2|9|77

Book. 17

$\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$

(1)

For I. D. M, I. M & W. M, B. M's taken as Knight's moves keeping the R-M's intact. Cases $n = 7, 9$.

~~$n = 7$ gives [illegible]~~.

$n = 9$. ~~$n = 7$~~

(i) I. D. M – 4 cases $\begin{pmatrix} 1 , 1 \\ 1 , -2 \end{pmatrix}$, $\begin{pmatrix} 1 , -2 \\ 1 , 1 \end{pmatrix}$, $\begin{pmatrix} 1 , -3 \\ 1 , -2 \end{pmatrix}$, $\begin{pmatrix} 1 , -2 \\ 1 , -3 \end{pmatrix}$.

(a) X (c) X (b) (d)

	(1,2)	(-1,2)	(1,-2)	(-1,-2)
(2,1)	×			×
(-2,1)		×	×	
(2,-1)		×	×	
(-2,-1)	×			×

of these (a) gives $ab' - a'b = -3$ so no use for $n = 9$ } These cases give 5 cases for $n = 5$

(c) $ab' - a'b = 3$ "

(b) r.d is $y = c$, l.d is $2x - 5y = d$; $8 - 5y = d \rightarrow d = 8, 3, -2, -7$ … $-27, -32$

$(-2, 1)$ $j - 2x + y = 2$ $j \rightarrow (i, j) = ($ …

$\begin{pmatrix} -2 , 2 \\ 1 , -2 \end{pmatrix}$ $\left.\begin{matrix} i - 2x + 2y = 2 \\ j + x - 2y = 2 \end{matrix}\right\}$ $(x, y) = (2, 2) \rightarrow (i, j) = (2, 4)$

$\begin{pmatrix} 1 , -1 \\ -2 , 1 \end{pmatrix}$ $\begin{matrix} i + x - y = 2 \\ j - 2x + y = 2 \end{matrix}$ | $(x, y) = (2, 2) \rightarrow (2, 4)$.

$10 + 30 + 50 + …$

$13 + 14 + 15 + 11 + 12 = 65$

$13 + 15 + 12 + 14 + 11 = 65$ ✓

$13 + 8 + 3 + ~~10~~ 8$

$\begin{pmatrix} 2 , -2 \\ -1 , 0 \end{pmatrix}$ $\begin{pmatrix} 1 , -1 \\ 2 , -3 \end{pmatrix}$ $\begin{pmatrix} -1 & 1 \\ 2 , -3 \end{pmatrix}$ | $\begin{pmatrix} 1 , -1 \\ 2 , -4 \end{pmatrix}$ $\left.\begin{matrix} -x + 3y = c & (3) \\ 3x - 5y = d & (3) \end{matrix}\right)$ $+ 23 + 18 = 65$

$\begin{pmatrix} 2 , -2 \\ 1 , -2 \end{pmatrix}$ $\begin{matrix} x = c \\ 3x - 4y = d \end{matrix}$ ‖ | $1 + 2 + 3 + 4 + 5 = 15$ X

$\begin{pmatrix} 1 & 1 \\ 1 , -2 \end{pmatrix}$ $\begin{matrix} i + x + y = 2, 3 \\ j + x - 2y = 2, 3 \end{matrix}$ | $(x, y) = (2 2)$ $(x, y = (3, 3) \rightarrow (i, j) = ~~(-4, 5)~~ ~~(3, 5)~~ = (-3, 6) = (4, 6)$

$\rightarrow (i, j) = (-2, 4) = (3, 4)$

$(3, 4) \rightarrow$ as reached.

24	8	17	1	15
7	16	5	14	23
20	4	13	22	6
3	12	21	10	19
11	25	9	18	2

$8 + 43 + 58 + 20$

$+ 1 + 46 + 63 + 21 = 260$

$(2, -1)$

$\begin{pmatrix} 1 , -2 \\ 1 , 1 \end{pmatrix}$

2	19	6	23	15
18	10	22	14	1
9	21	13	5	17
25	12	4	16	8
11	3	20	7	24

$-1, 2)$

$i + x - 2y = 2, 3$

$j + x + y = 2, 3$

$(x, y) = (2, 2)$, $n = 7$, $(x, y) = (3 3) \rightarrow (i, j) = (6, -3)$ $(6, 4)$

$\rightarrow (i, j) = (4, -2) = (4, 3)$ as reached.

(2)

$\begin{pmatrix}2, & -1\\ 1, & 1\end{pmatrix}, \begin{pmatrix}1, & 1\\ 2, & -1\end{pmatrix}, \cancel{\begin{pmatrix}2 & -3\\ 2, & 1\end{pmatrix}}, \cancel{\begin{pmatrix}-1 & 3\\ 2, & -1\end{pmatrix}}, \begin{pmatrix}2, & -3\\ 1, & -3\end{pmatrix}, \begin{pmatrix}-1, & 3\\ -2, & 3\end{pmatrix}.$

