merry-go-rounds, roller-coasters and the like, and encouraged to set up and analyze corresponding mathematical models. This wonderfully written and well-illustrated article has some irresistible quotes. "The standard of the work produced by the students has improved dramatically over the years . . . it may be that my expectations of them have changed". ". . . (this course) is not just suitable for sixth formers; I have learned a great deal myself", and finally—"Why not visit an amusement park yourself? Try all the rides. Find some problems to solve. Have fun. That is what mathematics is all about".

The last section, for the tertiary level, begins with the most difficult aspect of the teaching of mathematical modelling—how to think up good problems? The article does illustrate a number of interesting ones—refraction through variable media, modelling of the performance of teachers (a sure winner for generating snide comments), rainfall run-off, reservoir contamination, etc. Almost each one of the subsequent articles is devoted to a specific, relatively advanced theme, presented in elaborate detail. These include water quality modelling, spread of the epidemic of rabies modelled in 1 and 2 dimensions, design of a ferry bridge, image compression techniques—just to name a few. All these could as well have been targeted for a pedagogical journal such as the *American Journal of Physics;* and so also the one on "how to, and how not to, use Laplace transforms". No section of this omnibus conference proceedings can be complete without philosophical musings, and the one included here give a very useful account of "good and bad modelling in mechanics".

On the whole, this book is a mixed bag. Sure enough, there is something in it for everyone connected with mathematical modelling, but the score is too broad for most readers. The price ($97 only) is stiff, especially when converted into Rupees, but not overtly high considering that these are conference proceedings—a notoriously pricy class of publications. For those who have the money, time and also the patience to wade through, this volume does contain many valuable nuggets.

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The self-avoiding walk by Neal Madras and Goldon Slade, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1992, pp. 442, SFr 108.

Self-avoiding walk—a path on a lattice that does not visit the same site—is simple to follow but to answer basic questions like, (a) how far an n-step self-avoiding walk typically travels from its starting point, or, (b) how many such walks there are, or many such important ones turns out to be quite difficult. This book provides an extensive survey of the results available to those questions. Though the main focus of the book is on the mathematically rigorous results it also keeps an account of the answers which are arrived through some nonrigorous methods and computer simulation and which are mostly accepted by the researchers in physics and chemistry communities and gives a nice comparison of those with the current state of rigorous result obtained.

The first chapter brings out the basic questions and conjectures about the self-avoiding walk. It also introduces many important notions like the two-point generating function, the bubble condition and bubble diagram, the inclusion–exclusion principle, all the critical exponent and Hammersley's classical work using the idea of concatenation for the existence of critical exponent $\mu = \lim_{N\to\infty} C_N^{1/N}$, where C_N is the number of N-step walk beginning at the origin. The ideas and techniques introduced in the first chapter gets culminated deeply in Chapters 3, 4, 5, 6 and 7. In Chapter 3, the 1961 work of Hammersley–Welsh on the upper bound of C_N is discussed. This is improved by Kestern in 1964. Hammersley–Welsh's bound is still the best available for two dimensions and Kesten's for three and four dimensions. Bounds for the number of self-avoiding polygons are also proved here. The fourth chapter is concerned with Ornstein–Zernike's theory of decay of the two-point function. Here, probabilistic renewal theory came into picture while introducing bridges. In Chapters 3 and 4, subadditivity plays an important role in a subtle or direct way. Most of the proofs here are due to Chayes and Chayes. In Chapter 5, the lace expansion is derived using inclusion–exclusion principle and points out and some of its important application on lattice trees and animals and on percolation, whereas the sixth chapter uses the lace expansion technique which is due to Brydges and Spencer (1985) as a main tool to prove the results on more than four dimensions. Most of these proofs and results are originally due to the authors themselves. These results, as pointed out, resemble the results of random walk and the proofs also use many techniques, used for the related results of random walk. The seventh chapter is devoted to Kesten's pattern theorem which guarantees that a pattern would occur some proportion of times on all N-step self-avoiding walk, except for an exponentially small fraction, if it can occur several times on a self-avoiding walk and its immediate application in the form of ratio limit theorem for the C_N and related quantities. In the ninth chapter we get an extensive survey of various Monte Carlo algorithms that have been used to simulate self-avoiding walk. Rigorous analysis of ergodicity properties and autocorrelation times are the main focus here. Chapters 8 and 10 are devoted to some assorted additional results like the upper bound for a critical exponent, comments on the walk with geometrical constraints, infinite bridge, weakly self-avoiding walk and 'true' self-avoiding walk while Chapter 2 discusses the various applications for the self-avoiding walk in physics and chemistry, and the nonrigorous results due to computational or other methods, including Flory, Fisher, de Gennes.

This book is very much self-contained and is quite accessible to graduate students and researchers in mathematics, physics and chemistry interested in this area, though some more explanation of various concepts or techniques which arise from some specific discipline, would have been better for readers of other disciplines. Unfortunately, some of the topics in Chapter 10 which are related to the random walk have been discussed very briefly, but overall, the book covers a wide and extensive area to describe the main results and applications of self-avoiding walk. Notes and comments at the end of each chapter are quite instructive and point out to appropriate references. Bibliographic references and the notational index are quite extensive, if not full. Since the authors themselves are on the forefront of research in this area, this book, quite expectedly, has become a well instructive and authoritative account for the reader.

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Basic category theory for computer scientists by Benjamin C. Pierce, The MIT Press, 55, Hayward Street, Cambridge, Massachusetts 021 142, 1991, pp. 102, $ 17.95.

Category theory is the most general and abstract branch of pure mathematics and almost every structure in mathematics can be regarded as a category, often in several different ways. It fulfils the computer scientist's quest for a purer view of functions: a theory of functions in themselves, not a theory of functions derived from sets[1]. It provides a general framework for the theories of programming, which form the basis for the design and definition of programming languages and their associated software engineerig methods. However, from the viewpoint of a computer scientist, the real problem has been identified to be the fact that category theoretic definitions and their properties are so much more complicated than those that computer scientists are used to. Even apparently simpler definitions are usually expressed in terms of an alternation of universal and existential quantifiers. The existential quantifiers often take the complicated forms: "There exists exactly (or at most) some x such that. . . ." *This complication and unfamiliarity of the formulae make it difficult to acquire and develop the kind of intuitive skill at pattern matching that makes algebraic calculation such an effective method of mathematical reasoning*"[2].

Category theory can also be thought of as the science of the social behaviour of mathematical structures, whereby mathematical structures are meant such entities like groups, topological spaces, lattices and similar objects with internal structure[3]. By *social behaviour* is meant that one is not so much interested in the how of each such entity is structured internally, but with the how of the interaction of the object with other objects. In particular, no distinction is made between the objects belonging to the different 'species', if the objects cannot be distinguished on the basis of their external behaviour.

It is in this perspective, we feel, that the book should be perceived and received by a computer scientist. The author, being a computer scientist himself, declares that the book is an outcome of his efforts at learning category theory, while he was a graduate student. The background of the author is indeed reflected to a certain extent in the description of the category theoretic concepts in the tutorial introduction in the first two chapters. The various definitions are explicated in terms of the entities familiar to a computer scientist. However, in our opinion, the author would have done well to discuss the hurdles and the mental blocks that he had experienced, when he first embarked on a study of category theory, for we feel that the narration of such experiences will help the computer science readership, many amongst whom still feel that category theory is an arcane and esoteric branch of mathematics with little or no relevance to the practice of theoretical computer science. The tutorial introduction to category theory thus falls short of that intuitive appeal any computer scientist would be looking for in a text of this kind.

In the third chapter, the author reviews some of the application areas of category theory in computer science, most notably in the areas of the theory of programming languages. The author does well in introducing the problem and its category theoretic formulation. But the description again falls short of a comprehensive treatment when the solution procedure is being outlined. The author leaves the reader with a reference to the works of the original proponents of the solutions. It is here, that, we feel, the author could have been more elaborate, for, in our opinion, this is the most interesting part of the *junction between category theory and computer science*[4]. For, it has come to pass in present-day computer science, that it is the extensional or observational behaviour that is of much significance.

The bibliographical survey presented by the author is indeed quite extensive and does provide the interested reader with valuable pointers to 'where to look further'.

In the final analysis, we feel that the author could have taken a few more pages to explain in more detail the *whys* and the *hows* of the application and the applicability of category theory to computer science. Such a description would have been welcome.

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Linear algebra by Gunadhar Paria, New Central Book Agency (P) Ltd, 8/1, Chintamani Das Lane, Calcutta 700 009, India, 1992, pp. 232, Rs 55.

The reviewer feels that the book under review is not suitable for students as it is not self-contained. Neither it is useful for self-study as it is very confusing, nor will it meet the long-felt need of students for a fresh textbook on linear algebra which would have been a good companion for self-study as was claimed by the author.

The basic definitions and results (which should be there in any book on first course on linear algebra) regarding matrices and determinants are not developed in this book. Instead, the author prefers to refer to his own book on matrix and tensor (author refers to his own works only) which I have not seen. Another shortcoming is that it provides neither a bibliography nor an index.

This book has several serious errors including conceptual, logical, notational, typographical (and probably many others which I might have not noticed). Some of these are listed (not exhaustive) below:

- Why call a vector 'multi-dimensional vector'?
- Why call a line (as one usually studies from high school) a 'hyperline' (see page 8)?
- Definition of vector space and that of linear vector space on page 13 is not rigorous; in fact, it is not even correct, *e.g.*, in the definition of linear vector space, it is written that: "*the quantity* λ *being a properly chosen arbitrary scalar.*"
- The statements of the theorems are not clear. Probably they are titles of the subsections that the author would like to choose. For example, on page 35 is is written that:

  "**THEOREM 1.** *Norm of any degree* (*standard form*)."
- The definition of a right coset on pages 82–83 is not correct.
- On page 84, in example 1, it is written that: "*In* R^2, *let* $V = (x, y)$ *be a vector space, and let* $W = (x, y)$ *where* . . . ." This is very confusing as the notation (x, y) in R^2 is the standard notation for a point in the cartesian plane in coordinate geometry which all the students learn from higher secondary onwards.

Without spending (wasting!) much space and time, let me conclude this review with an advice to the student community to keep away from such a book; there are many excellent books on linear algebra available even in Indian print.

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Measures of noncompactness and condensing operators by A.S. Potapov, B.S. Sadovskij and A.E. Rodkina Birkhauser Verlag AG, P.O. Box 133, CH-4010, Basel, Switzerland, 1992, pp. 264, SFr 128.

There is a large class of practical problems arising in physical, biological, engineering and technological sciences which can be modelled by linear and nonlinear differential and integral equations. In operator theoretic setting such equations can be described by an operator equation of the form

$$x + Tx = y \tag{1}$$

in an appropriate Banach or Hilbert space.

If T is a compact operator then one can use the theory of rotation of compact vector fields, the Schauder–Tikhnov fixed point principle, Fredholm–Riesz–Schauder theory of linear equations to analyse the existence uniqueness and other information on the properties of solutions of such equations. However, there do exist concrete examples among the differential and integral equations typically of form (1), where the operator T under consideration falls short of being compact. The degree of noncompactness of the set is then measured by means of functions called measures of noncompactness. Instead of compactness of T one then looks for its condensing characteristic under which the image of any set is in a certain sense more compact than itself.

It turns out that condensing operators have properties similar to the compact ones earlier described which are very useful tools for investigating solvability of operator equations involving such operators. This book is aptly aimed to give a systematic description of such properties of condensing operators.

In Chapter 1, authors introduce the measure of noncompactness (MNC)–Hausdorff measure of noncompactness γ, the Kuratowskii measure of noncompactness α and the measure of noncompactness β. The known properties of Kuratowskii MNC and Hausdorff MNC are stated and proved, including the derivation of a number of formulas that compute directly the value of Hausdorff MNC in some concrete spaces. Having defined MNC, authors study condensing operators in Banach spaces and its generalisation to include the notion of ultimately compact operators and K-operators.

In Chapter 2, Fredholmness of bounded linear operators on Banach spaces is discussed, with respect to MNCs. This is done as follows. Let $L\,(E_1, E_2)$ denote the space of bounded linear operators from E_1 to E_2 and BE denote the Banach space of all bounded sequences $X = (x_1, x_2, \ldots, x_n, \ldots)\ x_n \,\varepsilon\, E$. Denote by KE the closed subspace of BE, consisting of relatively compact sequences. Denote by E^+ the quotient space BE/KE with the natural linear operation and norm. Then $C \,\varepsilon\, L\,(E_1, E_2)$ induces an operator from BE_1 to BE_2: $CX = (Cx_1, Cx_2, \ldots, Cx_n, \ldots)$, $X = (x_1, x_2, \ldots, x_n, \ldots)$. $C \,\varepsilon\, L\,(\mathrm{BE}_1, \mathrm{BE}_2)$ and $C\,(\mathrm{KE}_1) \subset \mathrm{KE}_2$ and hence this gives rise to an operator $C^+ \,\varepsilon\, L\,(E_1^+, E_2^+)$ by the formula

$$C\chi = CX + KE_2\ (\chi = X + KE_2).$$

One of the important theorems stated and proved is

Theorem (Fredholm criterion): The operator $C \,\varepsilon\, L\,(E_1, E_2)$ is Fredholm iff C is bijective (*i.e.,* it is injective and $C^+E_1^+ = E_2^+$).

