Gravitation

Rough

1. Introduction.

Newton's law $\gamma \frac{mm'}{r^2}$ or $\varphi = \frac{m}{r} = \int \frac{\rho\, dV}{r}$

Nothing had ~~a~~ happened to necessitate a revision of this law.

Though many reasons had arisen for doubting basic Newtonian Concepts.

2. Special Theory:

Two basic postulates

(1) ~~That~~ Physics to be invariant with respect to a set of observers in uniform translatory motion.

(2) Basic ~~A~~ geometric structure is the homogeneous Euclidean ~~for~~ 4-space

~~This~~ Because of uniform motion of fundamental observers no force field could be infused into the system. ~~The~~ The theory supplied a back grand universe to ~~be~~ ~~grounder~~ which various natural forces had to work ~~submit~~. ~~A~~ Particles, charges, forces were extraneous agents to be superposed ~~introduced~~ over this back grand

3 How to Generalise this theory?

How about a set of observers with any ~~gen~~ arbitrary general relative motion! Since this set is an integral part of the system, the cause which gives

Rise to this arbitrary relative motion must also be an ~~integral~~ internal ~~p~~ ingredient of the theory. It is clear that a ~~covariant~~ physical theory covariant under any general non uniformly moving observers must ~~be a~~ contain in its structure the description of a natural force.

Einstein proceeded to generalise his special theory by ~~generalising~~ stipulating

(1) Invariance under arbitrary transformations

(2) Background non Euclidean.

He restricted non Euclidean background to Riemannian background and found that natural force which his theory described was gravitation.

Naturally he was led to believe that if the Non Euclidean background was not chosen as Riemannian But non Riemann non Euclidean, he would get some other force of nature. This explains various attempts at so called unified field theories

This explains why the so called "General" ~~4.~~ Theory of Relativity is essentially a theory of Gravitation.

4. Description of Einstein's theory of gravitation:

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

$$R_{ij} - \frac{1}{2} g_{ij} R = \kappa T_{ij} \qquad \kappa = -\frac{8\pi G}{c^4}$$

In space not occupied by matter

$R_{ij} = 0$.

Newtons' theory as a first approximation

Rigorous static solution of $R_{ij} = 0$ ~~was~~ of Schwarzschild's solution — Perhaps the only exact solⁿ which has been generally useful. Three crucial tests.

~~Newton's theory as a first approximation~~

Now the structure of the field equations contain a feature which gives Special Relativity as a special case. This is done through the concept of osculating Euclidean plane of an ~~ra~~ Riemannian Space and the Principle of Equivalence —

Thus the principle that no ~~any~~ physical effect can be propagated with a velocity $> c$ is ~~being~~ organically incorporated in the gravitation theory. The gravitational effects must therefore be propagated with a finite velocity.

The first non-Newtonian feature

~~Eins~~ The question arises: Do gravitational waves exist?

Mathematically the answer is obtained if we find ~~an undulatory~~ regular solutions of

$R_{ij} = 0$ which asymptotically ~~regular everywhere and joined with a~~ tending to spherical waves at large distances ~~and they~~ have ~~regular~~ no singularities! They must be continuously ~~joined with a regular~~ joined to a solution of $R_{ij} - \frac{2}{2} g_{ij} R = -\frac{8\pi G}{c^2} T_{ij}$ in the region containg the sources.

This programme is unfulfilled.

Einstein gave an approximation method by which the theory was linearized

$$g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$$

$$\phi_\mu^\nu = \delta_{\mu\tau} h_{\mu}^{\nu} - \frac{1}{2}\delta_\mu^\nu h_\alpha^\alpha$$

$$\Box \phi_\mu^\nu = \frac{-16\pi G}{c^2} T_\mu^\nu$$

$$\phi_{\mu,\nu}^\nu = 0$$

A derivation of this can be found in any standard Text book. An excellent derivation is given by Einstein and Rosen J. Franklin Inst. 223 43, (1937).

Pauli and Fierz P.R.S. 173, 211 (1939) showed that the wave equation satisfied by a particle of zero rest mass and spin 2 is exactly

$$\Box \phi_\mu^\nu = 0$$

$$\phi_{\mu,\nu}^\nu = 0$$

A plane wave solution

$$g_{22} + g_{33} = 0, \quad g_{23} = 0$$

$$\text{or} \quad g_{22} = g_{33} = 0 \quad g_{23} \neq 0$$

This is the 1st approximation of what?

Fock 1951 Text book gives a second approximation

Trautman used Synge argument to show that use of energy pseudo tensor t_μ^ν in connection with 1st approximation is faulty. Because this t_μ^ν is just $-T_\mu^\nu$ of the Synge argument tensor of 1st approximation

How do we know that gravitational waves do exist and are propagated with the fundamental vel.?

Lichnerowicz worked out the characteristic surfaces of the equations $R_{ij}=0$. These are surfaces over which g_{ij} second derivatives of g_{ij} are discontinuous.

If coordinates are so chosen that a surface of discontinuity Σ gets the equation $x^4=0$, then g_{ij}, $g_{ij,k}$, $g_{ij,\alpha\beta}$, $g_{ij,\alpha 4}$ are continuous.

$g_{ij,44}$ is discontinuous.

$$\left.\begin{aligned} R_{\alpha\beta} &= \tfrac{1}{2} g^{44} g_{\alpha\beta,44} + F_{\alpha\beta} \\ R_{\alpha 4} &= -\tfrac{1}{2} g^{4\beta} g_{\alpha\beta,44} + F_{\alpha 4} \\ R_{44} &= \tfrac{1}{2} g^{\alpha\beta} g_{\alpha\beta,44} + F_{44} \end{aligned}\right\}$$

so $R_{ij}=0$ do not determine second derivatives uniquely because $g_{i4,44}$ do not occur at all

~~If~~ but we must take essential discontinuities.

Coordinates can be chosen (Gaussian coords. $g_{\alpha 4,4}=0$, $g_{44}=\pm 1$) so that $g_{i4,44}=0$ so we are interested only in 6 $g_{\alpha\beta,44}$

If the above equations are not to determine
$g_{\alpha\beta,hh}$, we must have

$$\bar{g}^{hh} = 0$$

$$\therefore \quad g^{ij} f_i f_j = 0$$

If $f = 0$ ~~and~~ Σ is a null surface

e

II

1. <u>Recapitulation</u>: ~~A~~ A Reasonably acceptable approximate solutions needed

   1. Linearised theory cannot be trusted.

   2. ~~Are~~ Do approximate solutions really correspond to some exact soln. or are they spurious?

2. Exact Solutions : : No spherically symmetric solution exists.

   ∴ Cylindrical Symmetry.

   <u>Einstein – Rosen Solution</u>

<u>Symmetry requirements.</u>

(1) Field symmetrical about any plane $x^2 =$ constant

(2) . . . $x^3 =$ constant

∴ $g_{12} = 0$, $g_{32} = 0$, $g_{42} = 0$

$g_{13} = 0$ $g_{43} = 0$

Single 2 & single 3 cannot occur

$x^1 = 0$ axis of rotation

x^1 runs from 0 to ∞

(x^1, x^2) meridian plane

x^3 angular coord.