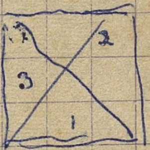
$\cancel{\begin{pmatrix}-2 & 3\\ 1 & 1\end{pmatrix}}, \begin{pmatrix}-2, & 1\\ 1, & 1\end{pmatrix}, \begin{pmatrix}-1, & -1\\ 2, & -1\end{pmatrix}, \begin{pmatrix}-2, & 3\\ 1, & -3\end{pmatrix}, \begin{pmatrix}+1, & -3\\ -2, & 3\end{pmatrix}.$

$\begin{pmatrix}2, & -3\\ -1, & 3\end{pmatrix}, \begin{pmatrix}-1, & 3\\ 2, & -3\end{pmatrix}, \begin{pmatrix}2, & -1\\ -1, & -1\end{pmatrix}, \cancel{\begin{pmatrix}+1 & 3\\ -2, & +3\end{pmatrix}} \begin{pmatrix}+1, & 1\\ -2, & 1\end{pmatrix}$

$\begin{pmatrix}-2, & 3\\ -1, & 3\end{pmatrix}, \begin{pmatrix}1, & -3\\ 2, & -3\end{pmatrix}, \begin{pmatrix}-2, & 1\\ -1, & -1\end{pmatrix}, \begin{pmatrix}-1, & -1\\ -2, & 1\end{pmatrix}$

$(\pm 1, 0), (\pm 1$

$(0, \pm 1)$



Keeping B.M's same, R.M's are too K.M's.

$4+31+44+40 = 26\,0$

17.

S.D.M: $(2,1)$, $\cancel{(-1,-2)}$

$i+2x+y=2$

$\cancel{j+x-2y}$

$\begin{pmatrix}2, & -2\\ 1, & -2\end{pmatrix}$ $\left.\begin{matrix} i+2x-2y=2\\ i+x-2y=2\end{matrix}\right\}$ $(x,y)=(2,2)$ $\rightarrow (i,j) = (2,4)$

$(-1, 1)$ $\left. i - x + y = 2 \right\}$ $(x,y) = (2,2)$

$(2, 4)$

$-7, -12, -17, -22,$

$2x \ldots \rightarrow d = -20, -18, -16, -14, -12, \quad \equiv 8, 3, 7, 2, 6, 1, 5, 0, 4.$

$-10, -8, -6, -4 \equiv 7, 0, 2, 4, 6, 8, 1, 3, 5$

ie $c = 4$, $d = 0, 1, 2, \ldots 8 \rightarrow (x,y) = (0,4), (1,4), \ldots (8,4)$

$\left.\begin{matrix} i+x-3y=4\\ j+x-2y=4\end{matrix}\right\}$ $(i,j) = (16,12), (15,11), (14,10), (13,9), (12,8), (11,7), (10,6), (9,5), (8,4)$

$\equiv (7,3), (6,2), (5,1), (4,0), (3,8), (2,7), (1,6), (0,5), (8,4)$

(d) u.d is $y = c$, l.d is $2x - 5y = d$, $\rightarrow$ identical values (i,j)

n = 7 – For case (b) u.d is $y = c$ & l.d is $2x - 5y = d$ or $x \equiv d$. giving only one value $(2,2)$ for (x,y) in the case $n = 5$, but for $n = 7$, we cannot take $x \equiv d$ so here also we get 7 values (i,j)

The same is true of (d)

$(2,1), (-2,1), (2,-1), (-2,-1)$

$(1,2), (-1,2), (1,-2), (-1,-2)$

$(2,1)$ & $(1,2)$,
$(2,1)$ & $(-1,-2)$

$(-2,1)$ & $(-1,2)$
$(-2,1)$ & $(1,-2)$

$(2,-1)$ & $(-1,2)$
$(2,-1)$ & $(1,-2)$

$(-2,-1)$ & $(1,2)$
$(-2,-1)$ & $(1,-2)$

$\begin{pmatrix}2, & -1\\ -1, & 3\end{pmatrix}$

$44 + 30 + 16 + 2 + 37 + 23 + 9$

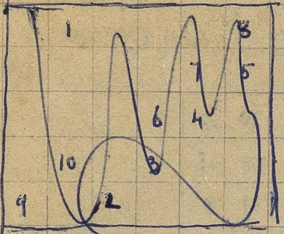
$= 162$

$\begin{pmatrix}1 & 1\\ 2 & -1\end{pmatrix}.$

$7 + 2 + 27 + 17 + 12 = 6\,0 \quad 13 + 3 + 18 + 8 + 23 = 6$

$\begin{pmatrix}1, & -3\\ 1, & -2\end{pmatrix}$ $i+x-3y=2 \to 3$, $j+x-2y=2 \to 3$ $(x,y)=(2,2), \to (i,j) = (6,14)=\underline{(1,4)}$ | $n=7, (x,y)=(3,3)$ $\longrightarrow (i,j) = (9,6) = (2,6)$

BR.17



8	1	24	17	15
5	23	16	14	7
22	20	13	6	4
19	12	10	3	21
11	9	2	25	18

associate ✓

18	21	4	7	15
25	3	6	14	17
2	10	13	16	24
9	12	20	23	1
11	19	22	5	8

associated ✓

$\begin{pmatrix}1, & 1\\ -1, & 2\end{pmatrix}$ $i+x+y=2$, $j-x+2y=2$ | $(x,y)=(2,2) \to (i,j) = (-2,0) = (3,0)$

