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# Space Time and Motion

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## 1. Introduction :

It was Poincare who stated that of all the changes which a material body can undergo, the simplest are the changes of position. Now position of a body is given in space, there is some lapse of time as the body undergoes a change in its position and whenever a body changes its position, we say that it is in motion. Thus space, time and motion are intricately interrelated. The history of Science records many painstaking efforts by Physicists, Mathematicians and Philosophers at a correct understanding of this relationship. Einstein's deep analysis of this relationship is at the root of his special theory of relativity of 1905 which opened up the way for the rapid development of Physics in the present century.

In what follows, we shall try to understand some of these basic notions of this relationship between space, time and motion as developed by Einstein.

## 2. Rest and Motion :

I am placing this duster on the table. Now I put myself this question : Is this duster at rest or is it in motion ? I see that it is at rest. But let us think a little and analyse this <sup>~</sup> motion of rest. The duster is at rest because its distances from the two edges  $ox$  and  $oy$  of the table do not change.

But what about the table ? Is it at rest ? The same reasoning can be applied again and one can say that the table is at rest because its distances from the walls of the room and from the floor of the room do not change. But what about the room ? The room is in the building built on solid earth. And what about the earth ? Well, the earth is moving round the Sun ! So what ! The earth is moving and with it the room is moving and with the room the table and the duster are moving !! We come back to our original question. Is the duster at rest ? or is it moving ? The analysis that we have employed above shows that it is not possible to give an unambiguous answer to the above question. One can always give a conditional answer like the following :

If you are on the earth, the duster appears to be at rest to you and if you are on the Sun, the duster appears to be moving ! Thus whether a body is at rest or in motion depends on the observer. Motion or rest are relative terms and a body may be at rest as seen by one observer, but it may be in motion as seen by some other observer.

3. Principle of Relativity :

This relative (not absolute) character of motion is known to thinkers, since the early days of history. Now science essentially deals with observations of Nature and with the laws governing natural phenomena. As motion has no absolute character, motion of an observer should have no effect on his observations of Nature. A phenomena of Nature should be describable in identical terms by two observers A and B irrespective of the

motion that either of them may have relative to the other.

Conversely, the lack of absolute character of motion or rest leads us to a criterion to judge whether a certain phenomenon is a phenomenon of Nature or no. If the phenomenon can be described by two observers A and B in a form which does not depend on the relative motion of the two observers then the phenomenon is a Natural phenomena and its description will find a place in science. This criterion is known as the Principle of Relativity and is known since the early days of science. But this criterion requires the testing of observations of Nature and our instruments of observations become sharper and sharper as the science advances. Thus every ~~new~~ new advance in science necessitates a fresh application of this criterion.

We shall now see how such a necessity arose at the end of the last century which *led* to a rethinking on the relationship between space, time and motion.

4. Propagation of Light :

Sound and Light are the two messengers of Nature. Many attempts have been made to understand the mechanism of propagation of light in all directions from its <sup>z</sup>source. Newton's corpuscular theory which postulated that light travels like bullets fired from a gun, was disproved by experiments and by the middle of nineteenth century *y* Huygen's theory of wave propagation of light was well established. According to this theory a source of light produces light waves which move outwards from the source in a way similar to water waves propagating in still water. *5*

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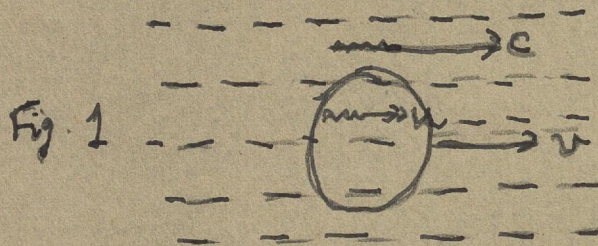
But then the question arose that a wave is produced and propagated in some medium, in what medium are the light waves produced and propagated ? This medium cannot be air or any material medium because there is no such material medium in <sup>ter</sup>instellar space and light from distant stars does reach us ! Whenever a scientist feels that some object or agency must exist but he does not know what it can be, he begins in his time-honoured way to formulate a problem . ' Suppose the object or agency exists. Call it x. Now let us find this x '. Physicists assumed that there is an all pervading medium in the universe and light propagates ~~as~~ waves in that medium. This medium was not called x but the name ~~ether~~ was given to it. The problem was to find x i.e. to think out and set up experiments to find properties of this postulated ether