~~Field symmetrical about any $x_2 =$ const~~

∴ g_{11}, g_{22}, g_{33}, g_{44}, g_{14} exist

<u>Coordinate conditions</u> x^1, x^4 such that

$$g_{11} = -g_{44}, \quad g_{14} = 0$$

$$R_{ij} = 0 \Longrightarrow \sigma_{11} - \sigma_{44} = 0 \quad \sigma = \cancel{\ell}\,(g_{22} \cdot g_{33})^{1/2}$$

What sort of transformations in (x^1, x^4) plane leave the system invariant ?

$$\frac{\partial \bar{x}^1}{\partial x^1} = \frac{\partial \bar{x}^4}{\partial x^4}, \qquad \frac{\partial \bar{x}^1}{\partial x^4} = \frac{\partial \bar{x}^4}{\partial x^1}$$

condition of consistency is

so that $\dfrac{\partial^2 \bar{x}^1}{\partial (x^1)^2} - \dfrac{\partial^2 \bar{x}^1}{\partial (x^4)^2} = 0$

$$\therefore \quad \sigma = a x^1 .$$

$$\beta_{11} - \beta_{44} + \frac{\beta_1}{x^1} = 0$$ — Eqn for cylindrical waves in 3-space

$$\alpha_1 = \tfrac{1}{2} x^1 (\beta_1^2 + \beta_4^2) - \frac{1}{2x^1}$$

$$\alpha_4 = x_1 \beta_1 \beta_4$$

$$\beta = \tfrac{1}{2} \log \left(\frac{g_{22}}{g_{33}}\right)$$

A Einstein and N. Rosen : J. Franklin Institute $\underline{223}$, 43, (1937)

No unambiguous way of loss of mass can be ascertained

The source being an infinite rod, it looks as if mass ~~to~~ is being pumped in at infinity.

In addition to cylindrical waves, several ~~plane fronted waves~~ exact solutions giving plane fronted waves have ~~been~~ obtained.

3. ~~4~~ Plane waves: Rosen 1937 proved that plane waves do not exist. How !

$$e^{2\beta} = \frac{g_{22}}{g_{33}} \qquad g_{22} g_{33} = r^2$$

$$e^{2\beta} (g_{33})^2 = r^2$$

$$e^{\beta} g_{33} = r e^{-\beta}$$

$$g_{22} = r e^{-\beta} = r^2$$

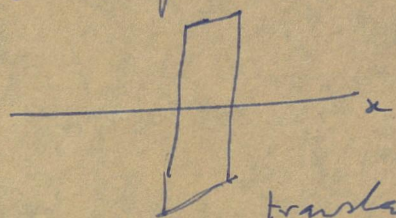
$$g_{--} \quad r e^{\beta}$$

$$ds^2 = - e^{\alpha} \left[(dx^1)^2 - (dx^4)^2 \right] +$$

Though it may not be proved, it is easy to ~~show~~ see since by tying to put ~~β=0~~ $g_{22} = g_{33}$, β is — static

Bondi and Pirani PRS 251, 519–533, 1957

defined plane symmetry as the same symmetry as that possessed by plane electromagnetic waves viz. admitting a 5-parameter group of motion



3-parameter group ~~taking oy unaltered, oz unaltered~~ translations along oy, translation along oz

and translations along ~~taking~~ t−x = const. ~~into themselves~~ taking t−x = const. into themselves

There are two null rotations which ~~leave~~ take t−x = const. into themselves. Shibata: J. Sc. Hiroshima Univ. 19, 101–112

Several exact solutions ~~were~~ have been known which are plane waves in this sense

Takeno H 1961 Scientific Reports of Research Inst. of Theo. Phys. Hiroshima Univ. no. 1

4. Bondi's metric: Pandya's derivation

Cylindrical and plane waves: Do they displace important physical characteristics of physically significant waves? General Relativity is a "peculiarly complete theory" and may ~~not~~ give misleading results for situations too far removed from physically reasonable. What about waves from bounded sources?

Spherical symmetry not permitted by Birkhoff.

1 Bondi worked out ~~an~~ case for axi-symmetric isolated system

Pandya worked out the case for an isolated system

Sachs 1962 " " the same case without any spatial symmetry

Bondi Burg Metzner PRS 269, 1962

Sachs P.R.S. 270, 103, 1962

Robinson Trautman 265, 463, 1962

P ~~Van~~ Pandya & Vaidya Proc. Nat. Inst. Sc A27, 620, 1961 ~~A22, 460, 1960~~
1963 Prog. Theor. Phys. 35, 129, 1963

5. Derivation of metric :

$v^\mu \sim (v^1, 0, 0, 0)$.

v_μ hypersurface orthogonal $(0, 0, 0, v_4)$

$\therefore$ ~~...~~ $v_\mu = g_{\mu\alpha} v^\alpha$ $\quad g_{11} = 0,\ g_{12} = 0,\ g_{13} = 0.$

Coordinate condition

$x^2 x^3$ such that $g_{22} = g_{33}$ $g_{23} = 0$ cf. Einstein Rosen

$$\therefore ds^2 = -B\left[(dx^2)^2 + (dx^3)^2\right] + C(dx^4)^2 + 2H\,dx^1 dx^4 + 2\lambda\,dx^2 dx^4 + 2\mu\,dx^3 dx^4.$$

~~It is possible to redu~~

$R_{11} =$ ~~$R_{22} = R_{33} = 0$~~ $R_{12} = 0,\ R_{13} = 0,\ R_{22} = R_{33} = 0\ R_{23} = 0.$

along with a transformation which leads to

$H = 1$ Shown by Bondi et al NOT done by us.

We assume $H = 1$.

$R_{22} + R_{33} = 0$

~~It is~~ $R_{11} = 0$ on $(u = c)$ hypersurface

$R_{12} = 0$ B, λ, μ
$R_{13} = 0$

6. News functions.

$R_{11} = 0 \longrightarrow$ introduces α ($\alpha x^2, x^3, x^4$)

γ $\quad$ " (x^2, x^3, x^4).

$R_{22} = 0$ $R_{13} = 0$ introduce ξ, η

$R_{44} = 0$ a relation between them

$R_{11} = 0$.

$R_{12} = 0, R_{13} = 0, R_{22} = 0$ $R_{23} = 0, R_{33} = 0$

These equations hold on the hypersurface t = constant. If we know the situation on a hypersurface, we easily get the complete knowledge of the outgoing waves. If any any new information is to be obtained it is through the arbitrary functions introduced. There are two arbitrary functions in the soln. These are news functions.

The solⁿ exhibited turns out again to have a singularity going to infinity and so - . .

$$ds^2 = -e^{\lambda} dr^2 - e^{\mu} r^2 d\Omega^2 + e^{\nu} dt^2 + 2a\,dr\,dt.$$

Two restrictions.

1 $\quad v^{\mu} = (0, 0, 0, v^4)$.

2. $\quad g_{14} = 0$.

$$\therefore ds^2 = -e^{\lambda} dr^2 - e^{\mu} r^2 d\Omega^2 + e^{\nu} dt^2.$$

$$= -e^{\lambda} dr^2 - R^2 d\Omega^2 + e^{\nu} dt^2 \qquad R = R(r, t)$$

$$\frac{\left(1 - \frac{2m}{R} + 2\frac{\dot R}{\dot u}\right) R'^2}{\left(1 - \frac{2m}{R} + \frac{\dot R}{\dot u}\right)^2}$$

$$e^{-\lambda} = \frac{\left(1 - \frac{2m}{R} + \frac{2\dot R}{\dot u}\right)^{-\frac{1}{2}}}{\dot u}$$

$$e^{-2\lambda} \dot R^2$$

$$T_{\mu\nu} = (\rho + p) u_{\mu} v_{\nu} - p g_{\mu\nu} + q k_{\mu} k_{\nu}$$

$$= M_{\mu\nu} + E_{\mu\nu}$$

Then $M_{\mu}{}^{\nu}{}_{;\nu} \neq 0$.

