Book. No. 19

Magic squares of order 15 by the N-method & C. M

$a+a' = 0$
$b+b' = -1$

$a = 1,\ b = 1.$
$a' = -1,\ b' = -2.$

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

$(i, j) = (2, 4)$ $\quad i = 2,\ j = 4.$

$(2+4,\ 4+4) = (6, 8) \equiv (1, 3)$

$(1, 2)$

$(a, b) = (3, 1) = (-2, 1)$ $\quad (1, 0)$

$a + a' = 1$
$1+b+b' = 0$

$2-2 = 0$
$4-4 = 0$ $\}\ (0, 0)$

$18 = 5 \cdot 3 + 2 + 1$
$y = 3,\ x = 2$

$i + ax + a'y =$

4, 1

$x = 4$
$y = 1$

$5+5+1 = 11$

$2 - 1 = 1$
$4 - 2 = 2$

(ii)

$y = 1,\ x = 2$

1	7	13	19	25
18	24	5	6	12
10	4	9	15	16
10	11	17	23	4

$2a + 3a' = 0$
$4 + 2b + 3b' = 3$
$2a + 3a' = 0$
$2b + 3b' = -1$

$x = 4,\ y = 0$
$i = 2,\ j = 4.$

$2 + 4 = 6 \equiv 1.$
$4 + 4 = 8 \equiv 3$

$8 + 5 + 1 = 14.$

$a + a' = 1$
$b + b' = 0$

$x = 4,\ y = 1.$

$\boxed{4+5+1}$

$4 + 10 + 1 = 15$

i to $yn + x + 1.$

$a = 1$
$b = 1$
$a' = -1$
$b' = -2$

$a + a' = 1$
$2a + 3a' = 0$

$x+1$ $\quad 5.$ $\quad n + x + 1.$
$6.$ $\quad 5+4+1 = 10$

$i + xa + ya',\ j + xb + yb'$

1	7	13	19	25
18	24	5	6	12
10	11	17	23	4
22	3	9	15	16
14	20	21	2	8

$2 + 3 \cdot 1 + 4(-1) = 1$
$4 + 3 \cdot 1 + 4(-2) \equiv 4$
7.

$24 = 5 \cdot 4 + 3 + 1.$
$y = 4,\ x = 3$

$y = 1,\ x = 1.$

$7 = 5 \cdot 1 + 1 + 1$
$i = 0,\ j = 4$

$i + xa + ya'$
$j + xb + yb'$

$a + a' = 1$
$4 + 1 + b + b' = 4$

(5)

(2)

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182	31	19	86	50	82	68	98	116	130	162	150	179	223	198
214	201 ~~140~~	185	37	23	9	56	85	72	105	119	133	153	137	166
140	172	218	204	191	40	27	15	59	88	63	92	106	124	156
128	159	146	175	222	210	194	43	18	2	46	79	66	95	112
101	115	132	165	149	178	213	197	181	34	21	5	52	83	69
87	75	104	118	123	152	136	169	216	200	187	38	24	11	55
14	58	78	62	91	109	126	155	142 ~~157~~	173	219	206	190	42	30
33	17	1	49	81	65	97	113	129	161	145	177	225	209	193
196	184	36	20	7	53	84	71	100	117	135	164	148	168	212
171	215	202	188	39	26	10	57	90	74	103	108	122	151	139
157	143	174	221	205	192	45	29	13	48	77	61	94	111	125
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	208	183	32	16	4	51	80	67	98

rank and associate

$91 = 6x + 7x$

(3)

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(4)

1, 2, 3 | 4, 6, 5 | 8, 9, 7 | 12, 10, 11 | 15, 13, 14

3, 1, 2 / 2, 1, 3 | 6 4, 5 | 7 8 9 | 12 | 14, 15, 13 / 13, 14

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

2, 1, 3 | 6 5 / 9, 4 | 9, 8, 7 | 10, 12, 11 | 13, 15, 14

~~3, 1, 2 | 6, 4, 5 | 7, 8, 9 | 11, 12, 10 | 13, 15, 14~~

1, 4, 8, 12, 15
2, 6, 9, 10, 13
3, 5, 7, 11, 14
~~(1, 15) (3, 14) (4, 7)~~
3, 6, 7, 11, 13
2, 5, 9, 10, 14.

1, 5, 8, 11, 15

3, 5, 9, 10, 13

$$x_1 + x_4 + x_7 + x_{10} + x_{13} = 40$$
$$x_2 + x_5 + x_8 + x_{11} + x_{14} = 40$$
$$x_3 + x_6 + x_9 + x_{12} + x_{15} = 40$$

$x_1 + x_{15} = 16$
$x_2 + x_{14} = 16$
$x_3 + x_{13} = 16$
$x_4 + x_{12} = 16$
$x_5 + x_{11} = 16$
$x_6 + x_{10} = 16$
$x_7 + x_9 = 16$

how arbitrary?

