May 20, 1997

Dear Dr Arun Venkataraman,

I am writing this in response to your querry to Prof. A. P. Gore. As per his suggestion, I have looked at your problem and come up with the following solution :

Kaplan - Meier Estimator of Survival function :

Suppose the distribution is discrete with jumps at finitely many specified time points $a_1 < a_2 ... < a_g$. Usually these points are equally spaced but this is <u>not</u> necessary condition. Let r_j be the number of individuals in view (still breeding) at time point a_j and d_j be the number who fail (stop breeding) at time point a_j. It is conventional to include in r_j any individuals who are censored at a_j. Let

$$\begin{aligned} F(t) &= \text{survival function} \\ &= 1 - \text{distribution function} \end{aligned}$$

$$\begin{aligned} \hat{F}(t) &= \text{Kapln} - \text{Meier estimator of } F(t) \\ &= \pi^{(t)}(1 - \frac{d_j}{r_j}) \qquad (1) \end{aligned}$$

where $\pi^{(t)}$ denotes product over j such that $a_j < t$. Using (1) we want to estimate mean (μ) of the distribution. It is given by

$$\hat{\mu} = \int_0^{\infty} \hat{F}(t)dt \qquad (2)$$

However when a_g (the largest observation) is censored we have a difficulty since $\hat{F}(t)$ does not approach to zero as $t \to \infty$ and hence the integral in (2) is infinite. To overcome this difficulty we compute restricted mean defined as

$$\hat{\mu} = \int_0^{L} \hat{F}(t)dt \qquad (3)$$

[References : (1) " Analysis of survival data " by D. R. Cox and D. Oakes (1984) Chapt. 4 p. 48, Chapman and Hall, New York. (2) "Survival Analysis' Millar R. G. (1981), p. 71, Wiley, New York.]

Calculation of the product limit estimator and mean for breeding tenure of dholes :

a_j	r_j	d_j	$1-\frac{d_j}{r_j}$	$\pi_{i\leq j}(1-\frac{d_i}{r_i}) = \hat{F}(a_j)$
1	6	0	1	1
2	5	0	1	1
4	4	2	0.5	0.5
5	2	0	1	0.5
6	1	0	1	0.5

It may be noted that $\hat{F}(0) = 1$. We compute restricted mean since the largest observation is censored.

$$\begin{aligned}\hat{\mu} &= \int_0^6 \hat{F}(t)dt \\ &= 1\times 0 + 1\times 0 + 4\times 1/2 + 4\times 1/2 + 5\times 0 + 6\times 0 \\ &= 4\end{aligned}$$

Thus the result agrees with that of referees computation. However, the result should be interpreted with caution as there are only six observations out of which 4 are censored.

If you need any clarification, you can send me e-mail on the address of Prof. A. P. Gore.

With regards,

Yours sincerely,
(Sudha Purohit)