[C is the cooling rate per unit ...

If nC is the cooling rate per particle]

[C - cooling rate per particle]

$C = C(T, n)$

$(n v^{\mu})_{;\mu} = 0$

$$u^{\mu} M_{\mu}{}^{\nu}{}_{;\nu}$$

$$+ nC =$$

$\rho = n + ne$ $\quad$ e is the specific internal energy ~~that does~~ over and above the rest energy

$$e_{,\mu} v^{\mu} = -C - p\left(\frac{1}{n}\right)_{,\mu} v^{\mu}.$$

$$u^{\mu} (E_{\mu}{}^{\nu}{}_{;\nu}) = -nC$$

If $e^{\lambda} = \left(1 - \frac{2m}{R} + \dot{R}^2 e^{-2\ell}\right)^{-1} R'^2$

then

$$-8\pi p R^2 \dot{R} = 2\dot{m}$$

$$8\pi \rho R^2 R' = 2m'.$$

Hence Pondunas.

oving Coordinates in Spherical Symmetry.

$$\nu^4 = \frac{1}{\gamma + V}$$

D

17/5/66

$$R_{44} = + \frac{2\,dm/dr}{r^2}$$

$$-8\pi q = \nu^4 \nu^4 R_{\mu\nu} = (\nu^4)^2 \left(\frac{-2\,dm/dr}{r^2}\right)$$

$$q = -\frac{dm/dr}{4\pi r^2}(\nu^4)^2$$

$$\lim_{r \to \infty,\, V=0} (4\pi r^2 q) = -dm/dr$$

$$L = 4\pi r^2 q$$

$$L_\infty = L(\gamma + V)^2$$

Lindquist, Schwartz and Misner

$$\cancel{k} \quad k^{1'} = \frac{\partial x^{1'}}{\partial x^\alpha} k^\alpha = k^1 = e^{-\lambda/2} = \sqrt{1-\frac{2m}{r}}$$

$$k^{4'} = \frac{\partial x^{4'}}{\partial x^\alpha} k^\alpha = x^{4'}_{,1} k^1 + x^{4'}_{,4} k^4 = 0.$$

$$k_{4'} = g_{4'\alpha} k^\alpha = g_{4'1} k^1$$

$$= \sqrt{1-\frac{2m}{r}}$$

For radially moving observers:

$$\cancel{\text{tho}} \quad \cancel{U = v^1, \quad v^4} \qquad V = \frac{dr}{dx} \qquad v^4 = \frac{du}{dx}$$

$$\cancel{v^1 +} \quad v_\mu v^\mu = 1$$

$$g_{\mu\nu} (v^\mu) v^\nu = 1 \qquad 2 v^1 v^4 + \left(1-\frac{2m}{r}\right)(v^4)^2 = 1$$

$$(v^4)^2 \left(1-\frac{2m}{r}\right) + 2Uv^4 - 1 = 0$$

$$v^4 = \frac{-U \pm \sqrt{U^2 + 1 - \frac{2m}{r}}}{1-\frac{2m}{r}}$$

$$= \left(1-\frac{2m}{r}\right)^{-1} (\gamma - U) = \frac{1}{\gamma + U}$$

Petrov Classification
~~What is Petrov's classification~~

1 Why classify? Pirani's 1957 Paper

Invariant formulation of Radiation Criteria

A null vector must be associated.

One of the ~~Eigen~~ Eigen vectors was null

∴ Radiation

2. Eigen Vector classification

3 Null vector Criteria of Debever

4 Spinor Criteria for Classification

So what?

What is Petrov's classification?

Equations $R_{ij} = 0$ are very complicated non-linear differential equations. It is next to impossible to obtain general solutions of these equations. Petrov wanted to see if ~~the~~ all possible solutions of these differential Equations could be set out into distinct classes. What he actually classified was the tensor R_{abcd} ~~under when~~ $R_{bc} = 0$

It turned out that ~~this~~ his method of classification associated certain vectors with R_{abcd} and in some of his classes these vectors were null vectors. Since gravitational radiation travels along null directions, Pirani saw an invariant link between such classes and existance of gravitational radiation in his pioneering paper of 1957. This led to the great interest shown by ~~Physicists~~ Relativists in this mathematical work of Petrov.

Ultimately it turned out that things are not as simple as Pirani thought in 1957 and that in short. Petrov's method of classification does not lead to any invariant def. of grav. radiation — . Still the study of classification has received a permanent place in the theory of gravitational radiation as we shall see in this lecture.

2. ~~Eigen vector~~ Matrix method of classification

J. L. Synge Comm. Dubl. Inst. Adv. Studies A, 15, 1964

Properties of

$$R_{abcd} = \tfrac{1}{2}\left(g_{ad,bc} + g_{bc,ad} - g_{ac,bd} - g_{bd,ac}\right)$$
$$+ g^{mn}\left([ad,m][bc,n] - [ac,m][bd,n]\right)$$

$$R_{bc} = g^{ad} R_{abcd} \qquad R = g^{bc} R_{bc}$$

~~$R_{[} \; R_{abcd} = R_{[ab}$~~

$$W_{abcd} = R_{abcd} - \tfrac{1}{2}\left(g_{ad} R_{bc} + g_{bc} R_{ad}\right.$$

$$\left. - g_{ac} R_{bd} - g_{bd} R_{ac}\right)$$

$$+ \tfrac{1}{6}\left(g_{ad} g_{bc} - g_{ac} g_{bd}\right) R$$

$$W_{abcd} = -W_{bacd} = W_{cdab} = -W_{dcab}$$

~~$= -W_{abdc}$~~

~~$= -W_{abdc}$~~

$$W_{a[bcd]} = 0$$

$$g^{ad} W_{abcd} = 0$$

$$W_{abcd} F^{cd} = \lambda g_{abcd} F^{cd}$$

$$g_{abcd} = -g_{ad} g_{bc} + g_{ac} g_{bd}$$

Petrov's Notation

$$W_{AB} F^{B} = \lambda g_{AB} F^{B}$$

$$x^4 = it$$

at an Event use coords so that

$$g_{ab} = \delta_{ab}$$

The method of classification is all algebraical. Then

$$g_{AB} \text{ diag}\{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\}$$

$$W = \begin{pmatrix} M & N \\ N & M \end{pmatrix}. \qquad M = \tilde{M} \quad N = \tilde{N}$$

$$tr \cdot M = 0 \quad tr \cdot N = 0.$$

$$F = (F_1 \, F_2 \, F_3 \; - \;)$$

$$WF = \lambda F$$

$$\det (W - \lambda I) = 0.$$

M real N imaginary

$G\,(F_1 \, F_2 \, F_3)$ real $H \; (F_4 \, F_5 - F_6)$ im

$$\begin{pmatrix} M & N \\ N & M \end{pmatrix} \begin{pmatrix} G \\ H \end{pmatrix} = \lambda \begin{pmatrix} G \\ H \end{pmatrix}$$

$$MG + NH = \lambda G, \qquad NG + MH = \lambda H$$

$$K = M + N \qquad J = G + H$$

$$KJ = \lambda J$$

K $\quad \det (K - \lambda I) = 0.$

	class	Eigen Values	Eigen Vectors	K
I	I	~~x~~ 3 distinct	3 non null	dia.
		~~$\lambda' \neq \lambda'' =$~~ $\lambda' \neq \lambda'' = \lambda'''$	J', J'' non null	dia
	II a		J' not null J'' null	$K \begin{pmatrix} \lambda' & 0 & 0 \\ 0 & \lambda''+1 & 1 \\ 0 & 1 & \lambda''-1 \end{pmatrix}$
II	II b	$\lambda' \neq \lambda'' = \lambda'''$		
	III a	$\lambda' = \lambda'' = \lambda''' = 0$	J' null, infinity of non null	
III	III b		One null E Vector	