$x_2 + x_{14} = 16$
$x_5 + x_{11} = 16.$

$x_2 = 2, x_{14} = 14$
$x_5 = 5, x_{11} = 11$

11 eqns between 15 quantities

Need not be very possible

$x_1 = 3, x_4 = 4, x_7 = 7$
$x_3 = 1, x_6 = 5.$

$x_1 + x_4 + x_7 + (16 - x_6) + (16 - x_3) = 40$

$x_1 + x_4 + x_7$

$x_3 + x_6 + 8$

	(1)	(2)													
(1)	2	1	4	6	5	7	8	9	11	10	12	15	14	13	3
(2)	4	6	5	7	8	9	11	10	12	15	14	13	3	2	1
(3)	5	7	8	9	11	10	12	15	14	13	3	2	1	4	6
(4)	8	9	11	10	12	15	14	13	3	2	1	4	6	5	7
(5)	11	10	12	15	14	13	3	2	1	4	6	5	7	8	9
(6)	12	15	14	13	3	2	1	4	6	5	7	8	9	11	10
(7)	14	13	3	2	1	4	6	5	7	8	9	11	10	12	15
(8)	3	2	1	4	6	5	7	8	9	11	10	12	15	14	13
(9)	1	4	6	5	7	8	9	11	10	12	15	14	13	3	2
(10)	6	5	7	8	9	11	10	12	15	14	13	3	2	1	4
(11)	7	8	9	11	10	12	15	14	13	3	2	1	4	6	5
(12)	9	11	10	12	15	14	13	3	2	1	4	6	5	7	8
(13)	10	12	15	14	13	3	2	1	4	6	5	7	8	9	11
(14)	15	14	13	3	2	1	4	6	5	7	8	9	11	10	12
(15)	13	3	2	1	4	6	5	7	8	9	11	10	12	15	14

A (nasik numbers) – Step (, (13,2)

Carrés magiques — Chap. I. to show is ~~equal~~ equivalent to the generalised I. D. M.

Chap I — deals with case p where p is prime. The essential things are (r_1) and (s): once these are chosen ~~magic~~ suitably the magic square is constructed. The magic square is constructed with ~~r_p~~ $r = a_1 b_1$ in the ~~position~~ cell $(1, p-1)$ and $s = c_1 d_1$ in the cell $(0, p-2)$, and the corresponding numbers are $pa_1 + b_1 + 1$ and $pc_1 + d_1 + 1$ (1 is added because in Carrés magiques the nos. in the magic square are from 0 to p^2-1 i.e. these numbers ~~are the~~ $pa_1 + b_1 + 1$ and $pc_1 + d_1 + 1$ are obtained after a_1 breakmoves followed by b_1 regular moves, and after c_1 breakmoves & followed by d_1 regular moves ~~of the~~ respectively) (of the G. I. D. M). Also in the squares of Carrés magique (C.M) $(i, j) = (0, ~~p~~$ using the scheme $\begin{pmatrix} a & a' \\ b & b' \end{pmatrix}_p$ of the G. I. D. M we have therefore

$$\left.\begin{array}{l} \cancel{a + b_1 a + a_1 a' = pa_1 + b_1 + 1} \\ \cancel{0 + d_1 a + c_1 a' = pc_1 + d_1 + 1} \end{array}\right\} \text{~~knowing p, a_1, b_1, c_1, d_1 these eqns give a, a'~~}$$

$$\begin{array}{l} 0 + b_1 a + a_1 a' = 1 \\ \cancel{0 + d_1 a + c_1 a} \\ 0 + d_1 a + c_1 a' = 0 \end{array} \left\{ \begin{array}{l} \overset{p-1}{a} + b_1 b + a_1 b' = p-1. \\ \\ \overset{p-1}{a} + d_1 b + c_1 b' = p-2 \end{array} \right\}$$

Knowing a_1, b_1, c_1, d_1 and p. These eqns give a, b, a', b' of the G. I. D. M.

Ex: (1) Fig. 7 of p. 40 of C.M.

Here ~~b_p~~ $r = 21$, $s = 43$, $p = 7$, $a_1 = 2$, $b_1 = 1$, $c_1 = 4$, $d_1 = 3$.

$a + 2a' \equiv 1$, $b + 2a' \equiv 6$.

$3a + 4a' \equiv 0$, $3b + 4b' \equiv 5$

leading to $2a' \equiv 3$, $b\,a' \equiv -2$, $a \equiv -2$

$3b' \equiv 13 \equiv 6$, $b' \equiv +3$, ~~$b = 2$~~ $b \equiv 1$

$a_1 p + x + 1$

$a_2 p + b_2 + 1$

$18 - 5 = 13$

$b + 12 = 6$

$b = -6$

$3b + 12 = 5$

$3b = -7 \equiv$

$(-2$ see p. 9.

$(-2, -2)$ $(-2)(-4)$ is regular move

$-2, -3$

$(-4, -5)$ a $(3, 2)$ is breakmove

4

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L. 15 · ~~226~~ 113
2 1695

(3)

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________________ *Examination.*

	105	135	60	210	15	30	0	120	195	45	75	90	165	180	150
	120	195	45	75	90	165	180	150	105	135	60	210	15	30	0
	150	105	135	60	210	15	30	0	120	195	45	75	90	165	180
	0	120	195	45	75	90	165	180	150	105	135	60	210	15	30
	180	150	105	135	60	210	15	30	0	120	195	45	75	90	165
×	30	0	120	195	45	75	90	165	180	150	105	135	60	210	15
7	165	180	150	105	135	60	210	15	30	0	120	195	45	75	90
	15	30	0	120	195	45	75	90	165	180	150	105	135	60	210
9	90	165	180	150	105	135	60	210	15	30	0	120	195	45	75
(10)	210	15	30	0	120	195	45	75	90	165	180	150	105	135	60
(11)	75	90	165	180	150	105	135	60	210	15	30	0	120	195	45
(12)	60	210	15	30	0	120	195	45	75	90	165	180	150	105	135
(13)	45	75	90	165	180	150	105	135	60	210	15	30	0	120	195
(14)	135	60	210	15	30	0	120	195	45	75	90	165	180	150	105
								105	135	60	210	15	30	0	120

(B) Step – (8,7)

(σ)

15	20	5	0	10
5	0	10	15	20
10	15	20	5	0
20	5	0	10	15
0	10	15	20	5

20	21	8	4	12
9	2	15	16	23
11	18	24	7	5
22	10	1	13	19
3	14	17	25	6

5	1	3	4	2
4	2	5	1	3
1	3	4	2	5
2	5	1	3	4
3	4	2	5	1

1	3	4	5	2
5	2	1	3	4
3	4	5	2	1
2	1	3	4	5
4	5	2	1	3

6	4	7	9	4
9	4	6	5	7
4	7	9	4	6
4	6	4	7	9
7	9	4	6	4

4	5	2	1	3
2	1	3	4	5
3	4	5	2	1
5	2	1	3	4
1	3	4	5	2

0	10	15	20	5
20	5	0	10	15
10	15	20	5	0
5	0	10	15	20
15	20	5	0	10

5	11	18	24	7
[illegible]				