11	~~11~~ 25	9	18	2
3	12	21	10	19
20	4	13	22	6
7	16	5	14	23
24	8	17	1	15

associate ✓

$\begin{pmatrix}1, & -2\\ -1, & -1\end{pmatrix}$ $i+x-2y=2$, $j-x-y=2$

$(x,y)=(2,2) \to (i,j) = (4,6) = \equiv (4,1)$

11	3	20	7	24
25	12	4	16	8
9	21	13	5	17
18	10	22	14	1
2	19	6	23	15

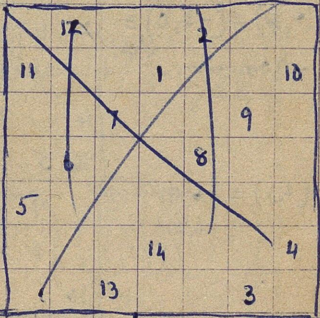
associate ✓

11	9	2	25	18
19	12	10	3	21
22	20	13	6	4
5	23	16	14	7
8	1	24	17	15

associated ✓

11	19	22	5	8
9	12	20	23	1
2	10	13	16	24
25	3	6	14	17
18	21	4	7	15

(associated) ✓



1+8+9+10+6

10	48	30	19	1	39	28
47	29	18	7	38	27	9
35	17	6	37	26	8	46
16	5	36	25	14	45	34
4	42	24	13	44	33	15
41	23	12	43	32	21	3
22	11	49	31	20	2	40

n = 7 associated, B.M. (2,−1)
R.M(1,1), (i,j) = (4,6)

40	3	15	34	46	9	28
2	21	33	45	8	27	39
20	32	44	14	26	38	1
31	43	13	25	37	7	19
49	12	24	36	6	18	30
11	23	42	5	17	29	48
22	41	4	16	35	47	10

R.M(1,1), B.M(−1,2), (i,j)=(6,4

associated

22	20	11	2			31
19	10	①			30	28
9	7			29	27	18
6				26	17	8
			25	16	14	5
		24	15	13	4	
	23	21	[illegible]	3		

R.M(1,1), B.M(−2,−1), (i,j) = (2,6)

×

a.d is $y = c = 3$.

$\begin{pmatrix} 1, & -3 \\ 1, & -2 \end{pmatrix}$ l.d $2x-5y=d$. $d=0,1, \ldots 6$

$(x,y) =$

$2x = 15, 16, \ldots 21$ $2x-15 = 0, 1, \ldots 6$.

$2x = 1, 2, \ldots 7$, $x = 4, \cancel{8}\,1, 5, 2, 6, 3, \cancel{7}\,0$

(~~2,2~~), (~~3,2~~), (0,3), (1,3), . . (6,3)

$i = (1-a-a')3$, $j = (1-b-b')3$

$i = 9$, $j = 6$, $(i,j) = (9,6) = (2,6)$.

$\begin{pmatrix} 1, & -2 \\ 1, & -3 \end{pmatrix}$ $i = 6$, $j = 9$, ie $(i,j) = 6,2)$

31	41	44	5	8	18	28
40	43	4	14	17	27	30
49	3	13	16	26	29	39
2	12	15	25	35	38	48
11	21	24	34	37	47	1
20	23	33	36	46	7	10
22	32	42	45	6	9	19

✓

R.M(1,1), B.M(−1,−2), (i,j) = (6,2)

19	10	1	48	39	30	28
9	7	47	38	29	27	18
6	46	37	35	26	17	8
45	36	34	25	16	14	5
42	33	24	15	13	4	44
32	23	21	12	3	43	41
22	20	11	2	49	40	31

✓

R.M(1,1), (B.M)(−2,−1), (i,j) = (2,6).

$\begin{pmatrix} 1, & 1 \\ -1, & 2 \end{pmatrix} (i,j) = (-3,0) \equiv (4,0)$

[illegible]

$\begin{pmatrix} 1, & -2 \\ -1, & -1 \end{pmatrix} (i,j) = (6,9) = (6,2)$

22	11			20	2	
	23	12	1		21	3
4		24	13			15
16	5		25	14		
	17	6		26	8	
		18	7		27	9
10			19	1		28

✓

R.M (1,−1), B.M (2,1), (i,j) = (4,0)

22		4.	16			10
11	23		5	17		
	12	24		6	18	
		13	25		7	19
20			14	26		1
2	21			8	27	
	3	15			9	28

✓

R.M (1,−1), B.M (−1,−2), (i,j) = ~~6~~. = (6,2)

$\begin{pmatrix} 1, & -3 \\ -1, & 2 \end{pmatrix} (i,j) = 9,0) = (2,0)$

$\begin{pmatrix} 1, & -2 \\ -1, & 3 \end{pmatrix} (i,j) = (6,-3) = (6,4)$

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

22 23 24 25 26 27 28

37 38 39 40 41 42 43 44 45

. . .