All for adiabatic law $p \propto \rho^{\gamma}$

Bondi $p \propto \rho^{4/3}$ Newtonian theory

$p \propto \rho^{k}$ $k > 4/3$ Radiating energy and contracting

rate of contraction determined by radiated energy

$p \propto \rho^{k}$ $k < \frac{4}{3}$ Energy has to be pumped in to cook the material to prevent collapse

$\left(m - \frac{m^2}{2r}\right) = E$

HFB^2 for Oppenheimer Snyder discussion

$\frac{dE}{dr} = \frac{m^2}{2r^2}$

Bondi — G. C. McVittie Adiabatic $p = 0$.

dependence of internal energy on pressure and density may be such that in the contraction there is a copious release of energy which must be radiated away which will lead to substantial decrease in mass

$E_1 - E_2 = \frac{m^2}{2r_2} - \frac{m^2}{2r_1}$

$= \frac{m^2}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$r_2 = kr_1$

$= \frac{m^2}{2}\left(\frac{1}{kr_1} - \frac{1}{r_1}\right)$

$= \frac{m^2}{2r_1}\left(\frac{1}{k} - 1\right)$

$r_1 + \Delta r_1 = kr_1$

$\Delta r_1 = (k-1)r_1$

$= \frac{m^2}{2r_1}\left(\frac{1-k}{k}\right)$

$\Delta E = \frac{m^2}{2r_1}\frac{\Delta r_1}{r_1 k}$

$= \frac{m^2}{2r_1^2}\frac{\Delta r_1}{k}$

FHN

Bonnor

G C McVittie Summary gives a reason why we must go to non-adiabatic cases

VII

1 Necessity of copious emission of neutrinos

Therefore amend Schwarzschild's ext. solution

2 Exterior soln $\qquad v^1, 0, 0, v^4$

$0\ 0\ 0\ u^4$

Vaidya 1953

" 1951

~~" 1954~~ $\qquad q = u_\mu k^\mu$

Raychaudhuri $\qquad = u_4 v^4$

Israel

$$ds^2 = -e^{\lambda} dR^2 - R^2 d\Omega^2 + e^{2\varphi} d\tau^2$$

$T_{\mu\nu} = \sigma k_\mu k_\nu \qquad k_\mu k^\mu = 0 \qquad u^4 = 1 \quad u_4 = u^4 g_{44}$

$\cancel{T^4_4}$ $\qquad u_4 u^4 k^4 k^4 = 1$

$v^\mu v^\nu T_{\mu\nu} = q \qquad k^4 = e^{-2\varphi}$

$v^\mu v^\nu \sigma k_\mu k_\nu = q$

$v^\mu v^\nu k_\mu k_\nu = 1 \qquad d\tau = dx^0 \quad (k^1)^2 \ \cancel{k^1}$

$k^4 = 1$

$v^4 v^4 k_4 k_4 = 1 \qquad e^{\lambda}(k^1)^2 - e^{\nu}(k^4)^2 = 0$

$v^\mu = \dfrac{dx^\mu}{d\tau} \qquad \cancel{\partial x^\mu} \qquad (k^1) = e^{(\lambda-\nu)/2}$

$$k^\mu \frac{\partial}{\partial x^\mu} = \frac{dx^\mu}{d\tau}\frac{\partial}{\partial x^\mu} = \frac{d}{d\tau}$$

$\cancel{T^4_1}$ $T_1^1 = q k_1 k^1$ $T_1^1 + T_4^4 = 0$

$T^{44} = q$

$u^\mu u^\nu T_{\mu\nu} = q$ $T_4^4 = q$

$T_{44} = q$

$(k_4)^2 = 1$ $k_4 k^4 = 1$

$k_4 = 1$ $k^4 = g^{44} k_4 = e^{-2\varphi}$ $k_1 k^1 = -1$

$-(k^1)^2 e^\lambda + e^{-4\varphi} e^{2\varphi} = 0$

$(k^1)^2 = e^{-\lambda + 2\varphi}$

$k^1 =$

$\cancel{T^{14} = q k^1 k^4 = q}$

u^1, u^4 $u^1 k$ $u^\mu u^\nu k_\mu k_\nu = 1$

$(u^1 k_1 + u^4 k_4) u^\nu k_\nu = 1$

$u^1 k_1 + u^4 k_4 = 1$

$(u^1 k_1 + u^4 k_4)^2 = 1$

$k_4 = 1$ Normalization.

$u_4 k^4 u_4 k^4 = 1$

$u^4 = 1,$ $u_4 = g_{44}$ $u^4 = e^{2\varphi}$

$(k^4)^2 e^{4\varphi} = 1$

$k^4 = e^{-2\varphi}$

$u^4 k_4 = 1$

$k_4 = 1$ $k^4 = g^{44} k_4$

$= e^{-2\varphi}$

$R^a \left(\frac{w^1}{1} \quad 0 \quad 0 \quad \frac{w^4}{1} \right)$

$m^a (0$

$\left(1-\frac{2m}{r}\right)^{-1}(w^1)^2$

$= \left(1-\frac{2m}{r}\right)(w^4)^2$

$w_\mu w^\mu = 0$

$w_\mu w^\mu = 0$

w^2 w^3

$\cancel{w^2}$ $\cancel{w^3}$ w^4

$\cancel{A w^2 + B w^3}$

$-r^2 (w^2)^2 - r^2 \sin^2\theta (w^3)^2 + \left(1-\frac{2m}{r}\right)(w^4)^2 = 0.$