Using this theorem authors obtain the Freedholmness of an operator C in terms of MNCs. One of the results in this direction is as follows:

Theorem (Sufficient condition for Fredholmness): Let $C = D + A$; D, $A \,\varepsilon\, L\,(E_1, E_2)$ and D is Fredholm. Suppose in the space E_1 there is given an MNC ψ and suppose that there exists $k < 1$ such that

$$\psi\,(AX) \leq k\,\psi\,(DX) \text{ for all } X \,\varepsilon\, B\,E_1.$$

Then C is Fredholm and its index is equal to the index of D.

There is a section devoted to relationship between the MNCs of the operator and its conjugate and also to the spectra of A_n and A where $A_n \to A$. There is a nice result stated in terms of normal MNCs which gives a necessary and sufficient condition for closeness of the parts of the spectra of operators A_n and A lying outside some disc.

In the fixed point theory and its applications an important role is played by the following closely related notions: degree of a map, rotation of a vector field and fixed point index of an operator. Chapter 3 of the book deals with the fixed point index of a condensing operator, its definition, properties and computation. This is then followed by derivation of fixed point indices of linear, asymptotically linear and Frechet differentiable γ-condensing operators. Further properties of the fixed point index and its generalisation to various classes of maps and to locally convex spaces are discussed in the remaining sections of this chapter. There is a separate section on the fixed point index of positive operators. The last chapter deals with the application of the concepts of MNCs and condensing operators to the theory of differential and integral equations. The main areas of applications are Cauchy problem for ordinary differential equation, Itoh's stochastic equation with deviating arguments, functional–differential equations of neutral type and Hammerstein integral equations.

The reviewer would have preferred to see a more systematic approach towards differential and integral equations. For example, Itoh's stochastic equation need not have been introduced right after Cauchy problem. There is a need to dwell more on the deterministic aspect before going over to stochastic analog. May be authors have had their own constraint which the reviewer is not aware of.

A book giving full treatment concerning condensing operators was very much needed and this job is well done in this book. All those working on applicable functional analysis would very much relish reading the book. The only prerequisite is the knowledge of elementary functional analysis.

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The world of Bohr and Dirac by N. Mukunda, Wiley Eastern Ltd, 4835/24, Ansari Road, Daryaganj, New Delhi 110 002, 1993, pp. 90, Rs 70.

The phrase 'pleasant duty' commonly used in our public functions is seldom meant seriously but this writer can honestly say that the duty (of reviewing) that he has been asked to perform is truly an enjoyable task. Prof. Mukunda is a physicist of great distinction, unfortunately not so well known as he deserves to be but held in high esteem in circles that really matter. A shy and retiring personality, he lives happily in his world of equations. However, bowing to public pressure, he has in recent years emerged from his cloister to write essays and deliver popular lectures on personalities as well as the texture of physics. The volume under discussion is a compilation of five lectures/essays prepared for quite different occasions; yet they blend beautifully to form a charming little volume, with a "unifying thread running through . . . involving both personalities and ideas."

The opening essay on Dirac is a tribute to the great master published soon after his death. Instead of the usual biographical memoir, one is offered a delicate portrait *via* a stimulating *bhashyam* on select papers of Dirac. As Mukunda says, "Dirac burst on the scene in late 1925. Thereafter he kept going like a house on fire, with a steady and staggering profusion of fundamental ideas and discoveries."

It all started with a visit by Heisenberg to Cambridge in July 1925, during which he delivered a seminar on the recent breakthrough he had achieved. Dirac could not attend the seminar but a couple of months later his thesis adviser Fowler passed on to him the proof sheets received from Heisenberg. At first Dirac did not pay much attention to what Heisenberg had to say but a week later he suddenly saw that it opened up a whole new world. Mukunda adds: "One of Dirac's key contributions in this phase was the exposure of the link between classical and quantum mechanics. This was the most beautiful expression of the Correspondence Principle [of Bohr] and, said Dirac, it had given him the most pleasure of all his discoveries."

Dirac's papers are noted for clarity and lucidity. They have a style of their own about which Mukunda says:

> Those familiar with the plays of Bernard Shaw are aware of the beautiful essays that appear at the end of the plays—rivalling the plays themselves for wit and insight. A somewhat similar statement could be made about the introductions to Dirac's papers. He developed his style of reviewing in his own way the most important recent developments in a particular area, expressing his opinions about problems and progress and putting things in perspective, before going on with a presentation of his own results in each paper.
>
> A collection of the introductory sections of his papers would be most interesting . . . .

One also learns that as early as 1931, Dirac had (essentially) introduced the concept of fibre bundles into physics, "decades ahead of the rest of the world." And in a paper in the *Reviews of Modern Physics* published in 1949, Dirac has this "remarkable sentence."

> I do not believe there is any need for physical laws to be invariant under these reflections, although the exact laws of nature so far known do have this invariance.

This is seven years before Lee and Yang questioned parity conservation in weak interactions.

I once heard a well-known French physicist describe Dirac's classic book *The Principles of Quantum Mechanics* as the book of the century. Mukunda goes even further. According to him, "There is little doubt that in the times to come it will be Dirac who will be remembered as the physicist of the century."

The Universe, they say, began with a bang. So did this century as far as the world of physics is concerned for it was in 1900 that Planck ushered in a revolution in human thought with his discovery of the quantum concept. Then followed a confusing but exciting period which culminated with the discovery of quantum mechanics, and Dirac's discovery of the relativistic wave equation (now appropriately named after him). Gamow once described this turbulent era as "thirty years that shook physics." Many were privileged to participate in this exciting drama (Dirac is reported to have said that it was a period when second-rate people could do first-rate work) but of these, two who stand out are Niels Bohr and Dirac; it is around these two central characters that the second essay is woven.

An essay on Bohr and Dirac is necessarily a study in contrasts for one cannot easily come across two strikingly different personalities. And yet in their own subtle ways they wonderfully complemented each other to weave a fabric of great beauty and marvellous texture.

There was a gap of almost a generation between Bohr (b. 1885) and Dirac (b. 1902). But, they worked in tandem "to accept and alter the fabric of classical physics to accommodate Planck's quantum of action." After a bright start in Denmark, Bohr landed in England around 1910 to pursue higher studies. He went to Cambridge to work with Sir J.J. Thompson the discoverer of the electron but the experience proved disappointing. Armed with a copy of *Pickwick Papers* (to improve his English), he then went to Rutherford's lab in Manchester where he struck the jackpot. Rutherford had discovered the atomic nucleus and the crying need of the hour was a decent model of atomic structure. Many ideas were floating around but none of any practical value. However, to Bohr the problems and possibilities immediately became clearer. He realised that "it was essential to bring in Planck's constant." And thus was born the celebrated Bohr model of atom. These days it is taught in high schools and one scarcely appreciates what a great conceptual advance it was when first proposed. In one sweeping stroke, as if by an edict, Bohr "suspended" classical mechanics in the realm of the atom, making way for "the application of Planck's idea to the dynamics of matter, which Dirac was to later describe as the most difficult first step. . . ."

The Bohr model was a sensation but after the first flush of success it was realised that it too had problems. A logically consistent framework was still lacking and Bohr alone could grope in the jungle, guided by his famous Correspondence Principle. The resolution of the crisis finally came in 1925 with Heisenberg's discovery of matrix mechanics. This was the spark that ignited Dirac who "soon elaborated, practically in isolation, his own version of quantum mechanics, giving it a particularly abstract and elegant structure."

Between 1925 and 1927 events moved in a high gear, with major contributions (apart from Dirac) by Heisenberg, Born, Jordan and Schröedinger. "During this period Bohr was in a sense watching from a distance, with a critical but approving attitude. . . . When the stage was set to find the meaning of the mathematical structure [of the new theory] Bohr reentered the scene." The reference here is to the philosophical interpretation of quantum mechancis which Bohr painstakingly developed and honed. Mukunda quotes Heisenberg to say: "Bohr was primarily a philosopher, not a physicist, but he understood that natural philosophy, in our day and age, carries weight only if its detail can be subjected to the inexorable test of experiment."

This then was the fundamental difference between Bohr and Dirac; whereas the former was primarily a philosopher (interested in natural philosophy, lately known as physics), Dirac was "a master craftsman in the art of theoretical physics." With the new mechanics, both were needed. A mechanics which cannot be put to practical use is of little value. Tools for manipulation had to be created which Dirac did with great dexterity, appealing to considerations of mathematical symmetry on the one hand and working in the abstract on the other. The result was a strange theory where quantities like position and momenta had to be represented by arrays instead of numbers as in classical physics. It needed a Bohr to provide insight into the structure of the theory and it came about because "of an intense occupation with actual phenomena, such that it was possible for him to sense the relationships intutively rather than derive them formally."

Dirac first met Bohr in May 1925 when the latter delivered a lecture in Cambridge. Mukunda does not discuss this but it is interesting to quote from Abraham Pais about what Dirac said referring to this visit by Bohr:

> People were pretty well spell bound by what Bohr said. . . . While I was very much impressed by (him), his arguments were mainly of a qualitative nature, and I was not able to really pinpoint the facts behind them. What I wanted was statements which could be expressed in terms of equations, and Bohr's work very seldom provided such statements.

In 1926 Dirac went to Copenhagen to spend time at Bohr's Institute. And about this visit Dirac later said (quote from Abraham Pais): "I admired Bohr very much. We had long talks together, long talks in which Bohr did practically all the talking."

Dirac and Bohr differed in writing styles too and about this Mukunda says:

> Dirac's writings have a characteristic and unmistakable directness, simplicity and beauty. Bohr on the other hand is much harder to read because each long sentence of his contains a great deal of thought in compressed form. He spent a lot of effort in the choice of each important word. Bohr's style of work was to have a junior collaborator sit at a desk and take down notes while he himself kept pacing up and down the room, forming and changing and reforming his phrases and sentences. Watching him at one such session, Dirac apparently said something to the following effect: "Professor Bohr, when we were young we were taught never to start a sentence until we knew how to finish it."

Mukunda is polite and kind in his references to Bohr. According to Pais, Einstein was exasperated by Bohr's writing and once said that Bohr thought clearly, wrote obscurely and considered himself a prophet!

From personalities the focus shifts, in essays three and four, to the style of modern physics. Here Mukunda is in his element. The essays are lucid and crystal clear and, in keeping with the demands of the occasion, shorn of mathematics. As such they can be read and enjoyed by one and all. I am not sure what the perception of the non-physicist reader would be but speaking for myself, as one who has stayed mostly at the ground level I found it an exhilarating experience to be lifted to a great height and given an overview of the subtle patterns that eludes one when one is preoccupied with 'nuts and bolts'.

In a sense, the style of modern physics, (which is the central theme of the third essay) was set by Dirac who recognised that the fundamental laws of Nature control a substratum of which we cannot form a mental picture and which must necessarily be dealt with through mathematical abstraction. And, as far back as 1931, Dirac clearly foresaw that the process of increasing abstraction would continue in the future as indeed has happened.

One important feature of theoretical physics today is the heavy reliance on symmetry. In practical terms, one exploits the machinery of what is called group theory. Once considered difficult and esoteric, group theory is now routinely taught at the graduate if not even lower levels. However, most of the applications invoke symmetry in a static sense. It is only in the study of elementary particles that the use of symmetry concepts has reached very sophisticated levels because there one is not merely dealing with space–time symmetries but the so-called internal symmetries as well. Symmetry is now used in a fundamental sense, in relation to a dynamical system with intricate degrees of freedom rather than being confined merely to geometric aspects. All this has made theoretical physics increasingly formal and to many forbidding as well. As Mukunda remarks, "It has been said that each generation of physicists feels that the next generation is too mathematical." One thing is certain. We can no longer assert as Lord Kelvin did during last century that understanding a physical system implies being able to build a mechanical model of it. Parenthetically one might also remark that theoretical physics has set the pace and style for many neighbouring disciplines as well, and many of the manipulative tools which were once the preserve of physicists are now freely used in disciplines like control theory, optical engineering and neural networks.

The mathematical structure of quantum mechanics receives special attention in the fourth essay because of its "rich and beautiful" structure. Quantum mechanics has been around for more than sixty years and has been amazingly successful in its practical applications. But when it comes to physical interpretation things are not so easy, in spite of Bohr's guidelines. The main reason for this is the fact that "many of its (*i.e.*, QM's) predictions run counter to intuition developed from 'common sense' ". The reader is given a rapid but interesting tour of the nuances of quantum mechanics including the difficulties of interpretation. And when it is time to conclude, Mukunda leaves the reader pondering about Heisenberg's aphorism: If the mathematics is clear, the physics is not, and conversely!

The last essay deals with the interplay between physics and biology. At first one is struck by the title because Mukunda is known to carefully avoid lectures on topics far from his domain. One's curiosity is naturally aroused and down the line, the reason for this unusual foray by the author becomes clear.

In ancient times there was no sharp boundary between philosophy (in relation to spirituality) and natural philosophy or, in other words, between metaphysics and physics. In the post-Renaissance period there was a parting of company and scientists like Galileo and Newton were able "to free themselves from the metaphysical traditions of their time." A new style of investigation and enquiry into natural phenomena was introduced and, as one knows, science progressed by leaps and bounds leading eventually to that revolution in human thought, *viz.*, quantum mechanics. Now "an important statement [of quantum mechanics] is that an atomic system has no numerical properties of its own unless and until it is subject to experiment and observation. This has led to the idea that an external consciousness—of the experimental observer—is an essential part of the whole scheme of quantum mechanics."