#### 5. The Michelson-Morley Experiments

The vast interstellar space can be looked upon as an ocean of ether. And in this vast ether-ocean the earth and other planets move like fish in the sea. This comparison leads one to a number of questions like the following :

(1) The sea-water offers resistance to the motion of the fish Does ether offer any resistance to the motion of planets ? (2) Motion of fish and rapidly moving steam-ships produce temporary local disturbances in the sea. Will the motion of planets through the ether ocean produce similar disturbances in ether ? It is obvious that the first step in trying to answer these and several other such questions is to measure the velocity of some 'fish' moving through the ether-ocean and what other fish is easily assessible to us than our own Earth gliding through ether !

Michelson and Morley thought out an experiment to measure the velocity of earth through ether. It is easy to understand the principles underlying their experiment. Assume that the earth moves in ether with a velocity  $v$ . A ray of light has velocity  $c$  in ether. What will be its velocity as seen from the earth? Of course this will depend on the direction of light ray vis-a-vis the direction of earth's motion. If we call this velocity of light as seen from the earth as  $u$ , the simple, commonsense principle of relative velocities can be used to find  $u$ . When the light ray is moving in the same direction as the



earth it is clear from figure 1 that

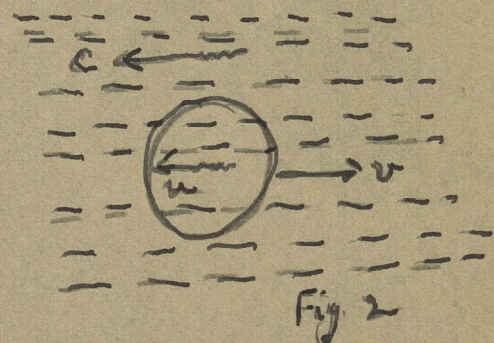
$$c = v + u \quad \text{or} \quad u = c - v$$

Similarly when the direction

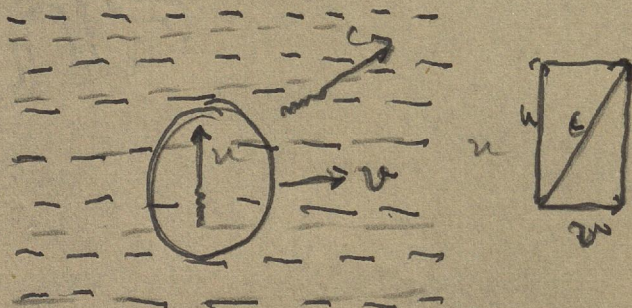
of light ray is opposite to the direction of earth's motion a reference to figure 2 show that

$$c = u - v \quad \text{or} \quad u = c + v$$

The case when light ray moves in a direction perpendicular



to the earth's motion is not that simple. One has to use Pythagoras



theorem to find  $u$ .