$W^\mu_{;\nu} w^\nu = 0$ $W_\mu W^\mu = 0$ $1-\frac{2m}{r}$

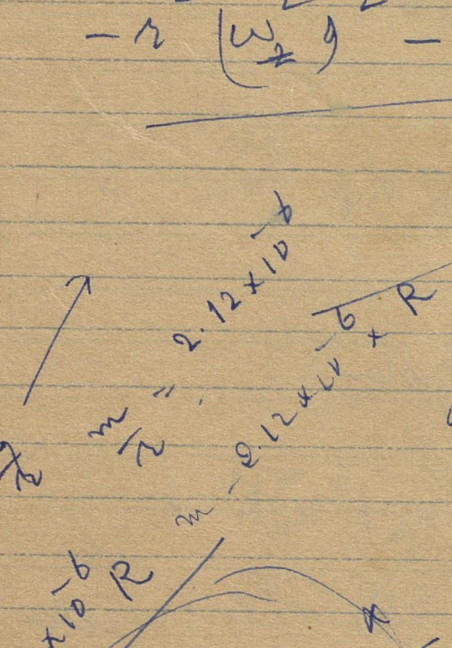
$\frac{m}{r} = 2.12\times10^{-6}$

$m = 2.12\times10^{-6}\times R$

$\frac{d^2u}{d\theta^2} + u = 3mu^2$

$u = \frac{1}{r}$ $\frac{1}{a}$

$a = 6.72\times10^{-6} R$



$\frac{1}{a} = \frac{3m}{a^2}$ $a = 3m$

$\frac{1}{a} = 3m$ $\frac{1}{3m}$

0. Recapitulation

1. Need for allowing flow of radiation

2. Einsteinian soln.

$$ds^2 = -e^{\lambda} dt^2 - R^2 d\omega^2 + e^{\nu} dr^2$$

References Vaidya 1943, 1951, Raychaudhury, Israel

$$T_{\mu\nu} = q\, k_\mu k_\nu$$

Normalization of k_μ, $k_4 = 1$

$$k^4 = g^{44} k_4 = e^{-\nu}, \quad (k^1)^2 e^{\lambda} = e^{-\nu} \quad k^1 = + e^{(-\nu-\lambda)/2}$$

$$k^1 \frac{\partial}{\partial r} + k^4 \frac{\partial}{\partial t} = e^{(-\nu-\lambda)/2} \frac{\partial}{\partial r} + e^{-\nu} \frac{\partial}{\partial t}$$

$$= e^{-\nu/2} \left[e^{-\lambda/2} \frac{\partial}{\partial r} + e^{-\nu/2} \frac{\partial}{\partial t} \right]$$

If $k^1 = e^{-\lambda/2}$, $k^4 = e^{-\nu/2}$

$$u_\mu u_\nu k^\mu k^\nu = u_\mu k^\mu \, (u_1 k^1 + u_4 k^4)^2$$

$$= \left(e^{-\lambda/2} u_1 + e^{-\nu/2} u_4 \right)^2$$

$$= e^{-\lambda} (u_1)^2 + e^{-\nu} \cancel{(u_4)}^2 + 2 e^{(-\lambda-\nu)/2} u_1 u_4 = 1$$

$$= - e^{-\lambda} (u_1)^2 + e^{-\nu} \cancel{(u_4)}^2$$

$$2 e^{-\lambda} (u_1)^2 + 2 e^{(-\lambda-\nu)/2} u_1 u_4 = 0$$

$u_1 = 0$ or $\cancel{e^{-\lambda/2} u_1 + e^{-\nu/2} u_4 = 0}$

$e^{\lambda} u$

$u_4 k^4 = 1$

$(u_4 k^4)^2 = 1$

$e^{\nu/2} k^4 = 1$

$k^4 = e^{-\nu/2}$

e^{ν}

$g^{44} (u_4)^2 = 1$

$e^{-\nu} (u_4) = 1$

$u_4 = e^{\nu/2}$

$T_{\mu\nu} = q\, k_\mu k_\nu$

Normalizing k_μ so that $u^\mu u^\nu \gamma_{\mu\nu} = q$

$u^1 = u^2 = u^3 = 0 \quad k^4 = e^{-\nu/2} \quad \therefore k^4 = e^{-\nu/2}, \ k^1 = e^{-\lambda/2}$

$k_1 = -e^{\lambda/2}$

$$k^\mu \frac{\partial}{\partial x^\mu} = e^{-\lambda/2}\frac{\partial}{\partial r} + e^{-\nu/2}\frac{\partial}{\partial t} = \frac{d}{ds}$$

$\cancel{T_1^4 = R} \quad T_1^1 = -q, \quad T_2^2 = 0, \ T_3^3 = 0$

$T_4^4 = +q, \quad T_1^4 = -e^{(\lambda-\nu)/2} q$

$R_1 = g_{11} k^1 = -e^{\lambda} e^{-\lambda/2} = -e^{\lambda/2}$

$$e^{\lambda} = \frac{1}{1-\frac{2m}{R}} \qquad \frac{dm}{ds} = 0 \qquad m = m(u)$$

$$T_1^1 + T_4^4 = 0 \quad \Longrightarrow \quad \frac{d}{dr}\left[m'\left(1-\frac{2m}{r}\right)\right] = 0$$

$$m'\left(1-\frac{2m}{r}\right) = f(m)$$

$$T_4^1 = \frac{2m'}{r^2} = e^{(\lambda-\nu)/2}\frac{2m}{r^2} \quad m' e^{\lambda/2} + \dot{m} e^{-\nu/2} = 0$$

$$e^{-\lambda} = \left(1-\frac{2m}{r}\right), \quad e^{\nu} = \frac{m'^2 e^{\lambda}}{\dot{m}^2} \quad \frac{m'^2}{m'^2 e^{-\lambda}} = \frac{\dot{m}^2}{f^2}\left(1-\frac{2m}{r}\right)$$

$m = m(r,t)$

$$ds^2 = -\left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 + \frac{\dot{m}^2}{f^2}\left(1-\frac{2m}{r}\right) dt^2$$

$u' m' + \dot{u}\dot{m} = 0$

$m = m(u)$

$u' dr + \dot{u}\, dt = 0$

$u' e^{-\lambda/2} + \dot{u} e^{-\nu/2} = 0 \qquad u = \phi(m) \quad \text{or} \quad m = m(u)$

$\cancel{\text{at}}\ du = u' dr + \dot{u}\, dt$

$$ds^2 = -\left(1-\frac{2m(u)}{r}\right) du^2 + 2\, du\, dr - r^2 d\Omega^2$$

Vaidya 1952

~~Ab~~

Sachs 1961 PRS A264, 309–338

Further references in Witten: Gravitation an Introduction to current Research

3. ~~Tensor Method~~ ~~Debever's~~ scheme

~~Elect~~ Electromagnetic analogue

If F_{ab} is null

$F_{ab}F^{ab} = 0 \quad F^{*}_{ab}F^{ab} = 0$

then $\exists\ k^a \ni k_a k^a = 0$

$k^a F_{ab} = 0 \quad k^a F^{*}_{ab} = 0$

~~Debever~~ (1958) proved a theorem that

Sachs' Statement of Debever's theorem

In every empty space-time $(R_{ij} = 0)$ $\exists$ at least one and at most 4 null directions k^a such that

$$k_{[a} R_{b]ij[c} k_{d]} k^i k^j = 0$$

Thus

$$\tfrac{1}{2}\, k_a R_{bij[c} k_{d]} k^i k^j - k_b R_{aij[c} k_{d]} k^i k^j = 0$$

$$k_a R_{bijc} k_d k^i k^j - k_a R_{bijd} k_c k^i k^j$$

$$- k_b R_{aijc} k_d k^i k^j + k_b R_{aijd} k_c k^i k^j = 0$$

One can classify the fields from a knowledge of rays

Is $R_{abcd} k^d = 0$? N 4 coincident rays

If no, take

$$R_{abc[d} k^c k_{e]} = 0$$

i.e. $R_{abcd} k^c k_e - R_{abce} k^c k_d = 0$? III 3, 1

Next $R_{abcd} k_e k^a k^c - R_{abce} k_d k^a k^c = 0$
~~or~~ and $R_{abcd} m_e m^a m^c - R_{abce} m_d m^a m^c = 0$ D 2, 2

$R_{abcd} k_e k^a k^c - R_{abce} k_d k^a k^c$ II 2, 11

$\cancel{- R_{bacd} k_e k^a k^c + R_{bace} k_d k^a k^c}$

$k_{[a} R_{b]ij[c} k_{d]} k^i k^j = 0$ I

There is a hierarchy. So that if any one is satisfied

[illegible] that follows are also satisfied.