True this point of view does not have universal acceptance. Nevertheless, for the first time, physics is directly confronted with the question of consciousness, leading to what might be called the "mind–matter question." At a somewhat lower level let us ask as to how humans acquire and accumulate knowledge. Here Mukunda refers to the contributions of the philosopher Immanuel Kant. Kant viewed knowledge as partly drawn from experience and partly a priori. His idea was that for science to be pursued there must exist "*synthetic,* **non-empty truths or knowledge that had** *necessarily* **to apply to actual experience but could not be** *derived* **from it.**"

This leads to another question: "If these synthetic truths are not the result of experience, where do they really come from? How is it that our minds already possess this machinery which then fits experience so well and precisely?" This takes Mukunda to Delbruck's book *Mind from matter*? in which Delbruck says:

> It appears that two kinds of learning are involved in our dealing with the world. One is *phylogenetic* learning, in the sense that during evolution we have evolved very sophisticated machinery for perceiving and making inferences about a real world. The second kind of learning involved in dealing with the world is *ontogenetic* learning, namely, the lifelong acquisition of cultural, linguistic and scientific knowledge.

In a nutshell, the a priori categories of thought which Kant invoked in order to justify Galilean–Newtonian physics are biologically evolved. However, biological evolution has turned out physical perception only to the "world of middle dimensions." Obviously our survival does not call upon us to be sensitive to happenings on an atomic scale, for example. But once science acquired "artificial senses" as Schwinger put it and began exploring the world of small dimensions, problems arose in reconciling the new experience with what we expect from common sense. As Gamow once said, "Our mind becomes so accustomed to these notions (of space, time and motion) that later on we are inclined to believe that our concept of the outside world based on them is the only possible one. . . ." When one considers what happens when objects move with speeds close to that of light or what goes on inside an atom, one is confronted with facts quite contrary to what our perception faculties have prepared us for. And if one delves deep into the philosophical foundations of quantum mechanics, one is faced with questions like: Does consciousness have any role to play in science? Is mind distinct from matter? Can life be understood within the framework of physics and chemistry? Ancient philosophers had their views about life but for the modern scientist this last question has become an all important one. Mukunda tells us that according to Bohr, "the understanding of life would require something beyond quantum mechanics and yet to be discovered, not within quantum mechanics itself." In other words, in Bohr's view the property of life and the understanding of routine cell functions in terms of physics and chemistry might be mutually exclusive or complementary.

What about Delbruck? Mukunda has this answer. "In his book (*Mind from matter?*) referred to earlier, describes the attempt (to see whether Bohr's idea was necessary to understand the life process) and comes to the conclusion—like some others before him—that the principle of complementarity is not necessary in this context. . . ." Obviously this and other questions relating to physics and biology can be argued back and forth endlessly and so Mukunda deftly winds up by transmitting his own "puzzlement" to the reader. The latter can't just put the book down and relax. There is homework to do—he has to think!

And now to the more mudane aspects without which "book criticism" is never complete. I would first of all like to compliment the publisher on good production values. One is so used to shabby printing and shoddy get up that a job well done deserves a pat. At the same time, I would enter a general plea that in this scientific age, publishers in India should equip themselves to handle mathematical symbols with ease. As an author with painful experience on this score, I found I was in good company for Mukunda too is having problems with ν and $\hbar$. The editor should have been more careful about proof correction to ensure that $P = \hbar\, k$ and not $h\, k$ as one finds on pages 30 and 31. I am sure the manuscript would have had these right. The book is embellished with nice portraits of eminent scientists but one wonders who is the artist who rendered them, or if the pictures have been reprinted from elsewhere, what the original source is.

Typical of Mukunda, he has taken the trouble to include a comprehensive bibliography as a "guide to further reading." Urging the reader to refer at least some of them, Mukunda adds: "Some of the books listed, while they have become classics of the literature, may be hard to come by. We have included them as a challenge to the more motivated reader"—the teacher peeping out!

The book is the second in the series of educational monographs being published by the Jawaharlal Nehru Centre for Advanced Scientific Research and clearly it sets a high standard for the authors to follow. I would expect a book of this class should be sold world wide and I am confident it would receive a good reception everywhere. I am therefore a little unhappy about the publisher's caveat that this book "is not to be sold outside the country to which it is consigned. . . ." I can understand this in the case of a low-cost reprint volumes but surely original monographs written in India fall under a different category. I do hope the publisher has some arrangement with his principals whereby this book (and others like it) can be marketed globally. I would like to take the opportunity to strongly urge the Indian Association of Physics Teachers to see that this volume is included in their low-cost book programme.

The appetite grows by what it feeds on. This book has whetted my appetite for more of a similar nature from the pen of Mukunda. May we expect another one at least, say on Srinivasa Ramanujan, Harish Chandra, Chandrasekhar and possibly Bhabha? Professor Mukunda, are you listening?

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Symmetries, gauge fields, strings and fundamental interactions, Vol. 1: Mathematical techniques in gauge and string theories by Tulsi Dass, Wiley Eastern Ltd, 4835/24, Ansari Road, Daryaganj, New Delhi 110 002, 1993, pp. 506, Rs. 450.

Quantum field theory was developed in trying to understand the quantum behaviour of systems with an infinite number of degrees of freedom. While it has proved to be very useful in many areas of condensed-matter physics, it displays its full elegance and power in the field of high-energy or elementary particle physics where it is combined with the special theory of relativity. Another important concept in high-energy physics is that of local gauge invariance and gauge fields. By the mid-70s, three of the four fundamental forces of nature (strong, weak and electromagnetic interactions) had been incorporated into a gauge theory called the Standard model. This is in complete agreement with all experimental observations so far. This success has led many field theorists to try to incorporate the force of gravity into a complete quantum theory of all particles and interactions. The most popular of all such models is called superstring theory.

All these developments have necessarily led to an increasing mathematical sophistication of the subject. While Lie group theory had already become an essential tool by the early 60s, other concepts like topology, instantons and magnetic monopoles became important during the 70s. Supergravity and superstring theories brought in the full arsenal of differential geometry. The BRST formalism was developed to deal with the more complicated kinds of field theories. All this means that a researcher in modern quantum field theory has to be conversant with a great variety of mathematical techniques. Unfortunately, there is no single source from which one can begin to learn so many different topics. Traditionally, high-energy physicists have had to learn these things in bits and pieces from a variety of sources including discussions with their colleagues.

Tulsi Dass has planned a three-volume series which will cover the basic mathematical foundations of gauge and string theories in Vol. I, the Standard model and related topics in Vol. II, and speculative subjects like Kaluza–Klein, supergravity and superstring theories in Vol. III. The idea seems to be to develop and then use unified language for presenting the entire subject. Volume I has now appeared and it does justice to this ambitious project. Besides all the items mentioned in the previous paragraph, this volume also deals with some of the standard quantum mechanics and field theory topics like symmetries, functional integration, constrained systems and multiple vacua in non-abelian gauge theories.

The pace is leisurely and a large number of simple illustrative examples and proofs are provided. Even basic notions like set theory, tensor analysis and manifolds are discussed so that someone with a minimal knowledge of modern mathematics and quantum field theory can go through the book on one's own. It is also possible to use parts of this book to teach a course on a particular topic in mathematics for M.Sc. Physics students. Each chapter ends with a list of books and papers for more advanced reading. Altogether a vast amount of material is covered. Though the book is somewhat steeply priced for individuals, it would be very useful for libraries to acquire the entire series for the benefit of students pursuing research in this area.

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Bertrand Russell and the origins of the set-theoretic paradoxes by Alejandro R. Garciadiego, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 264, SFr. 98.

It is ironical that the set-theoretic paradoxes of Cantor and Burali Forti were never meant to be paradoxes. They were merely contradictions arrived at in a *reductio ad absurdum* proof, attempting to prove the Trichotomy law did not hold for cardinal and ordinal numbers. It was at this time, in the last decade of the 19th century, that the British mathematician and philosopher Bertrand Russell discovered the paradox of the greatest cardinal number and his own argument of the class of all classes, which are not members of themselves. The mathematics scene at that time is described in Garciadiego's book, where he "reconstructs and reinterprets" the role played by Russell in the origin of the set-theoretic paradoxes.

Even as a teenager, Bertrand Russell had a dream of finding knowledge that one could consider certain. He was convinced that mathematics was more likely to be certain than any other form of general knowledge. Russell was taken up by the idea of two British thinkers, Clifford and Mill, who had tried to formulate philosophical systems that would contribute to a gradual social progress, individual freedom and human happiness, which included the advancement of science.

Russell joined Cambridge in October 1890 and spent three years studying mathematics. After the Tripos in 1893, he spent the fourth year studying philosophy. He wrote his dissertation "An essay on the foundations of geometry" under a neo-Hegelian influence. Neo-Hegelians were indifferent to contradictions, because their systems were nourished by them. It is then understandable why Russell would be discussing contradictions as something natural and innate in each science.

The deep effect of Cantor's ideas on Russell is emphasised throughout the book. Strangely, in 1896 Russell had even refused to discuss the possible mathematical uses of Cantor's theory of transfinite numbers. Cantor had proposed a second number class ω, $\omega+1$, $\omega+2$, . . ., beginning with the first number ω larger than any of the first class, that is, the ordinary natural numbers. Russell questioned the philosophical validity of Cantor's method. He argued that if the natural numbers were unlimited, there was not even a chance for ω to arise. To Russell, Cantor's transfinite numbers were impossible and self-contradictory.

In July 1898, Russell was working on a book on the principles of mathematics, which had been his "chief ambition ever since the age of eleven". By this time, his negative reaction to Cantor's work was slowly changing. This was probably due to: 1) his reading of Couturat's *De l'infini mathèmatique*, 2) Moore's rejection of neo-Hegelian philosophy, and 3) his own doubts about the role of symbolic logic in mathematics. In July 1899, he read Cantor again and began working on yet another draft of the principles of arithmetic which he now entitled "The Principles of Mathematics". The impact of Cantor's work is found throughout the manuscript. Russell explicitly mentioned that Cantor was the source of most of his definitions and theorems. Russell accepted Cantor's ω, but argued that there was a limit to the sequence of transfinite cardinal numbers. Russell's examination of Cantor's proof that there is no greatest cardinal number led him to formulate the contradiction of the class of all classes which are not members of themselves. "Some classes are members of themselves, some are not, the class of all classes is a class, the class of not-teapots is a not-teapot. Consider the class of all classes not members of themselves; if it is a member of itself, it is not a member of itself; if it is not, it is" (*The principles of mathematics*). By the end of May 1902, Russell finally finished his "Big" book on the principles of mathematics containing 8 parts and 59 chapters. It supported his thesis that pure mathematics could be derived from the principles of symbolic logic. However, Russell was disappointed with the book even before it was finally published in May 1903. Letters to colleagues and friends show his dissatisfaction, probably because he was unable to cope with the inconsistencies and provide a solution to them. He had set out to find a consistent explanation of the principles of mathematics, but the treatise when complete presented certain contradictions, which seemed even more difficult to explain and solve than the earlier paradoxes.

The book includes an appendix with Russell's correspondence "which reflect his emotional health" at the time of writing *The principles of mathematics* as well as those which exemplify the growth and development of his ideas.

The task of the author was made more difficult because of a number of contradictory facts, regarding the discovery of the set-theoretic paradoxes. The author painstakingly ferrets out the most convincing arguments and then arrives at his conclusions, some of them opposing Russell's own statements regarding historical facts. The book provides interesting reading and tells the reader all he wants to know about the set-theoretic paradoxes and Russell's role in their discovery.

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Space safety and rescue 1991 by Gloria W. Heath, American Astronautical Society, Univelt Inc., P.O. Box 28130, San Diego, California 92198, USA, 1993, pp. 260, $ 50.

This volume contains 16 papers presented at the Symposium of International Academy of Astronautics held at Montreal in conjunction with the 42nd International Astronautical Federation Congress, 1991. The papers are grouped under two sections—Safety and rescue in space and Space activities impact on the environment, followed by appendices, one of which summarises the international issues involving GEO debris.

Despite the four decades of familiarity with space activities, the subject matter of the book remains fascinating. The volume succeeds in driving home the point that the problems of space debris, collission chain reaction and the accident scenarios are no more imaginary but so much real as to demand a plan of action from the space club. More regulations and also more cooperation in space activities at the international level are bound to follow in the coming decades.

In the first part on Safety and rescue, the focus is on manned mission where the risk per person happens to be higher than any other transport and industrial activity. Space safety needs to be discussed in terms of both preventive actions and rescue contingencies. On the prevention side, human factor right from design, planning and certification to crew training and response assumes great importance. It is revealing to read that nearly 90% of all catastrophic failures occur during the ascent phase. Rescue options here are the use of ejection seats with or without encapsulation followed by smooth landing at a safe site away from the launch pad. But it is in the relatively low-risk phase of 'on-orbit' period or prolonged stay in space stations, that the options become technological challenges. Fascinating details of the specially constructed rescue modules and vehicles, hand-held maneuvering units for self-rescue during extra vehicular activities or the hull protection against meteoroids and debris by 500 kW lasers (obviously, SDI inspired) can be found in this volume. All these and a variety of statistics on space missions, accidents and performance record of rescue systems make an absorbing reading. Manned space missions themselves are wonderful, but successful rescue operations such as return of Apollo-13 or repair of Mir station border on fantasy.

Quoting Prof. Lust of ESA: *"The sheer immensity of space is the main obstacle to the recognition of space debris. The ocean too, for centuries was considered a bottomless sink and yet today we see the consequences of dumping toxic waste and of having accidents result in oil slicks"*. The environmental pollution caused by man is extending to space and the threat of collisions in space by uncontrolled growth of space debris is real.