113
½·15·~~226~~ 4 1
5 = 1695

½·15·16

~~45~~ 525

1575
120
1695

(2)

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Examination

195	45	75	90	165	180	150	105	135	60	210	15	30	0	120
135	60	210	15	30	0	120	195	45	75	90	165	180	150	105
45	75	90	165	180	150	105	135	60	210	15	30	0	120	195
60	210	15	30	0	120	195	45	75	90	165	180	150	105	135
75	90	165	180	150	105	135	60	210	15	30	0	120	195	45
210	15	30	0	120	195	45	75	90	165	180	150	105	135	60
90	165	180	150	105	135	60	210	15	30	0	120	195	45	75
15	30	0	120	195	45	75	90	165	180	150	105	135	60	210
165	180	150	105	135	60	210	15	30	0	120	195	45	75	90
30	0	120	195	45	75	90	165	180	150	105	135	60	210	15
180	150	105	135	60	210	15	30	0	120	195	45	75	90	165
0	120	195	45	75	90	165	180	150	105	135	60	210	15	30
[illegible]	[illegible]	135	60	210	15	30	0	120	195	45	75	90	165	180
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	60	210	15	30	0

~~$2a+a'=1,\ 2b+b'=6$~~

~~$4a+3a'=0,\ 4b+3b'=5$~~. $r = a_1 b_1,\ s = c_1 d_1$.

~~$a+2a'=1,\ b+2a'=6$~~

$x+yn$: $\left.\begin{matrix} y=a_1 \\ x=b_1 \end{matrix}\right\} \begin{matrix} y=c_1 \\ x=d_1 \end{matrix}$

$r=21,\ a_1=2,\ b_1=1$

$s=43,\ c_1=4,\ d_1=3$

$(i+xa+ya',\ j+xb+yb') = (i+b_1 a+a_1 a',\ j+b_1 b+a_1 b')$ $\quad [(i,j) = (0, p-1)$ $na\,(0,0)$

$(0+b_1 a+a_1 a',\ p-1+b_1 b+a_1 b')$

$$\left.\begin{matrix} (a+2a',\ 6+b+2b') = (1,6) \\ (3a+4a',\ 6+3b+4b') = (0,5) \end{matrix}\right\}$$

$a+2a'=1\ ;\ b+2b'=0$

$3a+4a'=0\ ;\ 3b+4b'=-1$

$2a' = 3 \equiv -4,\ a'=-2,\ a \equiv 5 \equiv -2$.

$2b' = 1 \equiv 8,\ b' = 4 \equiv -3,\ b=-1$

$u-u=1$ $\begin{pmatrix} a & a' \\ b, & b' \end{pmatrix} = \begin{pmatrix} -2, & -2 \\ -1, & -3 \end{pmatrix}$ $\quad \left.\begin{matrix} a+a' = -4 \equiv 3 \\ b+b' = -4 \equiv 3 \end{matrix}\right\}$

ie regular move is $(-2,-1)$ & breakmove is $(3,3)$ & this is verified to be correct from Fig. 7

Ex(2), Fig. 8, p. 41 [C.M], $p=7$

$r=11,\ s=32\ ;\ a_1=1,\ b_1=1,\ c_1=3,\ d=2$

$$\begin{array}{l|l} a+a'=1, & b+b'=0 \\ 2a+3a'=0 & 2b+3b'=-1 \\ a=3,\ a'=-2 & b=1,\ b'=-1 \end{array}$$

R.M is $(3,1)$, B.M is $(1,0)$. correctly as in Fig. 8

Ex(3), Fig. 9, p. 41 [CM] $p=5$ $\quad (i,j) = (0,4)$.

$r=11,\ s=32,\ a_1=1,\ b_1=1,\ c_1=3,\ d_1=2$

$a+a'=1,\ b+b'=0$

$2a+3a'=0,\ 2b+3b'=-1$

$a \equiv 3 \equiv -2 \,|\, a'=-2 \,|\, b=1,\ b'=-1$

$a+a'=1,\ b+b'=0$.

R.M is $(3,1) \equiv (-2,1)$

B.M is $(1,0)$ Correct as in Fig. 9

(1)

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Examination.

1, 2, 3 | 4, 5, 6 | 7, 8, 9 | 10, 11, 12 | 13, 14, 15

1, 2, 3 | 4, 6, 5 | ~~8~~ 9, 7 | 12, 10, 11 | 15, ~~14~~ 13, 14 nasik series: —— (1)

(15, 12, 8, 4, 1)
(14, 11, 7, 5, 3)
(13, 10, 9, 6, 2)

nasik all right

(10)

	6	5	8	9	7	12	10	11	15	13	14	1	2	3	4
	15	13	14	1	2	3	4	6	5	8	9	7	12	10	11
	5	8	9	7	12	10	11	15	13	14	1	2	3	4	6
	13	14	1	2	3	4	6	5	8	9	7	12	10	11	15
	8	9	7	12	10	11	15	13	14	1	2	3	4	6	5
x	14	1	2	3	4	6	5	8	9	7	12	10	11	15	13
1	9	7	12	10	11	15	13	14	1	2	3	4	6	5	8
	1	2	3	4	6	5	8	9	7	12	10	11	15	13	14
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	6	5	8	9

$(11, 32)_5$

00	11	22	33	44
43	04	10	21	
20	31	42	03	
02	13	24	30	
34	40	01	12	

a b
2a 2b
3a 3b
6a 6b.