Spinor Method: This method depends on the following relationship between spinors and null vectors and tensors.

A real null vector ξ^a corresponds to the product

$$\xi^A \bar{\xi}^{\dot{X}}$$

A skew symmetric tensor $F_{ab} = F_{[ab]}$

~~$\frac{1}{2}(F_{ab} - \ldots)$~~

For a real skew symmetric tensor

F_{ab} corresponds to $\varphi_{AB}\,\epsilon_{\dot{W}\dot{X}} + \bar{\varphi}_{\dot{W}\dot{X}}\,\epsilon_{AB}$

φ_{AB} being a symmetric 2-spinor

$$\epsilon_{AB} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

If W_{abcd} is a real Weyl tensor, it corresponds to a 4 symmetric 4 spinor Ψ_{ABCD}

$$W_{abcd} = \Psi_{ABCD}\,\epsilon_{\dot{W}\dot{X}}\,\epsilon_{\dot{Y}\dot{Z}} + \bar{\Psi}_{\dot{W}\dot{X}\dot{Y}\dot{Z}}\,\epsilon_{AB}\,\epsilon_{CD}$$

IV

Motivation: [illegible] In empty space we have

1 $R_{abcd} \neq 0$, $R_{bc} = 0$. For the Weyl tensor $W_{abcd} \neq 0$, $W_{bc} \equiv 0$. Hence even when $R_{ij} \neq 0$, one can work with Weyl tensor and classify the solutions. Thus with any solution of the field equations of general relativity the Weyl tensor stands in a particular relationship to the Null cone and there are rays for Weyl tensor.

∴ There is a possibility of there being gravitational radiation through matter.

2. Einstein began with a postulate that ~~The space time The condition at infinity has been the Riemannian space time~~
In order to get a unique solution of the partial diff. equations, in analogy with Newtonian concepts a condition at infinity was introduced viz. at large distances from the gravitating masses the gravitational field vanishes and ~~the~~ space time becomes asymptotically flat.

~~Now Later~~ This is essentially against the spirit of "completeness" of the gravitational theory. Another point against this "completeness" was the geodesic postulates. That this was removed in 1938-39 by EIH paper. The flat space-time condition was ~~[illegible]~~ removed in 1919 by developing a theory of the universe.

The flat spacetime condition assumes that there is a flat background over which gravitational fields are superposed.

Now we know that the geometry of the entire universe is not flat Euclidean but Riemannian, the line element being

$$ds^2 = dt^2 - e^{g(t)}\left[dx^2 + dy^2 + dz^2\right] \Big/ \left\{1 + k r^2\right\}^2$$

$k = +1, -1$ or 0

$\therefore$ Condition at infinity for gravitational waves has to change

$R_{abcd} \sim \frac{1}{r}$ Now R_{abcd} will not $\to 0$

$C_{abcd} \sim \frac{1}{r}$ But $C_{abcd} \to 0$

$\therefore$ When we speak of Weyl tensor

The condition is $C_{abcd} \sim \frac{1}{r}$ $R_{abcd} \not\to \frac{1}{r}$

3. For solution within the source region, ~~we~~ we must have some periodic charges (similar charges in time.

"completeness" again demands that there be gravitational radiation as the source of gravitational radiation

Linearized field

$\Box h_{\mu\nu} = 0$ $h^{\mu\nu}_{\ \ ,\nu} = 0$

~~The grav~~ If any other field interactions are free

$\Box h_{\mu\nu} = k\theta_{\mu\nu}$ $\theta^{\mu\nu}_{\ \ ,\nu} = 0$

~~This is~~ A linearized theory behaves like a linear flat background theory and so the interactions are not coupled but are to be superposed.

Now $\Box h_{\mu\nu} = 0$ produces a gravitational field. ~~It is~~ This field itself is ~~respon~~ affect the linear field

Since the matter stress tensor $T_{\alpha\beta}$ satisfies the condition $\theta^{\mu\nu}_{\ \ ,\nu} = 0$, it is expected to be included in $T^{\mu\nu}$. The appearance of stress tensor is to be expected in developing a theory of gravitation because stress and energy are the source of gravitational field. The gravitational field is expected to contribute some stress and energy as well.

~~Hence the field equation become~~

$$\Box h_{\mu\nu} = k(T_{\mu\nu} + t_{\mu\nu})$$

~~Gupta~~ Be for empty space $T_{\mu\nu} = 0$

Gupta derived $\Box h_{\mu\nu} = k t_{\mu\nu}$

from a Lagrangian with infinite no. of terms which when added reduce to

$L = R\sqrt{-g}$.

Article by Weber in Gravitation and Rel. Edited Chiu and Hoffman

S N Gupta Phys. Rev 96, 1683 (1952)

Does there exist a ~~Such a~~ $t_{\mu\nu}$ for due to travelling gravitational radiation?

W.A. Bergmann Int. N.?

~~Excellent~~

~~With these motivations~~

With these ~~motivations~~, are the problems that come up before us when we consider radiation through matter

[illegible]

[illegible]

$\cancel{w_c R_{[ij]}}$

$w_{[a} R_{bc]}, d_e = 0$

$w^a R_{abcd} = 0$

$R_{bc} = \tau w_b w_c$

$\cancel{ds^2}$ Einstein Rosen metric

$\beta_{11} - \beta_{44} + \frac{\beta_1}{x^1} = 0$

$\alpha_1 + \alpha_4 - x^1(\beta_1 + \beta_4)^2 \cancel{-} \frac{1}{2x^1}$

$ds^2 \quad \alpha_{11} + \cancel{\alpha_{14}} - (\beta_1 + \beta_4)^2 - 2x^1(\beta_{11} + \beta_{14})(\beta_1 + \beta_4) + \frac{1}{2x_1^2}$

$\cancel{\alpha_{14}} + \alpha_{44} \qquad - 2x^1(\beta_{14} + \beta_{44})(\beta_1 + \beta_4)$

$\alpha_{11} - \alpha_{44} - (\beta_1 + \beta_4)^2 - 2x^1(\beta_{11} - \beta_{44})(\beta_1 + \beta_4)$

$\alpha_{11} - \alpha_{44} - \beta_1^2 - \beta_4^2 - 2\beta_1\beta_4 + \frac{1}{2x^{1\,2}} = 0$