Beyond an altitude of 800 km, the self-cleaning effect of the Earth's atmosphere is missing. The low-Earth orbits in the range 900 to 1600 km and the geosynchronous orbits are being littered with debris which will assume serious proportions in the next 50–100 years. Of the objects in the most sought-after orbits, only a tiny percentage is active payloads the rest being inactive payloads, rocket bodies, launch and fragmented debris. In addition, there are enormous number of dust-like objects <1 cm which escape any radar observation, but are quite dangerous considering their hyper velocities of the order of 10 km/s.

Each one of the topics discussed under detection and quantification of orbital debris, analyses of possible collisions and their chain reactions and the counter measures and their effectiveness serves to enhance one's concern for orbital environment. The lone article on the impact of chemical propulsion on stratospheric ozone, toxicity, acid rain and green house effects—the concerns at low altitudes bring us down to earth with somewhat comforting conclusions. In all the measures towards rescue of man or environment in the space activity, there is a penalty in terms of payload mass or eventually in terms of high cost. Prevention is better than cure like in any other field of activity.

On the whole, this is a very useful collection of papers and references that the space scientists and technocrats may like to have in their shelves. For others, this may trigger their imagination of what is likely to unfold in distant space and yet of concern to those on Earth.

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Structural tools for the analysis of protein–nucleic acid complexes by D.M.J. Lilley, H. Heumann and D. Suck, Birkhauser Verlag AG, Klosterberg 23, CH-4010 Basel, Switzerland, 1992, pp. 470, SFr. 118.

The book is a collection of original and review articles on protein–nucleic acid interactions. Protein–nucleic acid interactions are responsible for the condensation of DNA on nucleosome core particles. They control gene expression and are involved in RNA maturation and translation. A large part of present research in molecular biology is concerned with protein–nucleic acid interactions. The articles in this volume are well organized and provide a balanced view of the developments in the field. On the whole, the book presents an overview of the work being carried out in the European subcontinent and somewhat underrepresents the work of the United States.

Three articles grouped together as the introductory chapters present methods used in elucidating nucleic acid structure or protein–nucleic acid interactions. Several articles in other sections also provide short descriptions of methodologies. These discussions on techniques are adequate to appreciate the validity of the results and are not comprehensive.

DNA structure and its various polymorphic forms are covered in four separate articles. As this field of investigation is fast expanding, the information provided covers only a part of the total activity. Protein–DNA interactions are covered in six articles. Although Suck (pp. 127–142) reviews the principal results obtained by X-ray crystallographic studies on protein–nucleic acid complexes, this section does not cover all systems that have been investigated in equal detail. Particularly, the work carried out in the US is underrepresented. Also, several notable publications such as the structure of the TATA-binding protein have appeared since the publication of this volume. It would have been worthwhile to include some articles on virus structures and the nature of protein–nucleic acid interactions seen in viral capsids in this section.

The most valuable portion of the book is the section on RNA–protein interactions and RNA structure. This section is also likely to have a much longer useful bench life in comparison with the other sections. The articles in this section cover the basic enzymology of RNA polymerases as well as studies on protein–RNA interactions in a number of systems. Theoretical methodologies that could be of use in predicting RNA folds, functional aspects of small nuclear ribonucleoprotein particles, studies on elongation factors, *t*-RNA synthateses, HIV reverse trascriptase, Qβ replicase, possibilities of RNA other than *t*-RNAs possessing well-defined three-dimensional structure are some of the topics covered in this section.

It is difficult to provide totally comprehensive coverage of the vast field of protein–nucleic acid interactions and their biological implications in a collection of articles running to less than 500 pages. However, most of the articles presented in this volume are very readable and provide a valuable source material for investigators interested in this area.

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The visual system from genesis to maturity by Roberto Lent, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 286, SFr. 158.

Despite significant advances in the study of various aspects of the visual system, the fundamental questions remain the same. We still do not understand how the diverse neural cell types of the visual system are generated and the regional patterns defined. Cellular and molecular mechanisms which guide developing axons to the synaptic targets and establish topographic maps in the visual pathway too await clearer understanding. Even less is known about how the formation and refinement of synaptic connections control the visual behaviour. The structural organisation and functioning of different parts of the visual system from photoreceptors to cortical columns and modules continue to hold mysteries which need to be unfolded.

Some of these issues have been brought to the reader once again in this book which is an outcome of a meeting held on the subject. The contributing authors have reviewed the current knowledge and the approaches being used by them to study these aspects providing interesting summaries and some directions to future development. With the new generation of jurnals such as *Current Opinion in Neurobiology* and *Trends* series which provide regular updates on the latest in the field from the experts, one wonders whether there is a niche for summaries such as these. There may be, if the book provides comprehensive coverage of the subject area in a volume such as this.

The book begins with an overview by Pasko Rakic describing briefly the schedule of developmental events in the visual system of the rhesus monkey starting from the retina to the visual cortex. The first seven chapters of the section on 'Genesis' are devoted to the retina which forms a relatively simple model system to study the basic principles of development and differentiation. Intracellular and molecular cytoskeletal-dependent mechanisms regulating photoreceptor differentiation in retinal culture system, regulation of glutamate decarboxylase enzyme expression by GABA in the avian retina, involvement of 9–0 acetylated ganglioside in neuronal migration and axonal outgrowth, role of activity and NMDA receptor in development and maintenance of the retinotectal topographic projection, role of afferent activity, glutamate receptor and intraretinal dendrodendritic influences on retinal ganglion cell development have been discussed. The generation of diverse cell types studied by following the cell lineage with recombinant retroviruses and developmental strategies of olfactory commissural and visual callosal fibres are the only two aspects covered with regard to the visual cortex development.

The second section of the book delves into diverse aspects of the mature visual system such as the organisation and possible functions of the catecholaminergic amacrine cells in the rhesus monkey retina, effects of GABA and glutamate analogs and antagonists on vertebrate eye movements after intracranial and intraocular drug administration as well as the existence of an inhibitory pathway between the nuclei of the optic tract (NOT) of both sides in the opossum which may play a role in the horizontal optokinetic reflex. Explanations have been sought for the mismatch between retinal ganglion cell density and cortical V1 magnification factor in lateral eyed mammals. Differences of functional architectural organisation of layers of cortical columns in striate and extrastriate areas, concepts of basic canonical microcircuits to understand the complex functions of neocortex, dynamic properties of single neurons in primary visual cortex that are related to completion phenomenon in the optic disc region, properties of neurons and their connections in the inferior temporal cortex of infant and adult macaque monkeys concerned with recognition and storage of information about visual form and colour as well as reorienting of visual spatial attention are some of the other problems that have been discussed.

The book provides an overview of vision from its inception to maturity. Though the book essentially compacts extensive knowledge on the visual system it may not satiate the appetite of those familiar with the state-of-the-art on the subject. However, for the general neuroscientists, opthalmologists and optometrists it is certainly a resource of current information in this field and would be a useful addition to the bookshelf.

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Formation and regeneration of nerve connections by S.C. Sharma and J.W. Fawcett, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1993, pp. 260, SFr. 148.

This book is a collection of papers presented at a meeting in Edinburgh, Scotland, in 1991 to commemorate the retirement of Prof. R.M. Gaze, a pioneer in the investigations of developmental neurobiology of visual system and the role model and inspiration to the large number of neuroscientists to continue in the same field. The main theme of the book is the state-of-art information on the mechanisms of development of visual pathway mainly in lower vertebrates like xenopus and Gold fish contributed by Prof. Gaze's erstwhile students and associates. A certain overlap and repetition are understandable as most of them are dealing with the same system and developmental mechanisms.

As befitting, the first chapter is by Jacobson who traces the evolution of perception of specificity and plasticity in relation to the nervous system from theory to scientific facts contributed by Gaze and his associates. The second chapter deals with the stages of development of retinal projection to the tectum and the factors controlling each stage. Role of different factors and cell death in the precise projection are discussed clearly in this chapter. Retinotopically ordered termination pattern in the tectum and thus the map formation are presented by Tylor in Chapter 3. The role of tract of postoptic commisure in the guidance of the optic fibres is elegantly demonstrated by experimental manipulation and HRP tracing. The above aspects of retinotectal projection are discussed in the *in vitro* system by Gooday in Chapter 4. According to him the order of projection is a multistaged process and precision increases gradually in the grouping of fibres which takes place at different places in the optic pathway. Role of glia at each stage is also mentioned. Chapters 5 and 6 deal with the plasticity in the binocular visual maps in the tectum and the role of visual sensory input and the NMDA receptors at the tectum in the organization of connections.

Chapter 7 is on experimental transplantation of mammalian retina into host brain and the functional response of the grafted retina to photic stimulation. Lund *et al* explain in this chapter that the developmental pattern in the lower vertebrates and mammals is the same and hence xenopus or Gold fish is a good model to study development though due to evolution of functional complexity finer structural modifications can be followed up only in the mammalian system. Fawcett has compared (Chapter 8) the topographic retinotectal map in frog and fish with that of birds and mammals. According to him the axonal path finding in lower vertebrates is more orderly because of slower growth and reduced input whereas the marked input and faster growth induce some discorded initial connections in mammals. One of the reasonings given for the wrong initial connections is to mark the ganglion cells for death so that final wiring and topography are pruned. In Chapter 8 Peters describes the mechanisms and details of the formation of vertical modules in the visual cortex of rat with the support of very elegant photomicrographs and electron-micrographs. Developmental theme is extended to the spinal cord sensory fibre projection in Chapter 10. Though physiological property remains the same at sensory neuronal levels there may be a difference in the synaptic transmission based on the modulatory activities at each level.

In Chapter 11 explants of Gold fish retina conditioned to injury are studied to compare the regenerative processes *in vivo* and *in vitro*. A previous conditioning accelerates the regenerative processes after a second injury by enhanced metabolic activities. Chapter 12 deals with comparative morphology of developing and regenerating retinal ganglion cells in birds and mammals and report that most of the qualitative features are similar and only in quantity there is a difference. Morphogenetic steps in anuran retina are described in Chapter 13 and the organization of the adult retina is based on the spatiotemporal sequence of cell generation which occurs at the ciliary margin. The same neuroepithelial cell is programmed for types, number and regional differences in distribution of the various neurons. Chapter 14 is on activity-driven sharpening of retinotopic connections at tectum in Gold fish. Blocking of NMDA receptors, sodium and calcium channels has elicited the information that the sharpening of final connections for function is dependent on the molecular mechanisms at the synapses. Ontogeny repeats phylogeny and the essential features of development of vertebrate eye have been established thus. Chapter 16 rationalises how simple mechanism of development paves the way to the understanding of the complex nature of development in vertebrates.

Chapters 15 and 17 are on mathematical modelling and computational neuroscience. Emphasis is laid on adopting collective activity of individual neurons or modular system of neuronal assemblies rather than sticking to neuron doctrine. Models of network are easier to make and are essential to test functional hypothesis. Willshaw (Chapter 17) quotes Prof. Gaze and cautions that repeated modification of the artificial model will be necessary based on the ever increasing and new experimental data in neuroscience.

In summary, this book deals with developmental neurobiology of the visual pathway of lower vertebrates and mammals by experts in the forefront of this area. This book provides sufficient stimulus to an aspiring neuroscientist to initiate studies in developmental neurobiology. To a confirmed neurobiologist it is a useful review to give new directions in researching developmental neurobiology.

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Neurotransmitter interactions and cognitive functions by Edward D. Levin, Birkhauser Verlag AG, P.O. Box 133, CH-4010 Basel, Switzerland, 1992, pp. 362, SFr. 168.

The book has effectively put together studies covered in 22 chapters and carefully examined the relationships among neurotransmitter systems operating in the basal forebrain that provide the basis for understanding the integrative neural functions underlying learning and memory. The rationale for this endeavour has been provided in the initial chapter which lays empahsis on considering the brain as an integrative organ. The next two chapters are devoted to detailed anatomic bases for the interactions between the cholinergic forebrain and its telencephalic targets including the input–output organization of transmitter—specific synapses such as GABA and glutamate, peptides—like substance P, enkephalin, somatostatin, neuropeptide Y, vasopressin, galamin and monoamines — noradrenaline, adrenaline, dopamine and serotonin. In subsequent chapters the book then proceeds to delineate the importance of both muscarinic and nicotinic cholinergic receptors along and in combination and their interactions at different levels with D_1, D_2 dopamine, α_1, α_2 and β noradrenaline and serotonin receptors in formulating the cognitive processes. Of particular significance is the suggested critical role of septohippocampal system and the prelimbic sector of the frontal cortex in the mnemonic function. The importance of neurotensin–acetylcholine, dopamine–acetylcholine, noradrenaline–acetylcholine, serotonin–acetylcholine functions in these areas *vis-a-vis* raphe and septal strucutres in formulating working representational memory *versus* reference dispositional memory has been examined critically by various authors. It is emphasized that such interactions can take many forms; they might be cooperative interactions or may involve a balance of opposing effects. Interactions may also be serial in nature, with one neurotransmitter system regulating the function of another. In addition, the form of interactions between the same two neurotransmitter systems can clearly differ depending on the particular behaviour involved in the anatomic site of interaction. Therefore, complex systems can often produce apparently conflicting results: while Normile and Altman suggested that serotonin blockers ameliorate deficits induced by clolinergic dysfunction, Ritcher–Levin and Segal, and Vanderwolf and Penara experiments suggested the opposite in that disruption of serotonergic activity potentiates impairments produced by induced cholinergic dysfunction. Such a possibility is also hinted by Olton and Peng in their chapter "Interactions of neurotransmitters and neuroanatomy: it's not what you do, it's the place that you do it".