Reference to figure 3, <sup>Shows</sup> show

that the

$$u^2 + v^2 = c^2$$

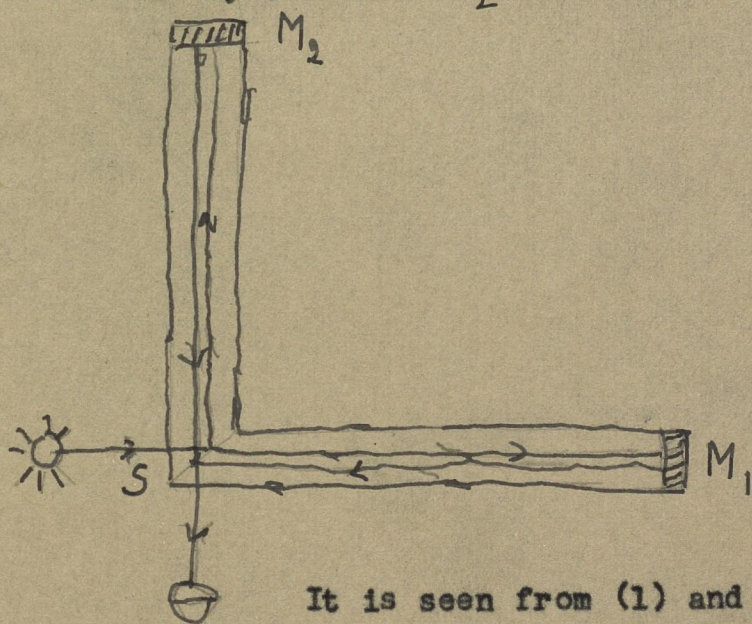
$$\text{or} \quad u = \sqrt{c^2 - v^2}$$

The simple mathematics given above is sufficient to understand the principles behind the M.M. Experiment. In this experiment a ray of light from a source S is sent in the direction of earth's motion

through a distance  $l$ , is reflected by a mirror  $M_1$  and returns to the starting position  $S$ . The time taken by this ray for its onward journey  $SM_1$  will be  $\frac{l}{c-v}$  and the time for its return journey  $M_1S$  will be  $\frac{l}{c+v}$ . Thus the total time  $t_1$  for to and fro journey in the direction of earth's motion will be

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2} = \frac{2l/c}{1-\frac{v^2}{c^2}} \quad (1)$$

Another ray of light is sent from  $S$  in a direction perpendicular to earth's motion. After traversing a distance  $l$  this ray is also reflected by a mirror  $M_2$  and retraces its path to reach the source  $S$ .



The time taken for to and fro journey at right angles to the earth's motion can be calculated to be

$$t_2 = \frac{l}{\sqrt{c^2-v^2}} + \frac{l}{\sqrt{c^2-v^2}} = \frac{2l}{\sqrt{c^2-v^2}} = \frac{2l/c}{\sqrt{1-\frac{v^2}{c^2}}} \quad (2)$$

It is seen from (1) and (2) that  $t_1 \neq t_2$ . Aim of the M.M.

experiment was to measure this time-difference and thus get an equation to determine  $v$ .

A stone was floated in a pond of mercury. Two mutually perpendicular arms, <sup>each</sup> of length  $l$  were fixed on the stone. The floating stone was arranged in such a position that one of the fixed arms will point in the direction of the earth's motion. The problem then reduced to sending a beam of light along one arm and getting it back by

reflection from  $M_1$  at the same time sending another beam along the other arm and getting it back and measuring the difference in the time <sup>taken</sup> ~~take~~ by the two. Well it is easy to give such a prescription but it is not so easy to carry it out. Firstly, in order to observe these light beams, the whole experiment must be performed in a dark room. And again the velocity of light is so large ( $3 \times 10^{10}$  cm/sec) that in order to get a measurable time difference the length of each arm would have to be enormously large (larger than the diameter of the earth!). But then there is a simplifying circumstance. Light is propagated in waves. In waves we have crests and troughs.

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Now if the times taken by light to travel along the two arms are different, it is possible in such a way to arrange matters that when a

crest of the first beam reaches us the second beam presents its <sup>trough</sup> trough to us and so when the two beams mix together they give zero oscillation to ether! The two beams together will then lead to darkness!! Michelson and Morley decided to measure the time difference by such a method of interference of the two beams of light.

They performed the experiment. What was the result of this experiment? They found that the two beams took the same time to travel along the two perpendicular arms! In terms of our formula they found that

$$t_1 = t_2$$

or

$$\frac{2l/c}{1 - v^2/c^2} = \frac{2l/c}{\sqrt{1 - v^2/c^2}} \tag{3}$$

But how can equation (3) hold?