0	6	12	18	24
17	23	4	5	11
9	10	16	22	3
21	2	8	14	15
13	19	20	1	7

2 2a
4 3a
b
8 1
03
05 6a.

0, 1, 4
23
23
32

	43	00	12	24
	04	11	23	30
03	10	22	34	41
14	21	33	40	02
20	32	44	01	13

$(11, 14)_5$

1	7	13	19	25
18	24	5	6	12
10	11	17	23	4
22	3	9	15	16
14	20	21	2	8

00, 11, 22, 33, 44
14, 20, 31, 42, 03
23, 34, 40, 01, 12
32, 43, 04, 10, 21
41, 02, 13, 24, 30

(-2,1), (-2,1), (-2,1), (-2,1), (-2,1).
(1,0)
$-2+a^2=1.$
$-1+b^2=0$

61
11, 22 6
671

$(11, 32)_5 \to \begin{pmatrix} -2 & 2 \\ -1 & 1 \end{pmatrix}$

14
26
31
20
31
28
27
22
25
28
29
34
26
27
30
29
22
22
32
31
22
30
25
16

627

3	1	-4
-7	0	7
4	-1	-3

3	5	7
4	9	2
8	1	6

8	1	6
3	5	7
4	9	2

-7, -4, -3, -1, 0, 1, 3, 4, 7.

-7, -4, -1.
-7, 4 1 7.
1, 4 1 3
-3, 0,

-7, -4, 3
-7, 0, 7.
-3, 4,
1,

4	-7	1
3	0	3
-1	7	-4

8, -7, 4
0, -1
-4, 7, -3

4, -7, 3
-1, 0, 1, 4
-3, 7,

$\frac{1}{2} \cdot p^2 \left(p^2(p+1) \cdot p^2 \right.$

$\frac{1}{2} \cdot \{ p^6 + p^2 - 2p^2 \}$

$\frac{1}{2} \cdot p^2 (p^4 - 1)$

31	43	00	12	24
42	04	11	23	30
03	10	22	34	41
14	21	33	40	02
20	32	44	01	13

00	11	22	31	43
11				
22				
33				
44				

00	11	22	33	44
12	23	34	40	01
24	30	41	02	13
31	42	03	14	20
43	04	10	21	32

1	7	13	19	25
8	14	20	21	2
15	16	22	3	9
17	23	4	10	11
24	5	6	12	18

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

43	00	12	24	31
04	11	23	30	42
10	22	34	41	03
21	33	40	02	14
32	44	01	13	20

24	1	8	15	17
5	7	14	16	23
6	13	20	22	4
12	19	21	3	10
18	25	2	9	11

24	31	43	00	12
30	42	04	11	23
41	03	10	22	34
02	14	21	33	40
13	20	32	44	01

15	17	24	1	8
16	23	5	7	14
22	4	6	13	
	10			
9				

43	31	24	12	00	44
04	42	30	23	11	33
10	03	41	34	22	22
21	14	02	40	33	11
32	20	13	01	44	

04, 03, 02, 01, 00

00	01	02	03	04
02	03	04	00	01
04	00	01	02	03
01	02	03	04	00
03	04	00	01	02

1	7	13	19	25
18	24	5	6	12
10	11	17	23	4
22	3	9	15	16
14	20	21	2	8

=

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3

(3,2)

+

0	5	10	15	20
15	20	0	5	10
5	10	15	20	0
20	0	5	10	15
10	15	20	0	5

(2,3)

Fig. 9, p. 41 (C.M)

Ex (ii), Fig. 10, p = 41 [CM]

$r = 11,\ s = 23$

$a + a' = 1,\quad b + b' = 0.$
$3a + 2a' = 0,\quad 3b + 2b' = -1$

$a' = 3 \equiv -2,\ a = 3 \equiv -2.$
$b' = 1,\ b = -1$

RM is (-2, -1)
BM is (1, 0). Corrected as in Fig. 10

Note: All the squares in Ex (1) – (4) are nasik

Conversely take a magic square with 1 in upper left hand corner & put it in the Tarry's numbered series form

Andrews: Fig. 14, p. 11 — Here RM is (2, 1)
BM is (1, -1).

$$\begin{pmatrix} a & a' \\ b & b' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$$

1	15	24	8	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3
9	18	2	11	25

(a)

$0 + 2b_1 - a_1 = 1. \qquad 4 + b_1 - 2a_1 = 4$
$0 + 2d_1 - c_1 = 0 \qquad 4 + d_1 - 2c_1 = 3$

$2b_1 - a_1 = 1. \qquad 3a_1 \equiv 1 \qquad a_1 = 2$
$b_1 - 2a_1 = 0. \qquad b_1 = 4$

$2d_1 - c_1 = 0 \qquad 3d_1 = 1,\ d_1 = 2$
$d_1 - 2c_1 = -1 \qquad c_1 = 4.$

$r = 24,\ s = 42$

00	24	43	12	31
42	~~30~~ 11	30	04	23
34	03	22	41	10
21	40	14	33	02
13	32	01	20	44

(b)

& this (b) put in decimal notation is the same as (a).
- This is associated and also nasik unlike Ex (3) & (4) above which are nasik but not associated

So we have found a new Tarry numbered series for p = 5

120 = 2.15.7.8

B.U.P.—J. 1240-5,000 Bks. × 6-7-68.

(0)

University of Bombay

______________________ Examination.

Latin square of order 15:

45

A Latin semi: — (A)

Primary no: 1, 2, 3, 4, 5 | 8, 10, 7, 9, 6 | 15, 12, 14, 11, 13)

Introd no: 0, 15, 30, 45, 60 | 105, 135, 90, 120, 75 | 210, 165, 195, 150, 180.