It is opportune to have a few chapters on the influence of GABA–benzodiazepine systems on the forebrain cholinergic activity in modulation of the memory processes as it has become easy to study these mechanisms/interactions with the availability of specific neurochemical connections between the two systems. Further, benzodiazepines represent one of the most widely prescribed class of pharmacotherapeutic agents and in addition to their therapeutic actions, these agents can also produce undesirable side effects, such as amnesia, confusion, depression and fatigue indicative of cognitive dysfunction. The last chapter by the editors themselves as an 'appreciation of the concert' of interactions of a number of neural systems that use a variety of neurotransmitters *via* multiple receptors which provide the bases for a complex system like cognitive function is indeed a treat to the reader.

To summarize, the book has highlighted the work of eminent scientists to clearly bring out the role of neurotransmitter interactions in cognitive functions. It also provides food for thought to open new vistas for the development of treatment for cognitive dysfunctions. The cited studies suggest that even transmitter systems relatively unaffected in disease like Alzheimer's may be important to consider for therapy. Rather than directing treatments at a system that is severely comprised by the disease, it may be a better therapeutic strategy to direct treatment at systems that are involved in the same functions as the affected system, but are still relatively intact.

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Book Reviews

Electrodynamic theory of superconductors by Shun-Ang Zhou (IEEE Electromagnetic Wave Series 34), Peter Peregrinus Ltd, The Institution of Electrical Engineers, P.O. Box 96, Stevenage, Herts SG1 2SD, UK, 1991, pp. 320, £42.

This introductory text containing 320 pages in three chapters is addressed to applied physicists, electrical and mechanical engineers and post-graduate students and provides a phenomenological understanding of electromagnetics, electrodynamics, mechanics and thermodynamics of type-II superconductors in a user-friendly manner.

There is marked emphasis on certain aspects of superconductivity which are often excluded from most books on the subject. These aspects are electromagneto-elastic deformations and acoustic anomalies, thermoelastic coupling, co-existence of superconductivity and magnetic order, and non-equilibrium superconductivity. These are relevant to the conditions of a superconductor subjected to thermal, mechanical and magnetic loadings and, therefore, are of considerable practical importance. Chapter III of about 80 pages is completely devoted to these aspects. In fact, it is this choice of topics that gives this book a distinctive character.

The rest of the book covers rather standard stuff. Chapter I (of over 100 pages) covers the basic principles of electrodynamics of deformable media. It makes this book self-contained. It could have, however, been shortened quite a bit. Chapter II is devoted to more or less standard (London) electrodynamics of superconductors and the phenomenological Ginzburg–Landau equations. The discussion of critical currents, flux pinning, ac losses and of the dynamics of Josephson junctions is good. Except for a brief discussion of the BCS theory, much of the treatment is macroscopic in keeping with the spirit of the book.

There are some minor typos. Perhaps the most amusing example is on p. 313 where 'van Vleck' has become 'van Vlack'.

The book is particularly recommended for engineers and outsiders who wish to be intelligent users of superconductors.

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Elements of ordinary differential equations and special functions by A. Chakrabarti, Wiley Eastern Limited, 4855/24, Ansari Road, Daryaganj, New Delhi 110 002, 1990, pp. 148, Rs 50.

This book consists of nine chapters and an appendix. Chapter 1 introduces the basic definitions concerning ordinary differential equations (ODEs) and linear ODEs in the main. Chapter 2 describes some elementary methods of solution of linear ODEs. Chapter 3 is concerned with the power series solution near an ordinary point of linear ODEs with analytic coefficient functions and Hermite differential equation is considered as the main example. Chapter 4 describes the Frobenius series method for solution near a regular singular point of linear homogeneous ODEs of the second order. Chapters 5, 6 and 7, respectively, give some useful properties of the special functions associated with the names of Legendre, Laguerre and Bessel. The last two chapters on existence and uniquess theory for solution of ODEs and eigenvalue problems are very brief and sketchy. The appendix lists some properties of the gamma function, the beta function, hypergeometric functions, the error function and the Chebyshev polynomials. The book may be suitable for an undergraduate course in engineering.

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Finite element methods in engineering science by C.T.F. Ross, Prentice-Hall, 66, Wood Lane End, Hemel Hempstead, Hertfordshire, HP2 4RG, England, 1990, pp. 520, $ 37.95.

The book is intended to introduce to a practical engineer or an undergraduate the concepts of FEM as applied to general problems in engineering science. The discussion throughout had been rather elementary but in consequence the lack of precision or indepth discussion leaves a serious reader perplexed. Even as the intent is to address the general problems of engineering science more than 90% of the treatment centres around problems of structural engineering. In fact, only Chapter 11 and a few pages in Chapters 12 and 13 discuss the problems in other fields. A heavy emphasis on an elementary treatment has obscured an insight into basic concepts. This is evidenced by a discussion of beams and frames for more than half the book (up to Chapter 6) without the reader (student?) ever getting an idea that a beam or frame element is the most basic of the finite elements! Lackadaisical treatment of the subject matter is abundant—natural co-ordinates of a triangle discussed on pages 221 and 292 will easily confuse a novice. Similarly, there is no reason why a grillage or a grid framework is to be discussed—not as part of beams and frames but after discussing the dynamic analysis by FEM. Even here the stiffness matrix is written directly without any reference to FEM analysis. Again, why was it necessary to postpone the introduction of axisymmetric elements to Ch. 12 long after the discussion of nonlinear problems? It can be claimed as a virtue that computer programs in BASIC have been made available throughout the book. On the contrary, a compendium of several programs each for a particular problem defeats the object of analysis through FEM, the basic concept of which is to provide a single standard means of analysis for all the different problems. Furthermore, in the present-day environment in which general-purpose FEM software packages are available, the user is hardly expected to write a program for his needs. On the whole, there is not much to recommend about the book being useful for an undergraduate or a practical engineer.

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Galois dream—group theory and differential equations by Michio Kuga, translated from the Japanese by Susan Addington and Motohico Mulase, Birkhauser Verlag AG, Klosterberg 23, CH 4010 Basel, Switzerland, 1993, pp. 150, SFr 52.

This book with a catchy title is an invitation to mathematics, or at least some part of the subject having algebraic, topological and function theoretic components, woven together in a nontrivial way. Evidently when first published in Japanese in 1968, students carried copies of it to show off, whether or not they could understand the contents! It consists of nineteen parts, or lectures given once a week; but they are of widely varying lengths, some as short as a couple of pages, and others much more substantial.

There are two rather distinct ways in which, to a user of mathematics like a physicist, the themes of group theory and differential equations come together. One is the study and exploitation of symmetry properties of a given differential equation, in order to simplify the problem of finding all, or large families of, its solutions. These symmetries naturally form a group, and in the continuous case pioneered by

Sophus Lie, very powerful tools in this direction have been developed over the decades. The present book is concerned with a different aspect, basically the topological understanding of the most appropriate space or manifold on which we should represent the solutions of a given differential equation. This is quite similar in spirit to constructing or 'inventing' a suitable Riemann surface to handle a multivalued analytic function.

The initial 'chapters' or weekly instalments are relatively light and easy to digest. The author covers, in an attractive style, such ideas as: basic set theoretic concepts and operations; partitions of sets based on equivalence relations; free groups; connectivity properties of manifolds or spaces, homotopic equivalence of curves and the fundamental group; and the case of simple connectivity. Many easily visualizable examples, with an occasional humourous touch, are given to clarify the points being made.

The author next moves on to the concept of one space being a covering of another. The notion of a covering map, covering transformations and the associated group, and the behaviour with respect to the fundamental groups of the two spaces involved, are all developed. These are attractive and instinctively appealing ideas which have an easy naturalness to their development. The construction of the universal covering space of a given connected space, and the Galois property for a covering relationship, are also tackled.

In the final portion, the attention turns to function theory. For definiteness the author works on suitable connected subsets of the complex plane. The basic questions analysed are: what is an analytic function, and how do we go about classifying its singularities? What are reasonable sets of operations we can perform on given sets of functions to lead to new functions? And if a (second order) differential equation with analytic functions for coefficients is given, how can we handle its 'space of solution' and what manifold is ideally suited to represent the values and singularities of a solution? These and related questions are discussed in some detail, though complete proofs of all stated theorems are not always presented.

Second-order differential equations for functions of a complex variable are very familiar to physicists and engineers alike. All the special functions of mathematical physics arise out of such equations. The traditional approaches are summarised in such classics as Jeffreys and Jeffreys, or Morse and Feshbach. Through such treatments one is familiar with the ways to handle possible multivaluedness of the resulting solutions, construction of independent solutions, the notorious 'logarithmic case' and so on. For one trained in this way, Kuga's book helps put all this in a nice perspective. Though not meant to be a thoroughgoing textbook, one is exposed to a useful collection of mathematical ideas with a lightness of touch. In this sense this is an invitation to the subject.

One note of concern must, however, be finally expressed. This book of some 150 pages is priced truly exorbitantly. The only way out seems to be for some gifted Indian mathematicians to take up writing in a similar style, to enthuse students and show them the pleasures of areas in mathematics, but at an affordable price.

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A mathematical treatment of dynamical models in biological science by K. Smitalova and S. Sujan, Prentice-Hall, 66, Wood Lane End, Hemel Hemsptead, Hertfordshire, HP2 4RG, England, 1991, pp. 200, $ 87.95.

It is probably only in the 'rarest of the rare cases' that one would object to the title of a book itself, especially for highly specialized and technical books. Unfortunately, however, the book under review falls into this category. Though the title suggests that the book is about models in biological sciences, the contents deal exclusively with mathematical ecology, and that too, exclusively with population dynamics. Personally, being at a centre for ecological sciences, and working mainly with mathematical models, I am strongly tempted to raise three cheers for this point of view. After all, it was not very long ago that one of the highest ranking scientists from the division of biological sciences at the institute had equated biology with the study of gene regulation (on something equally miniscule in scope). On this background, the mathematical ecology backlash from totally unexpected quarters, C'zechoslovakian mathematician, is all the more welcome. But, such extremist point of view serves very little constructive purpose. An honest and fair approach would have been to use the term population dynamics (or at least, mathematical ecology, as used in the very first sentence of the preface) in the title, instead of the misleading comprehensive 'biological science'.

The next serious objection has to be raised at the very first sentence of the preface itself—"Mathematical ecology as a biological science accumulates information concerning the evolution of biological communities". Certainly, mathematical ecology is much more than that; study of population changes, spatial patterns, impact of external influences, just to name a few. Secondly, the term 'evolution' is used traditionally in a very specific sense in biology, where it refers to (small or large) changes at the genetic level. In mathematics, and more specifically, in the study of dynamical systems, too the term evolution is used traditionally in a very specific sense; here, it simply means any change in the value of the variable under consideration with time. Unfortunately, the authors have from page 1 line 1, used exclusively the mathematicians' term. While they cannot be technically faulted, the use seriously interferes with the readability and clarity. Mere changes in population size should not be called 'evolution', when biological phenomena are described. This usage becomes even more confusing because the authors subsequently go on to describe changes in gene frequencies also.

Coming to the contents of the book (in a way a more traditional review would do), it first describes the growth of population of a single species. Both differential and difference equations are described, and the standard elaboration—age structure, delay, stochastic effects, etc.—are described at length, in a fully mathematical style and vocabulary. This chapter is followed by one on two species models, which lead to the next one on *n*-species communities. The last chapter has no biology at all, and deals with chaos and related mathematical matters. The preface describes this last chapter, with a perhaps unintentional honesty (crept in most probably during translation) as more pretentious!

One would have been willing to overlook the matters raised in the first two paragraphs had the book contained other points of interest. Regrettably, it is not so. It is not clear why and for whom this book was written. It seems mostly written for mathematicians, to enlighten them about the kinds of mathematical models used in ecology. On one hand, being accustomed to abstract representation, mathematicians probably do not need any connection between the symbols they manipulate, and the reality around them. On the other hand, if demonstrating the biological relevance of the models was the goal, the book comes nowhere near satisfying it. The ecological part, whenever it is described is at best grossly simplistic and naive, and at worst, misplaced and wrong. This lack of rigour, coming from mathematicians, is rather saddening. To quote a couple of typical examples (p.7). "The carrying capacity is the maximal population density which can be sustained for a long time in the given environment (How long?) or (p.2) "the population density describes the population only approximately since the population is too large to determine the exact number of its members". The accuracy of measurement (or the lack of it) surely is not the criterion of deciding between continuous and discrete models. A somewhat more distressing slip-up is on p.42, where two different equations are described, and they are said to have the same fixed points; a moment's scrutiny reveals the two equations to be identical any way!

A positive feature of the book is that it includes many post-1980 references. Quite understandably, a number of them are of European origin. A corresponding negative feature is the omission of some of the more standard works on theoretical ecology — *e.g.,* those by R. M. May. Perhaps for the same reason, the whole body of work related to host–parasite interactions, spread of diseases and the like receive scant attention. Perhaps to marginally compensate for it (though in a haphazard manner), topics such as game theory, and examples such as loss of plasmids from bacterial cells, are introduced, rather out of the blue. The index at the end of the book is just about one page, making it a more ritualistic than useful inclusion.