$+ 2\beta_1(\beta_1 + \beta_4) + \frac{1}{2x_1^2}$

$\alpha_{11} - \alpha_{44} + \beta_1^2 - \beta_4^2 + \frac{1}{2x^{1\,2}} = 0$

$$\beta_{11} - \beta_{44} + \frac{\beta_1}{x^1} = 0$$

$$\alpha_1 + \alpha_4 - x^1(\beta_1 + \beta_4)^2 - \frac{1}{2x^1} = 0$$

$$\alpha_{11} - \alpha_{44} + \beta_1^2 - \beta_4^2 + \frac{1}{2(x^1)^2} = 0$$

$\overset{\circ}{\alpha}, \overset{\circ}{\beta}$ empty

If

$\beta = \overset{\circ}{\beta}$

$\alpha = \overset{\circ}{\alpha} + f(t-r)$

$$e^{\alpha} = e^{2\gamma - 2\psi}$$

$$e^{2\beta} = \frac{r^2 e^{-2\psi}}{e^{2\psi}} = r^2 e^{-4\psi}$$

~~$\sigma =$~~

$x^1 = r$

$\beta = \log r - 2\psi$

$\beta_1 = \frac{1}{r} - 2\psi_1$

$\beta_{11} = -\frac{1}{r^2} - 2\psi_{11}$

$\psi_{11} - \psi_{44} + \frac{\psi_1}{r} = 0$

$e^{-4\psi}$

$\psi = -\frac{1}{2}\beta + \log r$

$\gamma = \frac{\alpha}{2} + \psi$

$\alpha = 2\gamma - 2\psi$

$\beta = \log r - 2\psi$

$\overset{\circ}{\beta} = \log r - 2\overset{\circ}{\psi}$

$\overset{\circ}{\beta}$

$\alpha = 2\gamma + \beta - \log r$

~~$= 2\overset{\circ}{\gamma} + \overset{\circ}{\beta}$~~

$\overset{\circ}{\alpha} = 2\overset{\circ}{\gamma} + \overset{\circ}{\beta} - \log r$

$\alpha = 2\overset{\circ}{\gamma} + \overset{\circ}{\beta} - \log r + 2f$

$= \overset{\circ}{\alpha} + f$

Simplest particular case

$$e^{\beta} \quad \overset{\circ}{\beta}_0 = \log r$$

$$\overset{\circ}{\alpha}_0 = 0$$

$$g_{22}\, g_{33} = r^2$$

$$\frac{g_{22}}{g_{33}} = e^{2\beta}$$

$$\cancel{g_{22} = r^2 e^{2\psi}}$$

$$\cancel{g_{33} = e^{-2\psi}}$$

$$g_{22} = e^{2\psi}$$

$$g_{33} = r^2 e^{-2\psi}$$

$$e^{2\beta} = \frac{e^{4\psi}}{r^2} \qquad e^{2\psi} = r e^{\beta}$$

$$e^{2\gamma} = r e^{\beta} \cdot e^{\alpha}$$

$$e^{\alpha} = e^{2\gamma - 2\psi} \qquad e^{2\gamma} = e^{\alpha} \cdot e^{2\psi}$$

$$\cancel{r e} \quad x^1 = r; \qquad r e^{\beta} = e^{2\psi}, \qquad e^{2\gamma} = r e^{\beta} \cdot e^{\alpha}$$

$$\psi_{11} - \psi_{44} + \frac{\psi_1}{r} = 0$$

$$\gamma_1 + \gamma_4 - r(\psi_1 + \psi_4)^2 = 0$$

$$\gamma_{11} - \gamma_{44} + \psi_1^2 - \psi_4^2 = 0$$

$$R_{ij} = \tau\, w_i w_j$$

does it not correspond to

$$\Box\, R_{ij} = t_{ij}$$

flying radiation

t_{ij} being the self energy [illegible] gravitation radiation

Gravitational radiation in cosmological models

$ds^2 = e^{H(t)} [\quad]$

=

V 18-5-66

1 § Introduction

2 Schwarzschild's solution and singularity.

3 Mathematical transformations

4. Physical Nature

1 Introduction:

$\frac{2GM}{c^2}$

~~Quasi~~ Radio sources $\frac{2GM}{c^2}$ 3C 48 and 3C 273

appear to be objects with very small diameters yet whose total energies lie in the range of 10^{60} to 10^{61} ergs. How can such high energies be present in the bodies of such small size?

Hoyle and Fowler suggested that such high energies can be stored up in the form of gravitational potential energies of highly compressed masses. For a spherical mass M of radius R this is $\frac{3}{5}\frac{GM^2}{R}$ for constant density but $\frac{3}{5-n}\frac{GM^2}{R}$ for a polytrope of index n. For order of quantities we say it is $\sim \frac{GM^2}{R}$. If $M = 10^6 \odot$ then $P.E. = 10^{60}$ ergs if $R = 3.8 R_\odot$! $\rho_{average}$ 2.57×10^4 (White dwarf)

If $M = 10^8 \odot$ radius is $3.8 \times 10^4 R_\odot$

$\rho_{average} = 2.57 \times 10^{-6}$

(Much less stringent requirement physically)

How could so large a mass get into so small a volume

For a given mass & if the radius is ... in equil. we have

$$ds^2 = \left(1 - \frac{2m}{r}\right)^{-1}$$

$r = 2m$

We shall first consider the natural limit to radii of configurations in GRG

Then ... which lead to contractions under gravitation and finally contraction leading to ~~annihilation~~ of energy

$\left(1-\frac{2m}{r}\right)(dt-dr)$ $+2(dt-dr)\,dr$ $-\left(1+\frac{2m}{r}\right)dr^2$ $-2\,dt\,dr\left(1-\frac{2m}{r}\right)$ -2

2. Schwarzschild's solution

$$ds^2 = \left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 + \left(1-\frac{2m}{r}\right) dT^2$$

Have you a singularity at $r = 2m$?

$r = 2m$ is a math artifact

Mathematically yes because $g_{ij} \to \infty$ at $r = 2m$ and $r = 0$

Can you transform this singularity

$$\cancel{du =} \quad u' g^{11} \quad u'\left(1-\frac{2m}{r}\right)^{1/2} + i\left(1-\frac{2m}{r}\right)^{-1/2} \to \infty$$

$$\cancel{ds^2 = \left(1-\frac{2m}{r}\right.}$$

$$ds^2 = -r^2 d\Omega^2 + \left(1-\frac{2m}{r}\right) du^2 + 2\,du\,dr$$

What about $r = 2m$?

$$\cancel{(r-2m)^{1} \quad g^{ij} \quad f_{,i} f_{,j}}$$

g^{11}.

Ultimate transformation $T = t + 2m \ln(r - 2m)$

If $u = t - r$

$$ds^2 = +\left(1-\frac{2m}{r}\right) dt^2 + \frac{4m}{r} dr\, dt - \left(1-\frac{2m}{r}\right) dr^2 - r^2 d\Omega^2$$

$$= -dx^2 + dy^2 + dz^2 - dt^2 + \frac{2m}{r}(dr + dt)^2$$

It has only a singularity at $r = 0$

Eddington 1924 Nature 113, 192

D Finkelstein Phys. Rev. 110, 965 (1958)

So mathematically there is a $r=0$ only.
But haven't we transformed the singularity to the transform $T = t + 2m\log(r-2m)$!
Or Finkelstein's solution is the given solution and you get a singularity in Schwarzschild solution because you use an irregular transformation for a regular metric!