5	8	10	7	9	6	15	12	14	11	13	1	2	3	4
14	11	13	1	2	3	4	5	8	10	7	9	6	15	12
8	10	7	9	6	15	12	14	11	13	1	2	3	4	5
11	13	1	2	3	4	5	8	10	7	9	6	15	12	14
10	7	9	6	15	12	14	11	13	1	2	3	4	5	8
13	1	2	3	4	5	8	10	7	9	6	15	12	14	11
7	9	6	15	12	14	11	13	1	2	3	4	5	8	10
1	2	3	4	5	8	10	7	9	6	15	12	14	11	13
9	6	15	12	14	11	13	1	2	3	4	5	8	10	7
2	3	4	5	8	10	7	9	6	15	12	14	11	13	1
6	15	12	14	11	13	1	2	3	4	5	8	10	7	9
3	4	5	8	10	7	9	6	15	12	14	11	13	1	2
15	12	14	11	13	1	2	3	4	5	8	10	7	9	6
4	5	8	10	7	9	6	15	12	14	11	13	1	2	3
12	14	11	13	1	2	3	4	5	8	10	7	9	6	15

not Latin in lower regions

a semi-Latin

(12)

Perhaps (A) is not a Latin semi:

135/12/147 ×

(A) – Step. (7, 8)

1, 2, 3, | 4, 5, 6 | 7, 8, 9 | 10, 11, 12 | 13, 14, 15

(1, 8, 15), (1, 9, 14), (1, 10, 13), (1, 11, 12)
(2, 7, 15), (2, 8, 14), (2, 9, 13), (2, 10, 12), ~~(2, 11)~~
(3, 6, 15), (3, 7, 14), (3, 8, 13), (3, 9, 12), (3, 10, 11)
(4, 5, 15), (4, 6, 14), (4, 7, 13), (4, 8, 12), (4, 9, 11),
(5, 4, 15), (5, 6, 13), (5, 7, 12), (5, 8, 11), (5, 9, 10)
(6, 3, 15), (6, 4, 14), (6, 5, 13), (6, 7, 11), (6, 8, 10),
(7, 2, 15), (7, 3, 14), (7, 4, 13), (7, 5, 12), (7, 6, 11), (7, 8, 9),
(8, 1, 15), (8, 2, 14), (8, 3, 13), (8, 4, 12), (8, 5, 11), (8, 6, 10), (8, 7, 9),
(9, 1, 14), (9, 2, 13), (9, 3, 12), (9, 4, 11), (9, 5, 10), (9, 7, 8)
(10, 1, 13), (10, 2, 12), (10, 3, 11), (10, 5, 9), (10, 6, 8)
(11, 1, 12), (11, 3, 10), (11, 4, 9), (11, 5, 8), (11, 6, 7),
(12, 1, 11), (12, 2, 10), (12, 3, 9), (12, 4, 8), (12, 5, 7), ~~(12, 6)~~
(13, 1, 10), (13, 2, 9), (13, 3, 8), (13, 4, 7), (13, 5, 6), ~~(13, 6, 5)~~, ~~(13, 7, 4)~~, ~~(13, 8, 3)~~, ~~(13, 9, 2)~~, ~~(13, 10)~~
(14, 1, 9), (14, 2, 8), (14, 3, 7), ~~(14, 5, 6)~~, (14, 4, 6), ~~(14, 5, 3)~~
(15, 1, 8), (15, 2, 7), (15, 3, 6), (15, 4, 5),

Take 5's one from each triplet

24

~~1, 4, 7, 10, 13~~ ×
(1, 5, 8, 11, 15)
(1, 5, 8, 10, 15)
~~(1, 6, 8, 9, 12)~~
~~2, 4, 9, 10, 12~~
~~(2, 5, 7, 9~~, 10, 14
2, 4, 11, 13

(13)

(14) With $r = 24$, $s = 42$ we have also found an associated series. Can it be that $r = ab$, $s = ba$ gives such a square in general ($a \neq 0$, $b \neq 0$ ∵ of pleine)

Try $r = 23$, $s = 32$ → this leads to repetitions

00	23	41	14	32
32	~~44~~			
14				
41				
23				

Obviously here the conditions ($r = a_1b_1$, $s = a_2b_2$; $a_1 + a_2 \not\equiv 0$, $b_1 + b_2 \not\equiv 0 \pmod 5$) are not satisfied (cf. p. 36 bottom)

Try $r = 13$, $s = 31$

00	13	21	34	42
31	44	02	10	23
12	20	33	41	04
43	01	14	22	30
24	32	40	03	11

→

1	9	12	20	23
17	25	3	6	14
8	11	19	22	5
24	2	10	13	16
15	18	21	4	7

nasik all right

but not

associated

$r = 34$, $s = 43$

00	34	13	42	21
43	22			
31		44		
24			11	
12				

won't be associated:

(33) → this should be 44 = 31 + 13 (22 + 22)

So it looks $r = 24$, $s = 42$ is the only possible one giving associated nasik series with 1 in the top left hand corner

~~43~~ 34

(9)	7	12	10	11	15	13	14	1	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
(10)	2	3	4	6	5	8	9	7	12	10	11	15	13	14	1
(11)	12	10	11	15	13	14	1	2	3	4	6	5	8	9	7
(12)	3	4	6	5	8	9	7	12	10	11	15	13	14	1	2
(13)	10	11	15	13	14	1	2	3	4	6	5	8	9	7	12
(14)	4	6	5	8	9	7	12	10	11	15	13	14	1	2	3
	11	15	13	14	1	2	3	4	6	5	8	9	7	12	10

(A) – Step (7, 8)

Set another magic series

2, 3, 1 | 9, 14, 4 | 6, 7, 12 | 13, 11, 8 | 10, 5, 15 — (2) magic series

15, 30, 0 | 120, 195, ~~45~~ | 75, 40, 165 | 180, 150, 165 | 135, 60, 210

(18) C.A, p. 41 – Remarks that $(11, 23)_p$ form orthog nasik series for all prime p 73