6. The negative result.

The above equation (3) implies that  $1 - v^2/c^2 = \sqrt{1 - v^2/c^2}$ . Which number can be equal to its own square root ? 0 and 1 are two such numbers. But in (3)  $1 - v^2/c^2$  comes in the denominator, so it cannot be 0. Therefore it must be 1. Thus  $1 - v^2/c^2 = 1$  or  $v = 0$ . The velocity of the earth is zero. The earth does not at all move in ether. But nature should not show any preference to earth and so we expect that the velocity in ether of any planet will be zero. This is really funny. All planets are at rest in ether.

Many attempts were made to explain this negative result of M.M. experiment. We shall note <sup>one</sup> here ~~are~~ such attempt by Lorentz. (He obtained a correct result in an incorrect way.) He argued that light took the same time to travel along the two arms because though we took the length of the two arms to be equal they were not really equal. The arm in the direction of earth's motion had actually become a little shorter. Suppose that ~~is~~ its length was  $l'$  then the equality of  $t_1$  and  $t_2$  would give

$$\frac{2l'/c}{1 - v^2/c^2} = \frac{2l/c}{\sqrt{1 - v^2/c^2}} \quad \text{i.e.} \quad l' = l\sqrt{1 - v^2/c^2}.$$

This last equation gives the length of the contracted arm. But why should the arm in the direction of earth's motion contract in length ? Well, because it is only on that assumption that one can explain Michelson-Morley's experiment.

It is clear that such assumptions and explanations are not satisfying. It was Einstein who first realised that the reasons for the negative result of this experiment were deep-rooted. They

were to be found in our faulty concepts of space, time and motion.

We had written down the following three simple relation between  $c$ ,  $u$  and  $v$  by looking at figure 1,2 and 3.

$$c = u + v, \quad c = u - v, \quad c^2 = u^2 + v^2.$$

They, (at least the first two of them) follow from 'common sense' <sup>or the Newtonian notion</sup> notion of space, time and velocities. Einstein analysed these 'common sense' notions and found that they were logically faulty and then he set out to ~~see~~ revise them.

7. Measurement of Time :

We shall try to follow Einstein. The first step is to forget about all 'common sense' notions about space, time and motion and begin with a clean slate. One thing is certain. The basic concepts are of those of space and time and the concept of motion or velocity is <sup>derived</sup> ~~frtived~~ from these and so depends on them. Thus the concepts of distance-  
<sup>derived</sup> measurements and time-measurements should be first formulated in an independent manner.

As we have seen earlier, the Newtonian concept of time was that of a 'true even flowing time' the same for all observers. According to this concept to measure time difference between any two events is similar to measuring space-distance between any two places on a high-way. We have mile-stones fixed on a highway. <sup>If</sup> a place A is at the mile-stone 6.7 and a place B at 7.9 then the distance AB is  $7.9 - 6.7 = 1.2$  miles. Well, similiary according to Newton there is a highway of time along which time-stones are fixed. <sup>An</sup> event X happened at 1946.82 and an event Y happened at 1967.93. Then the time-interval between the two events is 11.11 year $\Delta$ .

Einstein tells us to forget about the above analysis and try to think out a logical way of measuring this time interval. We know that time is measured by clocks. If an event X happens at A at

clock-time  $t_1$  and another event ~~X~~<sup>Y</sup> happens at A at the time  $t_2$ , then the time interval ~~at~~ between the two events is  $t_1 - t_2$ . This seems to be quite sound.