Let us discuss the nature of the null surface $r = 2m$

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \frac{4m}{r}\,dr\,dt \mp \left(1+\frac{2m}{r}\right)dr^2 - r^2 d\Omega^2$$

$r > 2m$ t-axis is time like. Static

$r < 2m$ you could transform to S. Form and see that t-axis is time like

$$\cancel{ds^2 = dx^2 + dy^2 + dz^2 + \frac{2m}{r}dr^2 + \frac{4m}{r}\,dr\,dt + \left(1-\frac{2m}{r}\right)dt^2}$$

metric is non static

$$\left(\frac{ds}{dt}\right)^2 = \left(1-\frac{2m}{r}\right) - \frac{4m}{r}\overset{v}{\frac{dr}{dt}} - \left(1+\frac{2m}{r}\right)v^2$$

ND

$r > 2m$

$\frac{2m}{r} < 1$

$$\left(1+\frac{2m}{r}\right)v^2 + \frac{4m}{r}v - \left(1-\frac{2m}{r}\right) = 0$$

$$\frac{-\frac{4m}{r} \pm \sqrt{\frac{16m^2}{r^2} + 4\left(1-\frac{4m^2}{r^2}\right)}}{2\left(1+\frac{2m}{r}\right)} = \frac{-\frac{4m}{r} \pm 2}{2\left(1+\frac{2m}{r}\right)}$$

for $r > 2m$ only one way traffic

$r < 2m$ two way traffic

$$v = \frac{-\frac{2m}{r}+1}{1+\frac{2m}{r}},\quad \frac{-\frac{2m}{r}-1}{1+\frac{2m}{r}} = -1$$

Once within Schwarzschild sphere, the object cannot be seen!

2. ~~Kruskals~~ Kruskal's coordinates

$$ds^2 = -16m^2 e^{-x}\, dv\, dw + 4m^2 x^2\, d\Omega^2$$

$x = r/2m$ in region $vw \leq 1$

~~vw~~ $\quad -vw = (x-1)\, e^{x}$

M. Kruskal Phys. Rev **119**, 1743 (1960)

R.W. Fuller and J.A. Wheeler Phys. Rev. ~~114~~ 128 919 (1962)

$(x-1)\, e^{x}$

$(1-x)\, e^{x} \leq 1$

$$-\frac{w}{v} = e^{T/2m}$$

$$-vw = (x-1)\, e^{x} \qquad x = \frac{r}{2m}$$

$x \quad 0 < x < \infty, \qquad 0 < T < \infty$

$-\frac{w}{v}$ in $(1, \infty)$

Schwarzschild's surfaces have been drawn in Schwarzschild and Lindquist, Schwarzschild and Wheeler

Phys. Rev. 137, B1364 (1965)

Physical nature of this singularity

1. Messages can go in, can't come out
If an object within $r \leq 2m$ already existed then
2. We would have a cancelers gravitational field. The object producing the field cannot be observed!

When we looked in the direction of clusters above, we saw the Sun!

This is highly repugnant!

3. Can an object do the rope trick and disappear withdraw in front of our eyes to within the Schwarzschild radius?

How would this look like?

Red Shift

The pours appears to us slower and slower.
$$ds^2 = \left(1-\frac{2m}{r}\right)^{1/2} dt^2.$$

ds^2 between arriving two successive radii, $dt \to \infty$.

For an observer on the surface contraction takes place at a finite rate for us, outsiders it takes infinitely slowly!

~~One has an~~

~~Energy~~

4. One has an impression that objects like this for which $r < 2m$ will be very massive

This is not quite true. Let $m = \frac{4}{3}\pi r^3 \rho$
$$\frac{2m}{r} = \frac{8\pi}{3} r^2 \rho \geq 1$$
$$\rho = \frac{3}{8\pi r^2}$$

for large enough r we can get small densities

Energy How can you build up a star from particles in a diffused state at ∞.

$$\int_{\infty}^{a} \frac{m}{r^2}\,dr = \left[-\frac{m}{r}\right]_{\infty}^{a} = -\frac{m}{a}$$

$$\frac{3}{5}\frac{\mu m^2}{R}$$

$$ds^2 = -\left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2 d\omega^2 + \left(1-\frac{2m}{r}\right) dt^2$$

$$u'\left(1-\frac{2m}{r}\right)^{1/2} + \dot{u}\left(1-\frac{2m}{r}\right)^{-1/2} = 0$$

$$du = u'\,dr + \dot{u}\,dt$$

$$dt = \frac{du}{\dot{u}} - \frac{u'}{\dot{u}}\,dr$$

$$ds^2 = -\left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2 d\omega^2 + \left(1-\frac{2m}{r}\right)\left[\frac{du}{\dot{u}} - \frac{u'}{\dot{u}}dr\right]$$

$$= -dr^2\left[\frac{1}{1-\frac{2m}{r}} - \frac{u'^2}{\dot{u}^2}\left(1-\frac{2m}{r}\right)\right]$$

$$- r^2 d\omega^2 + \left(1-\frac{2m}{r}\right)\frac{du^2}{\dot{u}^2} - \frac{2u'}{\dot{u}^2}\left(1-\frac{2m}{r}\right) du\,dr$$

$$+ \cancel{\left(1-\frac{2m}{r}\right)}\frac{du^2}{u'^2\left(1-\frac{2m}{r}\right)} - \frac{2u'}{u'^2\left(1-\frac{2m}{r}\right)} du\,dr$$

$$u'\left(1-\frac{2m}{r}\right) = -1$$

$$+ \left(1-\frac{2m}{r}\right) du^2 + 2\,du\,dr$$

$$ds^2 = -r^2 d\omega^2 + \left(1-\frac{2m}{r}\right) du^2 + 2\,du\,dr$$

$$u = t - r$$
$$du = dt - dr$$

$$= -r^2 d\omega^2 + \left(1-\frac{2m}{r}\right)(dt-dr)^2 + 2(dt-dr)dr$$

$$= -r^2 d\omega^2 + (dt-dr)\left[\left(1-\frac{2m}{r}\right)dt - dr + 2dr\right]$$

$$\left[\left(1-\frac{2m}{r}\right)dt + \left(1+\frac{2m}{r}\right)dr\right]$$

$$\left[(dt+dr) - \frac{2m}{r}(dt-dr)\right]$$

$$-r^2 d\omega^2 + dt^2 - dr^2 - \frac{2m}{r}(dt-dr)^2$$

$$0 = 1 - v^2 - \frac{2m}{r}(1-v)^2$$

$$(1-v)\left(1+v - \frac{2m}{r}(1-v)\right) = 0$$

$$v = 1 \qquad v\left(1+\frac{2m}{r}\right) = -1 + \frac{2m}{r}$$

or

$$v = -\frac{1-\frac{2m}{r}}{1+\frac{2m}{r}} \qquad r < 2m$$

$$T = t + 2\log(r-2m)$$

$$\left(\frac{ds}{dt}\right)^2 = (1-v)\left(\frac{\cancel{1+}\,1-\frac{2m}{r} + v\cancel{\left(1+\frac{2m}{r}\right)}}{1+\frac{2m}{r}}\right)$$

$$\left(\frac{dr}{dt}\right)^2 \left(1-\frac{2m}{r}\right)^2$$

$$1+\frac{2m}{r} \qquad -\frac{\frac{4m}{r}}{1+\frac{2m}{r}}$$

$$1 - \frac{\frac{4m}{r}}{1+\frac{2m}{r}}$$

$$(1-v)(v-\alpha)$$