22	20	11	2			
	23	21	12	3		
		24	15	13	4	
			25	16	14	5
6				26	17	8
9	7				27	18
19	10	1				28

R.M (1,-1), B.M(-2,1), (i,j) = (2,0)

22				6	9	19
20	23				7	10
11	21	24				1
2	12	15	25			
	3	13	16	26		
		4	14	17	27	
			5	8	18	28

R.M (1,-1), (B.M) = (-1,2), (i,j) = (6,4)

$\boxed{n=9}$

$\begin{pmatrix}1, & -3\\ 1, & -2\end{pmatrix} \times a' = -3.$

$\begin{pmatrix}1, & -2\\ 1, & -3\end{pmatrix} \times b' = -3.$

$\underset{\times}{\begin{pmatrix}1, & 1\\ 1, & 0\end{pmatrix}} \times \overset{ab'-a'b=-3}{\underset{\times}{\begin{pmatrix}1 & 1\\ 1, & -2\end{pmatrix}}}, \underset{\times}{\begin{pmatrix}1, & -3\\ 1, & 0\end{pmatrix}} \times \begin{pmatrix}1, & -3\\ 1, & -2\end{pmatrix} \times$

$\underset{\times}{\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}} \underset{\times}{\begin{pmatrix}1 & 0\\ 1, & -3\end{pmatrix}}, \underset{\overset{\times}{ab'-a'b=3}}{\begin{pmatrix}1, & -2\\ 1, & 1\end{pmatrix}}, \begin{pmatrix}1, & -2\\ 1, & -3\end{pmatrix} \times$

no Cases for n = 9:

$\underset{\overset{\times}{ab'-a'b=3}}{\begin{pmatrix}1, & 1\\ -1, & 2\end{pmatrix}}, \underset{\times}{\begin{pmatrix}1, & 1\\ -1, & 0\end{pmatrix}}, \underset{\times}{\overset{\times}{\begin{pmatrix}1, & -3\\ -1, & 2\end{pmatrix}}} \underset{\times}{\overset{\times}{\begin{pmatrix}1, & -3\\ -1, & 0\end{pmatrix}}}, \underset{\times\ \times}{\begin{pmatrix}1, & 0\\ -1, & 3\end{pmatrix}}, \underset{\times}{\begin{pmatrix}1, & 0\\ -1, & -7\end{pmatrix}}, \underset{\times\ \times}{\begin{pmatrix}1, & -2\\ -1, & 3\end{pmatrix}}, \underset{\times\, ab'-a'b=-3}{\begin{pmatrix}1, & -2\\ -1, & -1\end{pmatrix}}$

I.D.M (n=7, B.M's are same & R.M → K.M.

$\overset{(7\ cases)}{\begin{pmatrix}2, & -2\\ 1, & -2\end{pmatrix}}, \underset{\times}{\begin{pmatrix}2, & -2\\ -1, & 0\end{pmatrix}}, \overset{(7\ cases)}{\begin{pmatrix}-2, & 2\\ 1, & -2\end{pmatrix}}, \underset{\times}{\begin{pmatrix}-2, & 2\\ -1, & 0\end{pmatrix}} \Bigg| \overset{(n)}{\begin{pmatrix}1, & -1\\ 2, & -3\end{pmatrix}}, \overset{(7\ cases)}{\begin{pmatrix}1, & -1\\ -2, & 7\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-1, & 1\\ 2, & -3\end{pmatrix}}, \overset{(7\ cases)}{\begin{pmatrix}-1, & 1\\ -2, & 1\end{pmatrix}}$

I.M $\overset{(n)}{\begin{pmatrix}2, & -2\\ 1, & -3\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}2, & -2\\ -1, & -1\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-2, & 2\\ 1, & -3\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-2, & 2\\ -1, & -1\end{pmatrix}} \Bigg| \overset{(n)}{\begin{pmatrix}1, & -1\\ 2, & -4\end{pmatrix}}, \underset{\times}{\begin{pmatrix}1, & -1\\ -2, & 0\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-1, & 1\\ 2, & -4\end{pmatrix}}, \underset{\times}{\begin{pmatrix}-1, & 1\\ -2, & 0\end{pmatrix}}$

W.M $\overset{(n)}{\begin{pmatrix}2, & -2\\ 1, & 1\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}2, & -2\\ -1, & 3\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-2, & 2\\ 1, & 1\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-2, & 2\\ -1, & 3\end{pmatrix}} \Bigg| \underset{\times}{\begin{pmatrix}1 & -1\\ 2, & 0\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}1, & -1\\ -2, & 4\end{pmatrix}}, \underset{\times}{\begin{pmatrix}-1, & 1\\ 2, & 0\end{pmatrix}}, \overset{(n)}{\begin{pmatrix}-1, & 1\\ -2, & 4\end{pmatrix}}$

the same cases as for n = 5 are possible for n = 7 also in all the three methods, and for ~~ea~~ each one of those cases there are seven possible positions of (i, j).

I.D.M for associated squares using i = (1 - a - a')3, j = (1 - b - b')3.