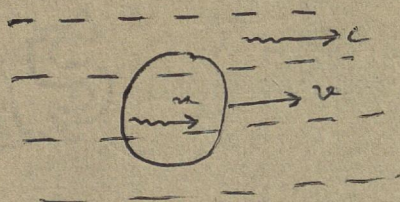
Now suppose the two events X and Y happen at two different places A and B. Then we must have a clock at A and a clock at B. Suppose the two events are again at  $t_1$  at A and  $t_2$  at B. Then is the time-interval between the two events  $t_1 - t_2$ ? Well, it will be so provided the two clocks keep the 'same time'. But how can one ~~assume~~ assume that the two clocks at different places keep the same time? Well, one can take the clock at B (say) and move it to A and check by one-self whether the two clocks keep the same time or not. But this requires 'moving' one clock. Concept of motion is to be derived from the concept of time measurement, but we need 'motion' in order to define time at two different places. This is something like moving in a circle.

One can think of several methods of comparing the time kept by two clocks at different places but in all these methods one will always have either to 'move' one of the clocks <sup>or</sup> to 'move' ~~an~~ an agent from one place to another. In order to define motion you need to measure time-difference between events at different places and in order to measure this time-difference, you need the concept of motion.

Einstein realised that the <sup>cl</sup> due to the negative result of Michelson - Morley experiment lay hidden in this <sup>or</sup> vicious circle. To break this vicious circle Einstein suggested a very simple way out. In order to make the measurement of time-interval between two events at two different places logically meaningful, we must preassume (i.e. assume in advance) a fundamental agent moving with a fundamental velocity. All motion <sup>con</sup>cept the motion of this fundamental agent (F.A.) is

derived motion. Then one can use this F.A. for comparing clocks at two different places and the method of measuring time becomes logically consistent.

Using this additional concept of a F.A. Einstein built up a logically consistent Dynamics and found that the result of Michelson Morley experiment leads one to conclude that this F.A. is light and that its velocity  $c$  in ether is the fundamental velocity. In order to see this more clearly we recall our figure 1. There we wrote down



the following :

" It is clear from figure 1 that  $c = v + u$  or  $u = c - v$  "

Our argument was the ' common sense ' arguement. But according to Einstein's analysis of the logic of these concepts we would get from the same figure  $c = \frac{v + u}{1 + uv/c^2}$ . Now this is not <sup>at</sup> all 'clear from figure 1'. When our equation was  $c = v + u$  we got  $u = c - v$  very easily . Let us now solve Einstein's equation

$$c = \frac{u + v}{1 + uv/c^2}$$

to find  $u$  . If we do it, ( it is simple algebra ! ) we shall find that  $u = c$  . So here lies the funniness of the result  $v = 0$  of M.M. experiment. The unlogical Newtonian dynamics led to the result  $u = c - v$  but the logical Einsteinian dynamics requires  $u = c$  . No wonder therefore that the interpretation of Michelson - Morley experiment on the logically faulty Newtonian dynamics will lead to the funny conclusion  $v = 0$  , - the planets are at rest

8. Concluding Remarks :

The added sharpness of our instruments of observation as represented by the interferrometer technique of Michelson and Morley

necessitated one more attempt to revise our concepts of space, time and motion. The deep probing by Einstein into the logical consistency of the then-prevalent ideas of time-measurement revealed great ~~crucies~~ <sup>crevices</sup> and thus he started on a course of reformulating these concepts. As a result of this reformulation, amongst other things, he found that Lorentz was right in saying that a moving rod gets shorter in the length in the direction of motion. Einstein proved from his logically consistent dynamics that

- (1) lengths of objects are not absolute, but depend on the motion of the observer measuring them .
- (2) Time interval between two events also depend on the motion of the observer measuring this interval.

He went much beyond this and deduced the new famous equation  $E=mc^2$  ~~E=mc^2~~, but this does not come under the subject of the present ~~article~~ <sup>talk</sup>.

The advance in experimental techniques led to a probing into the foundations of the theoretical structure. This in its turn gave rise to a series of conclusions which need experimental verification . This would naturally lead to a further sharpening of our instruments of observation and so the *caravan* of science moves on and on , its speed of growth developing in an exponential manner.