The description inside the cover page suggests that the book contains original presentation of very extended and deep problems, and a coverage of new results. Even if one assumes this to be true, the implications of these to biological systems are probably visible (and perhaps even self evident) only to mathematicians; lesser mortals cannot hope to grasp them. In fact, this book would be either enjoyable or useful (but unlikely to be both) to mathematical ecologists (mathematical dominant, ecology recessive) and theoretical biologists (more theoretical than biologists). Those dynamical systems theorists engaged in a game of one-upmanship with say number theorists or differential geometers would profit most from this book, to show off to these innocents that dynamical systems theory has so much to offer to ecology (and by implication to enviromental science, on to Agenda 21 and Rio Summit). They perhaps may not mind shelling out \$ 87.95. For others, not really a book one would wholeheartedly recommend.

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Men and women of space edited by Douglas B. Hawthorne, Univelt, Inc., San Diego, CA 92128, 1992, pp. 904+13, \$ 90.

This monumental reference book contains biographies of spacefaring persons from all over the globe. The exhaustive bibliographies of over 800 astronauts and cosmonauts who have actually flown and candidate members as well as those on ex-atmospheric suborbital flight that reach or exceed 80 km of altitude, are presented in an orderly fashion. This excellent book contains equally well-written introductory section summing up the systematic arrangement of bibliographies and five appendices. A chronological list of about 200 missions including a dozen X-15 suborbital flights is given in Appendix 5. Here, information such as prime and backup crew, place and time of launch as well as landing, mission duration for each of the mission are compiled. The first flight in the human history is made by Yuriy A. Gagarin (German S. Titov as backup) on Vostok-1 (Cedar) on April 12, 1961 at 9:06:54 am (Moscow time) from Baykonur Cosmodrome, Kazakhstan (in erstwhile USSR). Vostok-1 went around the Earth once and the flight lasted 1 hour 55 minutes. The last entry on space mission is STS-45/OV-104 Atlantis with seven crew which was launched on March 24, 1992, from Complex-39A, Kennedy Space Center, Fla, USA. The mission was in operation for 214 hours 10 minutes 26 seconds and it orbited the Earth 143 times.

The bibliographical entries on astronauts and cosmonauts are organized in alphabetical order in the first 831 pages of the book. These entries fall into the following categories: NASA astronauts and candidates; Manned orbiting laboratory pilot astronauts; X-20 DynaSoar pilot astronauts; X-15 military and civilian research pilots; Military astronauts-designees of the early 1960s; Shuttle payload specialists and passengers; International cosmonauts trained by USSR. The details under each of the bibliographies are elaborate. The first item of each entry states the nature of assignment of the corresponding astronaut/cosmonaut. For example, Rakesh Sharma (see p. 652): Indian cosmonaut–researcher (flight), Soyuz T-11/ Salyut 7/ Soyuz T-10, Soviet international program, the first Indian to make space flight. Similarly for Neil Armstrong: NASA pilot-astronaut (Group 2, 1962), the first person to walk on Moon; former X-15 civilian research pilot. The other entries are on: Date and place of birth, Nick name, Military affiliation, Marital status, Children, Education, Experience (illustrative), Space flight assignment, Sources (of collected information), and also some attributes like decoration and awards, recreational interests, physical description, place and circumstances of death (wherever applicable), writings. It is really a great pleasure to go through the above details of well-known and lesser known (owing to meager information in open literature) astronauts/cosmonauts.

The reviewer finds the book a very valuable addition to all libraries and useful to people from all walks of life. It can inspire a lot of youngsters in the years to come. It is hoped that the second edition will be equally delightful and will be able to meet the high expectations generated by this book.

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Spaceflight mechanics edited by R.E. Diehl, R.G. Schinnerer, W.E. Williamson, and D.G. Boden, Advances in Astronautical Sciences, Vol. 79, 2 parts, AAS Publications, Univelt, Inc., San Diego, CA 92128, USA, 1992, pp. 1293+18, $ 240.

The proceedings of the 1992 Spaceflight mechanics have been systematically organized in two volumes. The Conference hosted by the American Astronautical Society and AIAA was held at Colorado Springs, Co., on February 24–26, 1992. The Conference dwells on analytical and numerical aspects of astrodynamics and celestial mechanics, with emphasis on application to current and future space missions. In all, 85 papers were presented in twelve sessions. These sessions were: Orbit determination; Tethered satellite systems; Celestial mechanics; Optimization; Flexible body dynamics and control; Mars mission analysis; Earth-orbiting mission analysis/debris; Mission analysis; Numerical analysis. Many of the papers are descriptive and do not unduly bog down to elaborate mathematical treatment. Readers interested in probing further on the topics of their interest can access the accompanying detailed bibliography. Majority of the papers contained in the proceedings deal with the Earth and interplanetary mission analysis, orbit determination and allied topics. The remaining three sessions are devoted to spacecraft attitude control with flexibility taken into consideration.

The question of orbit determination has been addressed from time immemorial. The classical techniques which have emerged for over three centuries have relied heavily on innovative approaches for algorithm development and banked less on technological advancements. The modern methods try to bring into their realm latest technological developments such as, say, ground- and spacecraft-borne radars, and system theoretic concepts such as say, Kalman filter. This enhances and cultivates scope for new orbit determination algorithm and their applications to an unlimited horizon. However, the papers in this proceedings address only limited perspective on orbit determination of interplanetary spacecraft, multi-object tracking and Mars gravity modelling (inverse problem of orbit determination from space platform). A pair of papers analyze orbit determination error for spacecraft at L_1-libration point.

The session on celestial mechanics contains two abstracts and six full papers which are equally shared by orbital transfer and orbit analysis problems. The orbit transfer problems address both low and high thrust multi-impulse transfers. They use simplified equation of motion as well as tools that are based on Battin's universal functions, primer vector for fixed time optimality, Brachistochrone problem for bridging gap between high and low acceleration trajectories, respectively. The tools of orbit analysis apply essentially the well-known/time-tested methods to new contexts. In orbit analysis, numerical methods play important role. The two numerical methods are shown to yield high accuracy over a long period of orbit propagation. Montenbruck shows that the Taylor's series integration used for the first time is superior to the conventional methods like Runge–Kutta (R–K) method. However, a companion paper combines R–K with Enke's method and shows it to be more efficient than Cowell's method. Multiple impulse Lambert problem and propagation of covariance matrices during rendezvous are useful contributions that may find application in the near future. All but one paper out of eight in the session on optimization deal with the optimal trajectory control. The control is obtained by optimizing the appropriate performance costs *via* methods available in standard textbooks.

System theoretic studies on mission analysis are distributed in three sessions with like-minded titles. Here, the technical papers give vivid description of diverse topics from propulsion system to navigation to mission perspective of orbital transfer and rendezvous to Mars and interplanetary missions to launch missions, to name a few. Even though these papers lack mathematical finesse and innovative theoretical contributions, they can still inspire many a person since they describe several new avenues/possibilities from modern aerospace industry. The reviewer finds that the corresponding 32 papers which form the major portion of part II of the proceedings are the inspiring ones. It is possible that some of the concepts discussed here may find the light of the day while remaining ones may remain in books only.

Three sessions are also devoted to satellite dynamics and control. Two papers formulate the perturbed attitude motion, respectively, due to Mars atmosphere and chaotic attitude maneuver due to oscillation of small perturbing sub-bodies inside a satellite. Survey on time-optimal attitude maneuvers contain many useful references that may be useful to slew control of present multi-mission spacecraft. The feedback linearization is gaining some kind of acceptance at theoretician level in recent times. Spacecraft nonlinear control is one such example. During the past 25 years or so, tremendous progress has been made on the understanding of flexible body dynamics and control. In fact, the failure of the first American satellite was attributed to flexibility of four tiny turnstile antennae. Nonlinear control by feedback linearization technique is used here too for control of multipurpose communication satellite INSAT-II as an example by Modi and Ng. Elaborate simulation study amply demonstrates the feasibility of nonlinear control system. Modelling of dynamics of flexible space structure is done in many ways, prominent among them being continuum and discrete coordinate approaches. In the former case, model is represented by a distributed parameter system. In the latter case, large space structure is idealized as an assemblage of multibodies. The tethered satellite systems are special case of flexible spacecraft wherein flexibility of a tether holding the primary and the sub-satellite is prominent. Pioneering work on dynamics and control of tethered system is done by Misra, Modi, Kane and others. Some of the results of the luscious study have appeared in the proceedings.

In summary, the papers of this proceedings portray the current trend in global research on spaceflight mechanics. The reviewer finds refreshing the technical papers in the proceedings of both Spaceflight Mechanics conferences that were held thus far. It is a welcome addition to libraries having sizeable readership from aerospace community. However, owing to its price, individuals specially from third world countries like India will find it outside their reach.

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Soil analysis–Physical methods edited by Keith A. Smith and Chris E. Mallins, Marcel Dekker, Inc., 270, Madison Avenue, New York, NY 10016, 1991, pp. 632, $ 180.

Among the different materials one comes across, soil can be termed as one with maximum variability exhibiting complex behaviour. Unsaturated soil consists of solids, water and air. The water present itself exhibits different behaviour depending upon whether it is adsorbed or free, and so on. Soil mass exhibits visco-elasto-plastic behaviour. The behaviour keeps changing due to change in environment, cycles of wetting and drying, etc. Civil engineers and agricultural scientists are among those who are mainly concerned with soil.

It is the basic physical properties like particle size distribution, bulk density, water content, liquid and plastic limits which influence, be it the mechanical properties a civil engineer looks for, or the characteristics an agricultural scientist looks for. Hence these basic properties have been studied by both civil engineers as well as the agricultural scientists.

The treatise consists of thirteen chapters written by different authors. The subjects covered range from particle size analysis, density water content, Atterberg limits, permeability of saturated and unsaturated soils, etc. The penetrometer tests are used to understand tillage, resistance to root growth, etc. The engineering properties of soil are innovatively used to explain the behaviour of soil as an agricultural material.

The authors are to be complimented for explaining the complex soil behaviour in a lucid manner. The treatment is comprehensive and covers the recent developments also. It will serve as a good reference text for graduate students of civil engineering also.

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Logic of the genetic code: Conservation of long-range interactions among amino acids as a prime factor

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Abstract. Any statement on the optimality of the existing code ought to imply that this code is ideal for conserving a certain hierarchy of properties while implying that other codes may have been better suited for conservation of other hierarchies of properties. We have

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Logic of the genetic code: Conservation of long-range interactions among amino acids as a prime factor

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Abstract. Any statement on the optimality of the existing code ought to imply that this code is ideal for conserving a certain hierarchy of properties while implying that other codes may have been better suited for conservation of other hierarchies of properties. We have evaluated the capability of mutations in the genetic code to convert one amino acid into another in relation to the consequent changes in physical properties of those amino acids. A rather surprising result emerging from this analysis is that the genetic code conserves long-range interactions among amino acids and not their short-range stereochemical attributes. This observation, based directly on the genetic code itself and the physical properties of the 20 amino acids, lends credibility to the idea that the genetic code has not originated by a frozen accident (the null hypothesis rejected by these studies) nor are stereochemical attributes particularly useful in our understanding of what makes the genetic code 'tick'. While the argument that replacement of, say, an aspartate by a glutamate is less damaging than replacement by arginine makes sense, in order to subject such statements to rigorous statistical tests it is essential to define what constitutes a random sample for the genetic code. The present investigation describes one possible specification. In addition to obvious statistical considerations of testing hypotheses, this procedure points to the more exciting notion that alternative codes may have existed.

Keywords. Genetic code; mutation rates; evolution; frozen accident; stereochemical properties.

1. Introduction

The origin, organization and evolution of the genetic code have continued to be sources of fascination since the discovery of the code in the sixties. That it had to be at least a triplet-based code was dictated by the fact that four bases code for 20 amino acids. However, explanations put forward for specific assignment of codons to amino acids were based on diverse speculations. These have invoked the role of structural and stereochemical features (Woese 1965), the role of chance (Jukes 1966) following an initial adaptation phase—the 'frozen accident'—(Crick 1968), kinetic mechanisms involving hypercycles and bootstrapping (Eigen and Winkler-Oswatitsch 1981; Crothers 1982), and others. Some of the more recent studies have explored how deviations from the 'universal' code (e.g. as seen in mitochondria) are brought about (Osawa and Jukes 1989), while others have analysed the intriguing patterns of complementary hydropathy (Konecny *et al.* 1993), which has a bearing on the similarity between the two polypeptides encoded in the sense and antisense strands of DNA.

Another kind of approach to understanding the logic of the genetic code is to

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examine the sensitivity of the different properties of amino acids to point mutations in the DNA (e.g. Salemme *et al.* 1977). The code is expected to have evolved in such a way as to make the more important properties of amino acids less sensitive to mutations. Sitaramam (1989) has suggested an approach whereby one can examine to what extent the genetic code protects or conserves a particular property of amino acids. For a given rate of point mutation, the probability of conversion of any amino acid into any other can be computed from the genetic code (an explicit definition is given in the next section). For the 20 amino acids, one thus obtains a set of 400 conversion probabilities. These may be termed 'genetic distances' between the amino acids. The *effect* of one amino acid changing into another as a result of one or more mutations can be quantified, for a given property, by the absolute value of the difference between the numerical values of the property for the two amino acids. For every property, this leads to a set of 400 values; these may be referred to as the 'physical distances' between the amino acids for that property. Sitaramam (1989) has pointed out that the correlation coefficient between genetic and physical distances for any property of amino acids may be used to measure the robustness of the property to mutations. A large negative value of the coefficient implies that conversions that are more probable lead to relatively small changes in the value of the property, and conversely, conversions that lead to large changes have a low probability of occurrence (see also Di Giulio 1989). By examining the values of the correlation coefficients between the genetic and physical distances for various properties of amino acids, one can thus identify the important properties, i.e. *the properties that the genetic code is designed to protect.* Alternatives to correlation coefficient, such as mean square error (Haig and Hurst 1991), have also been subsequently proposed, though correlation coefficient seems to have an advantage in being scale independent.