(i, j) = (3,6), (3,6), (3,6), (3,6), (3,6), (3,6) for

I.M, $(i,j) = (3,9), (3,9), (3,9), (3,9), (3,9), (3,9) \equiv (3,2)$

W.M: $(i,j) = (3,-3), (3,-3), (3,-3), (3,-3), (3,-3), (3,-3) \equiv (3,4)$

I.D.M $\begin{pmatrix} 2, & -2 \\ 1, & -2 \end{pmatrix}$ u.d is $x = 3$ $\rightarrow 9-4y = d,\ d = 9,5,1,-3,-7,-11,-15$

$= 2,5,1,4,0,3,6.$

l.d is $3x-4y = d$ ~~$x\ d = 1$~~ $\rightarrow 3x-12 = d,\ d = -12,-9,-6,-3,0,3,6$

$= 2,5,1,4,0,3,6.$

$\therefore 3x-4y = 0,1,2,\ldots 6.$

$\rightarrow$ 7 cases

(n = 9) $\begin{pmatrix} 2, & -2 \\ 1, & -2 \end{pmatrix}, \begin{pmatrix} -2, & 2 \\ 1, & -2 \end{pmatrix}, \begin{pmatrix} 1, & -1 \\ -2, & 1 \end{pmatrix}, \begin{pmatrix} -1, & 1 \\ -2, & 1 \end{pmatrix}$

	3	6	9
2	5	8	
1	4	7	

$\left.\begin{aligned} i &= (1-a-a')4 \\ j &= (1-b-b')4 \end{aligned}\right\} \rightarrow (i,j) = (4,8), (4,8), (4,8), (4,8).$

Chap III, §1 to be recast, specially pp. 52, 54, 56

I.D.M:

$\begin{pmatrix} 1, & -1 \\ 1, & -2 \end{pmatrix} \begin{array}{l} i+x-y = 2 \\ j+x-2y = 2 \end{array} \Bigg|$ u.d is $y = c$, $y = 2$.

l.d is $2x-3y = d$, $4-3y = d \rightarrow d = 4,1,-2,-5,-8 \equiv 4,1,3,0,2$

$2x-6 = d \rightarrow d = -6,-4,-2,0,2 \equiv 4,1,3,0,2$

$\begin{array}{l} i+x-y = 2 \\ i+x-2y = 2 \end{array} \Bigg| \begin{array}{l} (x,y) = (0,2), (1,2), (2,2) (3,2), (4,2) \\ (i,j) = (4,6), (3,5), (2,4), (1,3), (0,2) \end{array}$

$\equiv (4,1), (3,0), (2,4), (1,3), (0,2)$

(n = 9) $y = 4,\ 2x-3y = d,\ 8-3y = d \rightarrow 8,5,2,-1,-4,-7,-10,-13,-16$

$\equiv 8,5,2,8,5,2,8,5,2.$

$2x-12 = d \rightarrow d = -12,-10,-8,-6,-4,-2,0,2,4$

$\equiv 6,8,1,3,5,7,0,2,4$

$d = 2,5,8.$

$y = 4$

$2x-3y = 2,5,8$ $\qquad 2x = 14,17,20 \equiv 5,8,2.$

$(x,4)$ $(1,4), (4,4), (7,4)$ $\qquad x = 7,4,1.$

~~$(i,j) =$~~ $\begin{array}{l} i+x-y = 4 \\ j+x-2y = 4 \end{array}$ $\qquad (i,j) = (7,11), (4,8), (1,5)$

$= (7,2), (4,8), (1,5)$

		8		
	2	9	4	
3	7	5	3	7
	6	1	8	
		9		

0,

~~$(2,0), (0,2), (2,0)$~~,

$(2,0) (-2,0), (0,2), (0,-2)$

$(-1,0)\ (1,0)$

$(0,-1), (0,1)$

$i + x - y = 4$
$i - x - y = 4$

D.M $\begin{pmatrix}1, & -1\\ -1, & -1\end{pmatrix}$ $\left.\begin{matrix}\text{u.d is } x = c\\ \text{l.d is } y = a\end{matrix}\right\}\rightarrow (x, 4) = (4, 4) \rightarrow (i, j) = (4, 12) = (4, 3).$

		3		
	2	7	6	
1	9	5	1	9
	4	3	8	
		7		

W.M $\begin{pmatrix}1 & -1\\ 1, & 1\end{pmatrix}$ Similar result.

4.DM $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ — one case $\begin{pmatrix}1, & -3\\ 1, & -1\end{pmatrix}$ 5 Cases, $\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix} \rightarrow$ one case $\begin{pmatrix}1 & -1\\ 1, & -3\end{pmatrix}$ 5 Cases. (2, 0)

B M = (2,0)   B M (−2,0)   B M (0,2)   B M (0,−2)

If n = 9 − (−2,0), (0,−2) are not valid ∵ a' ~~[illegible]~~ = −3, & b' = −3.

$\left.\begin{matrix}(i, j) = (-2, 2), = (3,2) \rightarrow \text{associative square for B M } (2,0)\\ (i, j) = (6, 2) = (1,2) \rightarrow \quad '' \quad \text{B M } (-2,0)\\ (i, j) = (2, -2) \equiv (2,3) \rightarrow \quad '' \quad \text{B.M } (0,2)\\ (i, j) = (2, 6) \equiv (2,1) \rightarrow \quad '' \quad \text{B M. } (0,-2)\end{matrix}\right\} n = 5.$

$\left.\begin{matrix}(i, j) = (4,3), (9,3), (3,4), (3,9) \equiv (4,3), (2,3), (3,4), (3,2)\\ \text{lead to associative squares}\end{matrix}\right\} n = 7$

		1		
	2	9	6	
3	7	5	3	7
	4	1	8	
		9		

(i, j) = (5,4) & (4,5) lead " for n = 9.