What about $(24, 42)_p$? $2+4 \not\equiv 0 \pmod 7$, $4+2 \not\equiv 0 \pmod 7$
$4-2 \not\equiv 0 \pmod 7$ i.e. nasik series

but this is not associated

Try $p=7$

00	24	41	65	12	36	53
42	66	13	30	54	01	25
14	31	55	02	26	43	60
56	03	20	44	61	15	32
21	45	62	16	33	50	04
63	10	34	51	05	22	46
35	52	06	23	40	64	~~30~~ 11

→

1	19	30	48	10	28	39
31	49	11	22	40	2	20
12	23	41	3	21	32	43
42	4	15	33	44	13	24
16	34	45	14	25	36	5
46	8	26	37	6	17	35
27	38	7	18	29	47	9

nasik all right

but not associated:

39 $r = 35$, $s = 53$.

~~42~~ 24.

ab ba

~~$6a \equiv 4$~~
~~$6b \equiv 6$~~

$6a \equiv 2$
$6b \equiv 4$.

$r = 53$, $s = 35$

24 42

24 13 $a = 5$.

31 35

Try $r = ~~53~~$, $s = ~~35~~$ 53

00	35	63	21	56	14	42
53	11	46	04	32	60	25
36	64	22	50	15	43	01
12	40	05	33	61	26	54
65	23	51	16	44	02	30
41	06	34	62	20	55	13
24	52	10	45	03	31	66

→

1	27	46	16	42	12	31
39	9	35	5	24	43	20
28	47	17	36	13	32	2
10	29	6	25	44	21	40
48	18	37	14	33	3	22
30	7	26	45	15	41	11
19	38	8	34	4	23	49

associated and nasik.

120	195	45	75	90	165	180	120	195	45	75	90	165	180
105	135	60	210	15	30	0							

525

201, 50, 83, 99, 172, 192, 160, 116, 150

X Learnt repetition with (A)

(9) - Step. (7,8). Should have taken (8,7)

5	1	3	4	6
4	6	5	1	3
1	3	4	6	5
6	5	1	3	4
3	4	6	5	1

(2,7)

1	2	4	6	5
6	5	1	2	4
2	4	6	5	1
5	1	2	4	6
4	6	5	1	2

(2,3)

6	3	7	10	11
10	11	6	3	7

repetition:

2	1	3	4	5
4	5	2	1	3
1	3	4	5	2
5	2	1	3	4
3	4	5	2	1

10	15	20	5	0
20	5	0	10	15
0	10	15	20	5
15	20	5	0	10
5	0	10	15	20

4	6	5	1	2
5	1	2	4	6
2	4	6	5	1
6	5	1	2	4
1	2	4	6	5

(3,2)

9	7	10	5	8

12	16	23	9	5
24	10	2	11	18
1	13	19	25	7
20	22	6	3	14
8	4	15	17	21

63

(17)

(18) So it works for p = prime, and if we take

for $p = 5$, $r = 24$, $s = 42$ ~~...~~ gives square $\underline{n}$ & $\underline{a}$ — only case — 2 cases

$p = 7$, $r = 35$, $s = 53$; $r = 26$, $s = 62$ " " — 5 cases

$p = 11$, $r = 57$, $s = 75$. (?) ie if $r = ab$, $s = ba$, $a + b \equiv 1 \pmod p$

$10r = 64$, $10s = 46$ → 10/10

Then for $p = 7$, both pleine(r)&(s), $r = 26$, $s = 62$ should work. Appears to work

because $6r = 51$, $6s = 15$ & right hand bottom term would be 66 complementary to 00

For $p = 11$, ($r = 2,10$, $s = 10,2$), ($r = 39$, $s = 93$), ($r = 48$, $s = 84$) should all work, ie 5 cases

$10r = 91$, $10s = 19$ ✓ | $10r = 82$, $10s = 28$ ✓ | $10r = 73$, $10s = 37$ ✓

For $p = 7$, $r = 26$, $s = 62$.

00	26	45	64	13	32	51
62	11	30	56	05	24	43
54	03	22	41	60	16	35
46	65	14	33	52	01	20
31	50	06	25	44	63	12
23	42	61	10	36	55	04
15	34	53	02	21	40	66

→

1	21	34	47	11	24	37
45	9	22	42	6	19	32
40	4	17	30	43	14	27
35	48	12	25	38	2	15
23	36	7	20	33	46	10
18	31	44	8	28	41	5
13	26	39	3	16	29	49

Nasik & associated;

195	45	15	90

111	140	68	219	22	42	10	131	210	58	89	91	167	183	154
135	208	59	76	92	168	184	156	110	143	69	217	27	40	11
155	113	144	67	222	25	41	15	133	209	46	77	93	169	186
13	134	196	47	78	94	171	185	158	114	142	72	220	26	45
188	159	152	147	70	221	30	43	14	121	197	48	79	96	170
44	1	122	198	49	81	95	173	189	157	117	145	71	225	28
174	187	162	115	146	75	223	29	31	2	123	199	51	80	98
16	32	3	124	201	50	83	99	172	192	160	116	150	73	224
97	177	190	161	120	148	74	211	17	33	4	126	200	53	84
212	18	34	6	125	203	54	82	102	175	191	165	178	149	61
87	100	176	195	163	119	136	62	213	19	36	5	128	204	52
63	214	21	35	8	129	202	57	85	101	176	193	164	106	137
55	86	105	178	194	151	107	138	64	216	20	38	9	127	207
139	66	215	23	39	7	132	205	56	90	103	179	181	152	108
206	60	88	104	166	182	153	169	141	65	218	24	37	12	130

nanu

C

20

Try for $p = 11$, ~~$x = 57$, $s = 75$~~ $r = 48$, $s = 84$

00	48	85	12	5,10	97	24	61	10,9	36	73
84	11	59	96	23	60	10,8	35	72	0,10	47
58	95	22	6,10	10,7	34	71	09	46	83	10
21	69	10,6	39	70	08	45	82	1,10	57	94
10,5	32	7,10	07	44	81	19	56	93	20	68
79	06	43	80	18	55	92	2,10	67	10,4	31
42	8,10	17	54	91	29	66	10,3	30	78	05
16	53	90	28	65	10,2	3,10	77	04	41	89
9,10	27	64	10,1	39	78	03	40	88	15	52
63	10,0	38	75	02	4,10	87	14	51	99	26
37	74	01	49	86	13	50	98	25	62	10,10