From such an analysis, Sitaramam (1989) demonstrated that the genetic code preferentially conserves the long-range nonbonded interaction energy. His approach, however, had two limitations. Firstly, the probability of a codon (triplet) undergoing one, two or three base changes was taken to be fixed at 1/9, 1/81 and 1/729 respectively. While this is a little better than considering rather *ad hoc* parameters like number of base changes needed (Di Giulio 1989) or exclusively single base changes (Haig and Hurst 1991), a more rigorous measure that takes into account probabilities of 0, 1, 2,... mutations would be more appropriate.

Secondly, the significance of the correlation coefficient was assessed using Fisher's z transformation (see e.g. Sokal and Rohlf 1981). This test implicitly assumes the existence of an infinite population of bivariate measurements from which a random sample is drawn. In the present context, what constitutes such a population? Indeed, the 400 values of physical and genetic distances are not independent. These considerations suggest that the formal application of the test of significance of the correlation coefficient may lead to a biased estimate of the p value.

These two limitations can be taken care of by allowing for various rates of base substitution and also by considering the population of all theoretically possible genetic codes (as done by Haig and Hurst 1991), from which a sample of codes can be drawn. The values of correlation coefficient (r) for each of the sample codes can be obtained, and the frequency distribution of these r values can be used to judge the statistical significance of the r value obtained for the existing genetic code.

We describe here an algorithm to compute the conversion probabilities for any intensity of mutation rate and also a Monte Carlo technique (Haig and Hurst 1991) for performing suitable tests of statistical significance.

2. Methods and models

2.1 *Computation of conversion probabilities*

Let y denote the average number of mutations (base substitutions) that a codon undergoes, say during DNA replication, or during the formation of germ-line cells which eventually give rise to the next generation. A given codon may undergo 0, $1, 2, \ldots, k$ such mutations with appropriate probabilities (defined later). As a result, the codon may code for the same amino acid (either because no mutations took place, or because multiple mutations restored the original triplet, or because the changed triplet coded for the same amino acid), or a different amino acid. The probability of conversion of, say, glycine to alanine is then defined as the probability that a triplet has mutated to one that codes for alanine, given that it originally coded for glycine. We compute the probabilities of conversion of amino acids from the conversion probabilities for codons. The details of the procedure are described below.

2.1a *Conversion probabilities for codons*: With reference to a given codon, say UGC, all the 64 codons can be classified into four groups:

(i) UGC	:	There is only one such codon
(ii) Differing from UGC in only one base	:	There are nine such codons
(iii) Differing from UGC in two bases	:	There are 27 such codons
(iv) Differing from UGC in three bases	:	There are again 27 such codons.

Let the groups be denoted by T0, T1, T2 and T3 respectively.

To begin with, the codon is in class T0. If it undergoes a single mutation, it moves into the class T1. If it now undergoes *one more* mutation, there will be three possible outcomes:

(i) The mutation may be at a different position than the first mutation. The codon then moves to class T2. The (conditional) probability of such an outcome is 2/3.
(ii) The mutation is at the same position as the first one, but the codon does not revert to UGC, i.e. it remains in class T1. The probability of such an outcome is $1/3 \times 2/3$, i.e. 2/9.
(iii) The mutation is at the same position as the first one, and the codon reverts to UGC, i.e. it moves into class T0. The probability of such an outcome is $1/3 \times 1/3$, i.e. 1/9.

In an analogous manner, one can compute the probability of a codon moving from any class to any other class as a result of a single mutation. The results can be expressed in the form of a square matrix M of order 4, such that the elements m_{ik} denote the probability of conversion from class Tk to class Ti. The matrix M is seen to be

$$M = \begin{bmatrix} 0 & 1/9 & 0 & 0 \\ 1 & 2/9 & 2/9 & 0 \\ 0 & 6/9 & 4/9 & 3/9 \\ 0 & 0 & 3/9 & 6/9 \end{bmatrix}$$

Let $P(N)$ be the column vector $(p_0, p_1, p_2, p_3)^{\mathrm{T}}$ whose elements denote the probabilities that the codon will be in the class T0, T1, T2 or T3 respectively after N mutations have taken place. It is seen that

$$P(N) = M^N \cdot U,$$

where U is the column vector $(1,0,0,0)^{\mathrm{T}}$. Since the possible numbers of codons in classes T0, T1, T2, T3 are 1, 9, 27 and 27 respectively, as N tends to infinity $P(N)$ tends to $P(\infty) = (1/64,\ 9/64,\ 27/64,\ 27/64)^{\mathrm{T}}$.

As defined earlier, the parameter y denotes the mean number of mutations experienced by a codon. Assuming mutations to be random events, the probability that a codon undergoes exactly k mutations ($k = 0, 1, 2, \ldots$) is given by the Poisson probability $(\mathrm{e}^{-y} y^k / k!)$.

Hence F, the column vector $(f_0, f_1, f_2, f_3)^{\mathrm{T}}$, whose elements denote the probabilities that a codon will be in the class T0, T1, T2 or T3, is given by

$$F = \mathrm{e}^{-y} \sum_{k=0}^{\infty} P(k) \cdot y^k / k!.$$

For any given value of y, the column vector F can be calculated as follows.

Let X denote the matrix of right eigenvectors of the matrix M, and B denote the diagonal matrix of eigenvalues. Then it can be shown that

$$M^N = X \cdot B^N \cdot X^{-1}.$$

The expression for F then becomes

$$\begin{aligned} F &= \mathrm{e}^{-y} \cdot X \cdot \sum_k B^k \cdot y^k / k! \cdot X^{-1} \cdot U \\ &= X \cdot C \cdot X^{-1} \cdot U, \end{aligned}$$

where C is a diagonal matrix with

$$c_{ii} = \exp(-y) \cdot \exp(b_{ii} \cdot y) = \exp((b_{ii} - 1) \cdot y),$$

where b_{ii} is the ith eigenvalue of the matrix M.

The eigenvalues of M are seen to be equal to 1, 5/9, 1/9 and $-1/3$, while the matrix X of (unnormalized) corresponding eigenvectors is seen to be

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 9 & 5 & 1 & -3 \\ 27 & 3 & -5 & 3 \\ 27 & -9 & 3 & -1 \end{bmatrix}$$

It can also be shown that X^{-1} is equal to $X/64$. Thus, for any value of y, the column vector F can be computed.

The probability of conversion z_{ik} for two codons i and k in a specified time interval, when the mutation rate is y, is then seen to be, from the above expressions,

$$z_{ik} = f_0 \quad \text{if the two codons are identical,}$$

$$z_{ik} = f_1/9 \quad \text{if the two codons differ in one base.}$$

This is because f_1 is the probability of changing to a codon differing in one base. There are nine different codons which differ from i in exactly one base, so to obtain the probability of i going to k, which is one of the nine possibilities, f_1 has to be divided by 9. Arguing in a similar manner, it can be seen that

$$\begin{aligned} z_{ik} &= f_2/27 \text{ if they differ in two bases, and} \\ &= f_3/27 \text{ if they differ in all the three bases.} \end{aligned}$$

Notice that $z_{ik} = z_{ki}$. It can also be seen that when $y = 0$ (no mutations), $z_{ii} = 1$ and $z_{ik} = 0$ for $i \neq k$. Also, as $y \to \infty$, $z_{ik} \to 1/64$ independently of i and k, as expected.

2.1b *Conversion probabilities for amino acids*: Let the degeneracies of two amino acids be n_1 and n_2, i.e. they are coded for by the codons $i_1, i_2, \ldots, i_{n_1}$ and $k_1, k_2, \ldots, k_{n_2}$ respectively. Assuming codon usage frequencies to be equal, q_{12}, the probability of conversion of amino acid 1 to amino acid 2 is

$$q_{12} = \frac{1}{n_1} \sum_{a=1}^{n_1} \sum_{b=1}^{n_2} z_{i_a k_b},$$

whereas

$$q_{21} = \frac{1}{n_2} \sum_{a=1}^{n_1} \sum_{b=1}^{n_2} z_{i_a k_b}.$$

Thus, q_{12} and q_{21} are not necessarily equal; they are equal only if $n_1 = n_2$.

2.2 *Physical distances*

A set of 20 representative properties of amino acids (Hutchens 1976; Prabhakaran and Ponnuswamy 1979) was used in the analysis. The physical distance between a pair of amino acids for a chosen property was defined to be the absolute value of the difference between the numerical values of the property for the two amino acids under consideration.

2.3 *Tests of significance for r*

As mentioned before, the test based on Fisher's z transform for determining the statistical significance of r is not very appropriate, owing to the non-independence and non-normality of the values of genetic and physical distances. The question we wish to ask is: how does the value of r obtained here compare with what may be obtained by *chance alone*?

To obtain the various values of r that might occur by chance, we need to consider alternative genetic codes. Unconstrained realizations of possible genetic codes (e.g. UUU, GGG, AAA and CCC coding for the same amino acid) are clearly inappropriate. We therefore keep the existing level of degeneracy and assignment of codons unchanged. Thus GCU, GCC, GCA and GCG are assumed to code for the same amino acid, but not necessarily alanine; AAA and AAG are assumed to code for the same amino acid, but not necessarily lysine; and so on. We thus keep the 20 sets of codons unchanged, and obtain realizations of alternative genetic codes by assigning the 20 amino acids randomly, one to each set of codons (Haig and Hurst 1991). For every such genetic code, one can compute the conversion probabilities, and correlate them with physical distances.

A FORTRAN program implementing the above approach has been developed. The necessary computational details are described here.

The genetic distances were stored in a 20 × 20 matrix G, and the physical distances in a matrix D. If these 400 elements are denoted by variables (in the program, arrays) X and Y respectively, the correlation coefficient r is given by

$$r = \frac{\overline{XY} - \bar{X} \cdot \bar{Y}}{s_X \cdot s_Y},$$

where the bar denotes the mean and s the standard deviation.

An alternative genetic code is simulated by randomly permuting the serial numbers of the amino acids in the genetic code. This amounts to generating a new 20 × 20 matrix, say G'. The elements of G' are obtained by appropriate permutations of the rows and columns of G. The array (say X') obtained from G' thus contains the same elements as in X, but in a different order.

We have made use of this fact, and linearly transformed the elements of the original matrices G and D such that each set of 400 values has mean zero and standard deviation unity. The computation of r is thus much more efficient, since r is simply $\overline{XY}$.

Generating all the possible genetic codes even under the constraint imposed here would involve 20! ($\cong 2{\cdot}4 \times 10^{18}$) permutations, which is unrealistic. We have restricted our analysis to between 500 and 1000 permutations. For each permutation, the array X' was generated by looking up the elements of G in an appropriate order, and r was computed. We also computed the mean and standard deviation of the values of r so determined. For a given property and a given G matrix, the mean and standard deviation stabilized by about 400 simulations, which indicates that this is an adequate sample size. We fixed our sample size to be 1000 simulations. For any property, we can now compare the r value for the existing genetic code with the distribution of r values obtained from the simulations.

3. Results and discussion

3.1 *Transition probabilities*

The codon transition probabilities p_0, p_1, p_2 and p_3 are given in table 1 as a function of y, the mutation rate. As expected, p_0 monotonically decreases as y increases, while p_1 increases initially with y, and then decreases as y increases still further. It is interesting to note that when the value of p_1 (single base change) is highest (for $y \cong 1$), the value of p_0 (probability of no change) is still appreciably high ($\cong 0{\cdot}20$ to $0{\cdot}40$), emphasizing the importance of taking into account the probability of 'no change' in estimating the effects of mutation. On the other hand, for higher values of y, probabilities of two- and three-base changes (p_2 and p_3) also become appreciably high. In the light of these results, considering single-base changes alone as an indicator of the effect of mutations, as in the earlier investigations (e.g. Haig and Hurst 1991), does not seem to be fully justified.

Table 1. Codon transition probabilities for no change (p_0), one base change (p_1), two changes (p_2) and three changes (p_3) for different values of y, the average number of mutations.

y	p_0	p_1	p_2	p_3
0·1000	0·90534	0·09154	0·00309	0·00003
0·2000	0·82058	0·16773	0·01143	0·00026
0·3000	0·74461	0·23073	0·02383	0·00082
0·4000	0·67648	0·28240	0·03930	0·00182
0·5000	0·61532	0·32435	0·05699	0·00334
0·7500	0·48818	0·39544	0·10677	0·00961
1·0000	0·39043	0·43128	0·15880	0·01949
2·0000	0·17405	0·41305	0·32674	0·08616
3·0000	0·08973	0·33210	0·40969	0·16847
4·0000	0·05348	0·26540	0·43903	0·24208
5·0000	0·03636	0·22016	0·44443	0·29905
7·5000	0·02120	0·16619	0·43430	0·37832
10·0000	0·01734	0·14894	0·42654	0·40719
20·0000	0·01564	0·14072	0·42193	0·42170
30·0000	0·01563	0·14063	0·42188	0·42187

Table 2. Amino acid properties examined (from Hutchens 1976; Prabhakaran and Ponnuswamy 1976).