(i, j) = (~~[illegible]~~(8,7), (7,8) " n = 15.

(2,0), (−2,0), (0,2), (0,−2)

4	1	6
3	5	6
4	5	2

$\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}, \begin{pmatrix}1, & -3\\ 1, & -1\end{pmatrix}, \begin{pmatrix}1 & -1\\ 1, & 1\end{pmatrix}, \begin{pmatrix}1 & -1\\ 1, & -3\end{pmatrix}$

1 1
−1 1

2, 0
0, 2
0, −1

(−1, 0)
(2, 0)

$\left.\begin{matrix}\text{u.d is } y = c\\ \text{l.d is } x = d\end{matrix}\right\}$ $\left\{\begin{matrix}\text{u.d is } x = c\\ \text{l.d is } y = d\end{matrix}\right\}$

x = 4, y = 4.

		9		
	8	1	6	
7	3	5	7	3
	4	9	2	
		1		

(0, 2)

$\begin{pmatrix}1, & -1\\ 1, & -2\end{pmatrix}$

2, 0
(0, 2)

		7		
	8	3	4	
9	1	5	9	1
	6	7	2	
		3		

−1,

(0, −1)
(0, 2)

$\begin{pmatrix}1, & [illegible]\\ -1, & 1\end{pmatrix}$

		7		
	4	3	8	
1	9	5	1	9
	2	7	6	
		3		

(8)

				5				
			4		10			
		3	16	9	22	15		
	2	20	8	21	14	2	20	
1		7	25	13	1	19		25
	6	24	12	5	18	6	24	
		11	4	17	10	23		
			16		22			
				21				

		9		
	6	1	8	
3	7	5	3	9
	2	9	4	
		1		

		9		
	8	1	6	
7	3	5	7	3
	4	9	2	
		1		

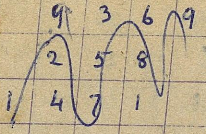
								9								
							8		18							
						7		17		27						
					6		16		26		36					
				5	46	15	56	25	66	35	76	45				
			4	54	14	55	24	65	34	75	44	4	52			
		3		13	63	23	64	33	74	43	3	53		63		
	2		12	62	22	72	32	73	42	2	52	12	62		72	
1		11		21	71	31	81	41	1	51	11	61		71		81
	10		20	70	30	80	40	9	50	10	60	20	70		80	
		19		29	79	39	8	49	18	59	19	69		79		
			28	78	38	7	48	17	58	27	68	28	78			
				37	6	47	16	57	26	67	36	77				
					46		56		66		76					
						55		65		75						
							64		74							
								73								

(1,4) ∧ (3,7)

$$\begin{pmatrix} 3 , & -1 \\ 1 , & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 \\ 4 , & -2 \end{pmatrix}$$

(9)

				8	9	~~9~~	
		3	[illegible]	8	1	6	
	2	[illegible]	7	3	5	7	3
				4	9	2	
		7			1		



		3
7	2	6
1	[illegible]	9
4	8	3

		7		
	4	3	8	
1	9	5	1	9
	2	7	6	
		3		

(-1, 0)

		1		
	4	9	2	
7	3	5	7	3
	8	1	6	
		9		

(0, 1)

		3		
	2	7	6	
1	9	5	1	9
	4	3	8	
		7		

		1		
	2	9	4	
3	7	5	3	7
	6	1	8	

		7	0	
	8	8 3	4	
9	1	5	9	9
	6	7	2	
		3		

(1, 0)

		3		
	6	7	2	
9	1	5	9	7
	8	3	4	
		7		

(1, 0)

		9 (-1, 0)		
	8	1	6	
7	3	5	7	3
	4	9	2	
		1		

(0, -1)

	9	9		
	6	1	(0, 1) 8	
3	7	5	3	7
	2	9	4	
		1		

(0, -1)