1	53	94	14	66	107	27	68	120	40	81
93	13	65	106	26	67	119	39	80	11	52
64	105	25	77	118	38	79	10	51	92	12
23	76	117	37	78	9	50	91	22	63	98
116	36	88	8	49	90	21	62	103	23	75
87	7	48	89	20	61	102	33	74	115	35
47	99	19	60	101	32	73	114	34	86	6
18	54	100	31	72	113	44	85	5	46	98
110	30	71	102	43	84	4	45	97	17	58
70	111	42	83	3	55	96	16	57	104	29
41	82	2	54	95	15	56	108	28	69	121

nasik & assured

$S = 671$

(1)	180	30	15	0	45	75	60	90	105	120	150	135	165	210	195
(2)	210	195	180	30	15	0	45	75	60	90	105	120	150	135	165
(3)	135	165	210	195	180	30	15	0	45	75	60	90	105	120	150
(4)	120	150	135	165	210	195	180	30	15	0	45	75	60	90	105
(5)	90	105	120	150	135	165	210	195	180	30	15	0	45	75	60
(6)	75	60	90	105	120	150	135	165	210	195	180	30	15	0	45
(7)	0	45	75	60	90	105	120	150	135	165	210	195	180	30	15
(8)	30	15	0	45	75	60	90	105	120	150	135	165	210	195	180
(9)	195	180	30	15	0	45	75	60	90	105	120	150	135	165	210
(10)	165	210	195	180	30	15	0	45	75	60	90	105	120	150	135
(11)	150	135	165	210	195	180	30	15	0	45	75	60	90	105	120
(12)	105	120	150	135	165	210	195	180	30	15	0	45	75	60	90
(13)	60	90	105	120	150	135	165	210	195	180	30	15	0	45	75
(14)	45	75	60	90	105	120	150	135	165	210	195	180	30	15	0
(15)	15	0	45	75	60	90	105	120	150	135	165	210	195	180	30

B (nasikayun. series) – Step (2, 13)

$x_2 + x_5 + 8 + (16 - x_4) + (16 - x_2) = 40$ ✓

$x_3 + x_6 + (16 - x_7) + (16 - x_4) + (16 - x_1) = 40.$

$x_1 + x_4 + x_7 = x_3 + x_6 + 8$

$x_4 + x_7 = x_6 + 10$

$x_2 = 2,\ x_5 = 5,\ x_8 = 8,\ x_{11} = 11,\ x_{14} = 14.$

3 2 1	4 6 5	7 8 9	11 10 12	15 14 13

$x_1 + x_4 + x_7 + x_{10} + x_{13} = 40$

$x_3 + x_6 + x_9 + x_{12} + x_{15} = 40.$

$x_1 + x_{15} = x_3 + x_{13} = x_6 + x_{12} = x_6 + x_{10}$

$= x_7 + x_9 = 16.$

$x_1 = 1,\ x_4 = 13,\ x_7 = 4,\ x_{10} = 12,\ x_{13} = 10$

1, 3, 4, 6, 7, 9, 10, 12, 13, 15

(1, 15)
(3, 13)
(4, 12)
(6, 10)
(7, 9)

15, 3, 6, 7, 9

1 3 4 6 7
15 13 12 10 9

$x_2 + x_5 + x_{11} + x_{14}$
$+ 8 = 40.$

~~1, 2, 3, 4, 5, 6, 7, 8~~
~~15, 14, 13, 12, 11, 10, 9, 8~~

2, 6, 10, 14

14, 15, 17, 18, 20.
50, 49, 48, 46, 44.

Symmetric nanti semis

4, 1, 6 | 7, 9 | 3 | 11 | 15

3, 4, 7, 11, 15
2, 6, 8, 10, 14
1, 5, 9, 12, 13

11, 4, 7, 15, 3
10, 6, 8, 14, 2
5, 12, 9, 1, 13

11, 10, 5	4, 6, 12	7, 8, 9	15, 14, 1	3, 2, 13

1, 2, 3, 4, 5, 6, 7, 8, 9

123 | 564 | 978

312 | 456 | 897

22

$15b+25b'=0$
$15b+9b'=-3$

$16b'=3$
$b'=5$

$15a+25a'=5$
$15a+9a'=0$
$16a'=5.,\ a'=6=-1.$
$3b=10\equiv 3\quad a=\quad 16a=3$

$\cancel{5a=-18\equiv}3 \quad 5a=3 \quad \cancel{a=10}\ 80$
$\underline{a=2}$
64

$a=5,\ a'=3.$
$6a'=-10\cdot$
$3a'=-5$

(28)

what about the fig in bottom

of pp. 16, 18, 20 accepts G.G.D.M? $2b'=-3b$
$=-12$
$b'=-6$

$\cancel{16a=3}$
$16b=1$

Fig on p. 16: $p=7,\ r=55,\ s=53,\ (i,j)=(0,6)$
in $(1,6)$, in $(0,5)$

$\cancel{5a+3a'=1}\ 5a+3a'=1,\ 3b+3b'=0$
$3a+5a'=0,\ 3b+5b'=-1.$

$a'=2,\ a=-1,\ b=-2,\ b'=1$

R.M is $\cancel{(2,1)}$ $(-1,-2)$, B.M is $(1,-1)$. Which is Correct.