1. Thermodynamic transfer hydrophobicity
2. Protein environment or bulk hydrophobicity
3. Polarity
4. Isoelectric point pH_i
5. p*K*, Equilibrium constant with respect to the COOH group
6. Molecular weight
7. Bulkness
8. Chromatography index
9. Refractivity index
10. Short- and medium-range nonbonded energy
11. Long-range nonbonded energy
12. Protein environment total nonbonded energy
13. Power to be at the N-terminal of alpha helix
14. Power to be at the C-terminal of alpha helix
15. Power to be at the middle of alpha helix
16. Power to adopt beta sheet
17. Power to adopt beta bend
18. Heat capacity
19. Absolute entropy
20. Entropy change for formation from elements

3.2 *Correlations between genetic and physical distances*

The 20 physical properties of amino acids (Hutchens 1976; Prabhakaran and Ponnuswamy 1979; Sitaramam 1989) we examined are listed in table 2. For a property, the physical distance between a pair of amino acids is, as mentioned before, the absolute value of the difference between the numerical values for the amino acids.

Table 3. Correlation coefficients for the 20 properties as a function of mutation rate *y*.

property \ y -->	0.10	0.20	0.30	0.40	0.50	0.75	1.00	2.00	3.00	4.00	5.00	7.50	10.0	20.0	30.0
1	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.30	-0.26	-0.21	-0.17	-0.10	-0.08	-0.07	-0.07
2	-0.30	-0.31	-0.31	-0.32	-0.32	-0.33	-0.33	-0.33	-0.28	-0.21	-0.14	-0.03	0.01	0.03	0.03
3	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19	-0.19	-0.18	-0.16	-0.13	-0.10	-0.06	-0.04	-0.03	-0.03
4	-0.21	-0.21	-0.21	-0.21	-0.20	-0.20	-0.20	-0.17	-0.12	-0.08	-0.04	0.02	0.03	0.04	0.04
5	-0.27	-0.27	-0.26	-0.26	-0.26	-0.26	-0.25	-0.22	-0.18	-0.13	-0.09	-0.03	-0.01	0.00	0.00
6	-0.30	-0.30	-0.30	-0.30	-0.30	-0.31	-0.31	-0.29	-0.24	-0.17	-0.11	-0.02	0.01	0.03	0.03
7	-0.28	-0.29	-0.29	-0.29	-0.29	-0.29	-0.29	-0.25	-0.18	-0.09	-0.01	0.09	0.13	0.14	0.14
8	-0.31	-0.32	-0.32	-0.33	-0.33	-0.34	-0.34	-0.34	-0.31	-0.25	-0.19	-0.10	-0.07	-0.05	-0.05
9	-0.29	-0.30	-0.30	-0.30	-0.31	-0.31	-0.32	-0.32	-0.29	-0.24	-0.19	-0.10	-0.07	-0.06	-0.06
10	-0.31	-0.31	-0.31	-0.30	-0.30	-0.30	-0.29	-0.25	-0.17	-0.09	-0.02	0.08	0.11	0.12	0.12
11	-0.32	-0.33	-0.34	-0.34	-0.35	-0.36	-0.37	-0.40	-0.40	-0.36	-0.32	-0.23	-0.19	-0.17	-0.17
12	-0.31	-0.32	-0.32	-0.33	-0.33	-0.34	-0.34	-0.35	-0.31	-0.26	-0.19	-0.09	-0.06	-0.04	-0.04
13	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.31	-0.26	-0.18	-0.09	-0.01	0.10	0.13	0.15	0.15
14	-0.26	-0.26	-0.26	-0.27	-0.27	-0.27	-0.26	-0.24	-0.20	-0.15	-0.10	-0.03	-0.01	0.00	0.00
15	-0.31	-0.31	-0.31	-0.31	-0.31	-0.30	-0.30	-0.27	-0.23	-0.18	-0.14	-0.08	-0.06	-0.05	-0.05
16	-0.30	-0.30	-0.30	-0.30	-0.30	-0.30	-0.30	-0.27	-0.23	-0.18	-0.13	-0.07	-0.05	-0.04	-0.04
17	-0.31	-0.31	-0.31	-0.31	-0.30	-0.30	-0.29	-0.23	-0.16	-0.10	-0.05	0.00	0.02	0.03	0.03
18	-0.26	-0.26	-0.26	-0.26	-0.26	-0.26	-0.25	-0.19	-0.11	-0.02	0.06	0.16	0.19	0.21	0.21
19	-0.31	-0.32	-0.32	-0.32	-0.33	-0.33	-0.34	-0.35	-0.33	-0.29	-0.25	-0.17	-0.14	-0.12	-0.12
20	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.13	-0.10	-0.04	0.01	0.06	0.12	0.13	0.14	0.14

The correlation coefficient between physical and genetic distances was computed for each property for 15 different values of the mutation intensity *y*, ranging from $y = 0{\cdot}10$ to $y = 30{\cdot}0$ (which represents $y \cong \infty$). The results are presented in table 3.

As mentioned before, a large negative value indicates that the property is less sensitive to mutational changes. For low rates of mutation, the value of the correlation coefficient (r) is negative for *all* the properties. Since the number of degrees of freedom is nearly 400, any value of r less (more negative) than $-0{\cdot}103$ is significant at 1%

level by a one-tailed test based on Fisher's z transform. By this criterion, even up to a mutation intensity $y = 3$, nineteen of the 20 properties (table 3, all except property 20, viz. entropy of formation) show significant negative correlation. In fact, 10 of the 20 properties have r less than $-0{\cdot}20$. By $y = 7{\cdot}5$, however, only properties 11 (long-range nonbonded interaction energy) and 19 (absolute entropy) retain such low values of r.

Variation of r with y indicates two broad patterns. For most of the properties, r increased (became less negative) with increasing y. For some of the properties, e.g. 2 (protein environment or bulk hydrophobicity), 8 (chromatography index), 11 (long-range nonbonded energy) and 19 (absolute entropy), r decreased with increasing y initially, and then increased with further increase in y. The minimum value of r corresponded to y between 1 and 2·5 for these properties. It seems that these properties are more resistant to change at an intermediate level of mutation than at low levels! The initial decrease in r for these properties was, however, small in magnitude (e.g. 0·08 or 25% for property 11, long-range nonbonded interaction energy).

Table 4 shows the proportions of r values in the simulations that were smaller than the value obtained for the existing genetic code. A number less than 0·05 indicates a 95% level of significance. It is seen from the table that even for the low mutation rate $y = 0{\cdot}50$, for two of the properties (5, pK for COOH group; 17, power to adopt beta bend) a *majority* of the simulation r values were *less* than the actual value! When the mutation rate was 3·0, the following properties also gave r values with the same feature: 4, isoelectric point; 5, pK for COOH; 7, bulkness; 10, short- and medium-range nonbonded energy; 13, power to be at the N-terminal of alpha helix; 18, heat capacity; 20, entropy of formation from elements. In other words, though the z test suggested that these values are significant (table 3), the rigorous randomization tests revealed a more than even possibility of obtaining such low values by chance. This result underscores the necessity of exercising care in the choice of tests of significance.

One can infer from table 4 that at low levels of mutation rate, the genetic code conserves properties 2 (protein environment or bulk hydrophobicity), 8 (chromatography index), 9 (refractivity index), 11 (long-range nonbonded energy), 12 (protein environment total nonbonded energy) and 19 (absolute entropy). In fact, for even lower rates of mutation, ($y = 0{\cdot}001, 0{\cdot}005, 0{\cdot}01, 0{\cdot}05$; detailed results not shown), these are the only properties that show a negative correlation coefficient significant at 1% level. For high rates of mutation ($y > 4$), only property 11, long-range nonbonded interaction energy, is seen to be significant at 1% level. The result obtained by Sitaramam (1989) thus remains valid under this rigorous scrutiny.

In a similar study but with a different (and more approximate) formalism, Di Giulio (1989) examined 18 physicochemical properties of amino acids, and showed that polarity was a highly conserved property, followed by molecular volume. In a subsequent study of four properties, Haig and Hurst (1991), with minimization of the mean square error as the design criterion and a Monte Carlo technique for testing significance, obtained evidence against molecular volume as a conserved property, but confirmed the observation for polarity (polar environment) and demonstrated that hydropathy was also a highly conserved property. On the other hand, our results, based on what we believe to be a more rigorous measure of conversion probabilities, run counter to these in that polarity does not seem to be a conserved property. In fact, 25% or more of the randomly generated codes show a higher degree of conservation for polarity than the natural code. This point needs further, detailed scrutiny.

Table 4. Probability that the correlation coefficient for a property from a random genetic code is *less* than that obtained from the existing genetic code, for different values of mutation rate *y*.

y --> property	0.100	0.200	0.300	0.400	0.500	0.750	1.000	2.000	3.000	4.000	5.000	7.500	10.00	20.00	30.00
1	0.113	0.108	0.084	0.101	0.087	0.093	0.086	0.096	0.090	0.130	0.140	0.208	0.219	0.228	0.207
2	0.004	0.002	0.002	0.003	0.004	0.003	0.003	0.010	0.030	0.114	0.268	0.544	0.637	0.643	0.673
3	0.244	0.215	0.222	0.239	0.217	0.246	0.219	0.245	0.243	0.274	0.332	0.328	0.422	0.420	0.436
4	0.463	0.456	0.510	0.513	0.517	0.555	0.535	0.578	0.562	0.586	0.623	0.574	0.648	0.613	0.633
5	0.765	0.738	0.752	0.716	0.745	0.692	0.632	0.555	0.547	0.519	0.509	0.540	0.554	0.566	0.541
6	0.031	0.029	0.022	0.022	0.022	0.039	0.036	0.099	0.237	0.375	0.438	0.566	0.533	0.610	0.600
7	0.043	0.048	0.035	0.054	0.043	0.085	0.090	0.267	0.554	0.691	0.798	0.845	0.850	0.870	0.852
8	0.006	0.012	0.006	0.012	0.007	0.009	0.005	0.003	0.009	0.010	0.028	0.108	0.159	0.194	0.183
9	0.000	0.003	0.001	0.001	0.000	0.000	0.002	0.008	0.033	0.091	0.157	0.291	0.285	0.329	0.326
10	0.372	0.353	0.358	0.382	0.398	0.382	0.429	0.565	0.690	0.772	0.808	0.844	0.854	0.879	0.875
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.006	0.013
12	0.001	0.002	0.006	0.004	0.004	0.004	0.003	0.001	0.011	0.024	0.088	0.205	0.293	0.313	0.336
13	0.094	0.113	0.127	0.139	0.133	0.177	0.217	0.401	0.624	0.746	0.840	0.916	0.929	0.933	0.926
14	0.114	0.133	0.150	0.142	0.150	0.180	0.179	0.273	0.331	0.381	0.438	0.490	0.507	0.524	0.505
15	0.320	0.295	0.295	0.308	0.296	0.292	0.308	0.331	0.334	0.317	0.328	0.327	0.322	0.315	0.312
16	0.191	0.176	0.202	0.199	0.192	0.226	0.205	0.268	0.296	0.324	0.317	0.341	0.384	0.368	0.371
17	0.704	0.728	0.735	0.756	0.748	0.767	0.769	0.814	0.823	0.793	0.736	0.689	0.659	0.650	0.654
18	0.354	0.395	0.381	0.436	0.445	0.511	0.543	0.703	0.824	0.876	0.905	0.909	0.928	0.937	0.941
19	0.010	0.002	0.003	0.008	0.001	0.004	0.004	0.005	0.003	0.005	0.019	0.066	0.079	0.095	0.082
20	0.293	0.326	0.353	0.418	0.415	0.528	0.559	0.740	0.761	0.770	0.797	0.816	0.802	0.816	0.811

We have examined three more aspects of these results.

(i) Since our main interest is in looking at change in a physical property when there is a change of amino acid, it may be worth looking at the correlation coefficient for the 380 pairs of values of genetic and physical distances, omitting the 20 cases where an amino acid does not change ('conversion to itself'). The results of such an analysis show that even for a low mutation rate of $y = 0{\cdot}1$, and even by Fisher's z transform-based test, 12 of the 20 values of correlation coefficient are not significantly different from zero. Properties 11 (long-range nonbonded energy) and 19 (absolute entropy) continue to have large negative values of r, however.

(ii) In addition to the correlation between the genetic distance (G) and the physical distance (D), we have examined the correlations between log(G) and D, G and log(D), and log(G) and log(D). All four correlation coefficients have by and large similar values. Here too, a large negative correlation coefficient is obtained only for the two properties 11 and 19 over the entire range of mutation rate examined.

(iii) For each property, one can compute the correlation coefficient between the genetic and physical distances for each amino acid separately, from the 19 values of probability of conversion to other amino acids and the 19 values of the physical distances from the other amino acids. For each property, one obtains 20 values of r, one for each amino acid (Di Giulio 1989). A property that is conserved by the genetic code should have all or most of these 20 values negative. We see that long-range nonbonded energy shows a negative correlation for at least 19 of the 20 values over the entire range of mutation rate. The only exception is tyrosine—perhaps a hint that it may not have been well integrated into the genetic code.

The approach adopted here can be extended in several ways. Clearly, a larger number of properties of amino acids should be examined. Secondly, one can try to unravel the logic of the genetic code in the two-letter code itself. Thirdly, a finer analysis may assign different relative probabilities of mutation for transitions and transversions. These investigations are in progress.

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