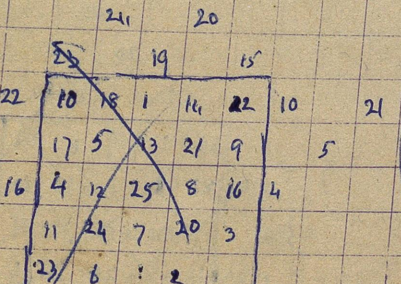
3

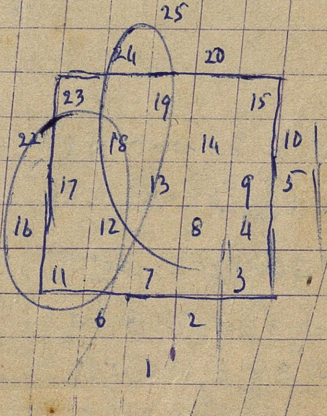
7	4	1	7	
	8	5	2	
	3	9	6	3

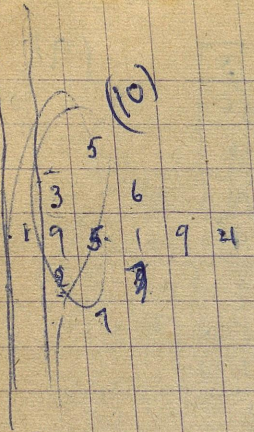
		2	
	1		
		2	25

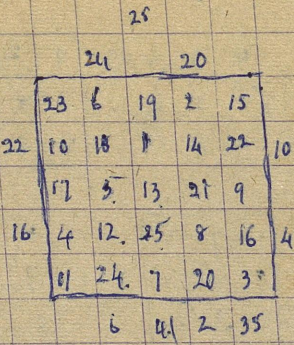
1	2	3
4	5	6
7	8	9

		9		
	8	1	6	21
7	3	5	7	③
	4	9	2	
		1		







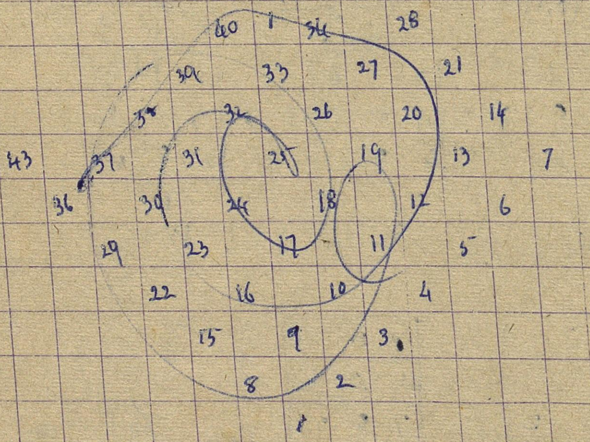


23	6	19	2	15
10	18	1	14	22
17	5	13	21	9
4	12	25	8	16
11	24	7	20	3

R.M (1,1). B.M (0,2)

		9		
	8	1	6	
7	3	5	7	3
	4	9	2	
		1		

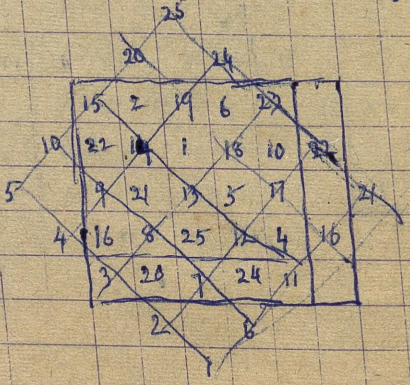
		9		
	6	1	8	
3	7	5	3	7
	2	9	4	



		1		
	4	9	2	
7	3	5	7	3
	8	1	6	
		9		

(0,-1), (0,1) = (0,-2)

కెలోజు ద్వ్యక్కం బంధ త్రి ||
నన్న మోల్ మోంతో కొడక త్రి ||
నన్న కొ పదుం కబాడం నర్లా ||
ఆరర్ బాద దదర్ బాద నల్లా |
నా నమ్మ విన్న క్క ఇల్లా ||
క్షౌకళ ప్రేవేల [illegible] ల్లి నివి సాను ||
విన్నల అనక్రైశా | పిల్లల
నరసింహన్నా హొదదవే పల్లి
నోగడ్ దాక్ష్మ సేవాన్నగా.



R.M (-1,1). B.M (0,2)

				1				
			2		6			
		3	20	7	24	11		
	4	16	8	25	12	4	16	
5		9	21	13	5	17		21
	10	22	14	1	18	10	22	
		15	2	19	6	23		
			20		24			
				25				

R.M (-1,-1), BM (0,-2)

				1				
			6		2			
		11	24	7	20	3		
	16	4	12	25	8	16	4	
21		17	5	13	21	9		5
	22	10	18	1	14	22	10	
		23	6	19	2	15		
			24		20			
				25				

R.M (1,-1), BM (0,-2) Islamic

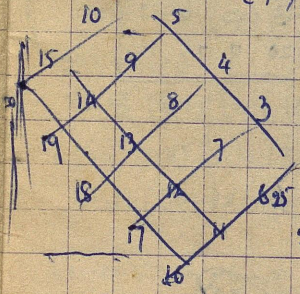
				5				
			4		10			
		3	16	9	22	15		
	2	20	8	21	14	2	20	
1		7	25	13	1	19		25
	6	24	12	5	18	6	24	
		11	4	17	10	23		
			16		22			
				21				

(1,1), (2,0)

				21				
			16		22			
		11	4	17	10	23		
	6	24	12	5	18	6	24	
1		7	25	13	1	19		25
	2	20	8	21	14	2	20	
		3	16	9	22	15		
			4		10			
				5				

21

(1,-1), ([illegible])(2,0)



				5				
			10		4			
		15	22	9	16	3		
	20	2	14	21	8	20	2	
		19	1	13	25	7		25
	24	6	18	5	12	24	6	
		23	10	17	4	11		
			22		16			
				21				

(-1,1), (0,-2)

			22		16			
		23	9	17	4	11		
	24	6	18	5	12	24	6	
		19	1	13	25	7		1
	20	2	14	21	8	20	2	
		15		9	16	3		
			10		4			
				5				

(-1,-1), (-2,0).