Fig on p. 18. $p=7,\ r=26,\ s=62$

$6a+2a'=1 \mid 6b+2b'=0$
$2a+6a'=0 \mid 2b+6b'=-1.$
$a=5,\ a'=3 \mid b=4,\ \cancel{b'=...}$
$a=-2$ ✓ $\mid b\equiv -3.\ b'=2$

R.M is $(-2,-3)$, B.M is $\cancel{(\ ,2)}$ $(1,-1)$ ✓ correct

$-16b'=1$
$a=-3a'$
$=-9$
$=-2$

$b'=-3b$
$\cancel{-16b}$
$16b=1.$
$\underline{b=4}$

Verifies both the results & our rules

Coronal taken

properties of the above figures

$-16b=$
$b'=-3b\quad b'=9\equiv 2$
$2b-18b=-1$
$16b=1.\ b=-3.$

Fig. 16 (top one): $p=7,\ r=24,\ s=42$

$4a+2a'=1,\ 4b+2b'=0$
$2a+4a'=0,\ 2b+4b'=-1$

$a'=1,\ a=-2\ \}\ b=0-1,\ b'=+2$

RM is $\cancel{(2,1)}$, BM is $(-1,-1)$
$(-2,-1)$, BM is $(-1,1)$ ✓ Correct

$a=-2a'$
$-6a'=1.\ a'=1.$

$b'=-2b$
$-6b=-1$
$6b=1.$
$\underline{b=-1}$

P.T.O

			X
00	12	44	
54	25	195	X
8	41	20.7	X
22	189	220	X
35	202	176	X
186	218	144	X
199	170	158	X
211	141	127	X
167	154	110	X
138	121	96	X
163	107	64	X
134	93	76	X
120	73	47	X
102	89	13	X
70	60	28	

$p.\ 20$ Case $p = 11,\ r = 48,\ s = 84$.

$(i, j) = (0, 10)$. 48 is in $(1, 10)$, & (84) in $(0, 9)$

$(i + 8a + 4a',\ j + 8b + 4b') = (1, 10)$

$(i + 4a + 8a',\ j + 4b + 8b') = (0, 9)$

$$\left.\begin{aligned} 8a + 4a' = 1, &\quad 8b + 4b' = 0 \\ 4a + 8a' = 0, &\quad 4b + 8b' = -1 \end{aligned}\right\}$$

$$a = -2a',\ \left.\begin{aligned} -12a' &= 1 \\ a' &= -1 \\ a &= 2 \end{aligned}\right\} \qquad \begin{aligned} b' = -2b,\ -12b &= -1 \\ b &= 1 \\ b' &= -2 \end{aligned}$$

R.M is $(2, 1)$, BM is $(1, -1)$ which is correct ✓

$a = -2a',\ -6a' = 1$,

$b' = -2b,\ -6b = -1$,

$\begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix}$ RM $(2, 1)$ BM $(-1, -1)$

$a' = -1,\ a = 2$

$b = 1,\ b' = -2$ $\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$

RM $(2, 1)$, BM $(1, -1)$ ✓

Reln between C.M. Chap I & G.I.D.M. Case p (prime).

Say, $r = y_1 x_1$, $s = y_2 x_2$ $(i, j) = (0, p-1)$.

r in $(1, p-1)$, s in $(0, p-2)$

i.e.

$0 + x_1 a + y_1 a' = 1,\ (p-1) + x_1 b + y_1 b' = (p-1)$

$0 + x_2 a + y_2 a' = 0,\ (p-1) + x_2 b + y_2 b' = p - 2$.

$$\left.\begin{aligned} x_1 a + y_1 a' &\equiv 1 \\ x_2 a + y_2 a' &\equiv 0 \end{aligned}\right\} \quad \left.\begin{aligned} x_1 b + y_1 b' &\equiv 0 \\ x_2 b + y_2 b' &\equiv -1 \end{aligned}\right\} \pmod{p}$$

For (b) on p. 11

$r = 24,\ s = 42$ $(i, j) = (0, 4)$
$\quad y_1 x_1 \quad y_2 x_2$

$$\left.\begin{aligned} 4a + 2a' &= 1 \\ 2a + 4a' &= 0 \end{aligned}\right| \left.\begin{aligned} 4b + 2b' &= 0 \\ 2b + 4b' &= -1 \end{aligned}\right\}$$

giving $a = 2,\ a' = -1,\ b = 1,\ b' = -2$

i.e RM is $(2, 1)$, BM is $(1, -1)$ which is correct

Given (x_1, x_2, y_1, y_2) we can find a, b, a', b' i.e R.M and BM of G.I.D.M

Given (x_1, x_2, y_1, y_2) we can find a, b, a', b' for $p = 5, 7, 11$.

& we have derived a number of special examples for $p = 5, 7, 11$.

Conversely – Given a magic square – Notice later that even if 1 be not in the top left hand corner, the Jarry numeral series works – Take Andrews Fig. 41 p. 17

15	3	16	9	22	15
2	20	8	21	14	2
19	7	25	13	1	19
6	24	12	5	18	6
23	11	4	17	10	23
	VII – Fig 41,	p. 57			

Middle row can be written as 11, 44, 22, 00, 33 & col. 4 as

41, 23, 00, 32, 14

Taking 33 element to right of 00, and 32 below 00, we can develop middle row & 4th col. going to right & backwards, going down & up-wards respectively with $r = 33,\ s = 32$ i.e. 33, 11, 44, 22, 00 (19, 7, 25, 13, 1)

and 32, 14, 41, 23, 00 (18, 10, 22, 14, 1)