$x = y$

$(0,4)$ ~~3(1,38~~

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

$(x,y) = (2,0), (3,0), (4,0), (0,0), (1,0) \quad c = 0.$

$(x,y) = (4,3), (0,3), (1,3), (2,3), (3,3) \quad c = 3$

$(x,y) = (1,1), (2,1), (3,1), (4,1), (0,1) \quad c = 1.$

$(x,y) = (3,4), (4,4), (0,4), (1,4), (2,4) \quad c = 4$

$(x,y) = (0,2), (1,2), (2,2), (3,2), (4,2) \quad c = 2$

$(x,y) = (2,0), (2,1), (2,2), (2,3), (2,4), \quad d = 2$

$(1,1), (2,0) \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$$\left.\begin{array}{l} \text{u.d is } 2y = c \;\; y = c \\ \text{l.d is } 2x = d \;\; x = d \end{array}\right\} \begin{array}{l} i + x + y = 2 \\ j + x - y = 2. \end{array} \quad (i,j) = (-2,2) \equiv (3,2)$$

$\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$

i.d is $3x = c$ or $x = c$

u.d is $x - 2y = d.$

$(2,0), (2,1), (2,2), (2,3), (2,4) = x,y$

$$\left.\begin{array}{l} i + 2x - y = 2 \\ j + x + y = 2 \end{array}\right\} (i,j) = (-2,0), (-1,-1), (0,-2), (1,-3), (2,-4$$

$\equiv (3,0), (4,4), (0,3), (1,2), (2,1)$

16	8	25	12	4
24	11	3	20	7
2	19	6	23	15
10	22	14	1	18
13	5	17	9	21

13	5	17	9	21
16	8	25	12	4
24	11	3	20	7
2	19	6	23	15
10	22	14	1	18

8	22	11	5	19
15	4	18	7	21
17	6	25	14	3
24	13	2	16	10
1	20	9	23	12

$(0,3), (0,2)$

$(0,1), (0,0), (0,4)$

$(2,2), (2,1), (2,0), (2,4), (2,3)$

$(2,2), (1,4), (0,1), (4,3), (3,0)$

([illegible], -2)

~~2~~ $x - 2y = -2, -7, -2, -2, 3$

$\equiv 3:$

$x - 2y = 1$

$x = 2y + 1.$

$(1,0), (3,1), (5,2), (7,3), (9,4)$

$\equiv (1,0), (3,1), (0,2), (2,3), (4,4) \rightarrow 2 + 9 + 11 + 18 + 25 = 65$

$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$(-2, -2)$

$-3x + 2y = d.$

$-3x + 12 = d$

$(0,4), (1,4)$

$y - 3x = d$

$2y = 5. \; y = 7$

$2y + 2 = 3x + 3$

(13)

$-3x+2y=d$

32	42					2	12	22
		6	16	26	~~36~~	~~37~~	47	
21	31	41					1	11
2			5	15	25	35	45	46
10	20	30	40	50				9
				4	14	24	34	44
8	~~18~~	~~19~~	29	39	49			
43					3	13	23	33
	7	17	~~27~~	~~28~~	38	48		

Vide p. 64 of Bk. 16. B M is not (1,0)

$$\begin{pmatrix} 1 & 2 \\ 4 & -5 \end{pmatrix}$$

$\begin{pmatrix} -1, & 1 \\ 2, & -1 \end{pmatrix}$ l·d is $x=c$

u·d $-3x+2y=d$ } not (0,1)

~~3x~~ $+3x-2y=d$

~~$3x=2y+d$~~

$-12+2y=d$

~~d = -12, -10, -8, -6,~~

$-3x+2y=d$ ~~= -6, 8, 1~~

$-3x+2y=2, 5, 8.$

$-3x+2y=2.$

$-3x+2=2,$

$-3x+$

$-3x+8=d.$ d = ~~0~~ 1. −8

d = 8, 5, ~~2~~, −1, −

d = 2, 5, 8.

(0,4), (1,4), - (8,4)

$-3x+2y=d.$ $-3+8$

(0,0), (0,8)

~~(0,9)~~ (8,

(3,0) (3,1) (3,8)

~~4 + 13 + 22 + 31~~ x, y = (0,4), ..

~~12~~ $2-2y=d.$ ~~d = 0, - - 8~~ d = 12, 10, 8, 6, 4, 2, 0, −2, −4

~~$3x-8=d,\ d=-8,-5,-2,$~~

$3x-8=d$

(0,k)

(4,0) 5, 16.

3 ~~8~~ 8, −1, 2, 5, 8, 11, 14, 17, 20.

5, 2, 8, 5, 2, -

$x=4$

$3x-2y=1, 4, 7.$

$3x=12$

3, 1, 8, 6, 4, 2, 0, 7, 5

~~3, 1, 8, 6, 4, 2, 1~~

= 1, 4, 7. $3x-2y=1, 4, 7.$

(4,1), (4,4), (4,7) $2y=11, 8, 5.$ $y=1, 4, 7.$

(x,y) = (4,1), (4,4), (4,7)

14, 41, 68

$3x=2y+1.$

(0,1) (3,1) ~~(6,1), (0,1)~~

10 + 13 + 16 (0,0), (1,1),

$3x=5$

~~l =~~ $l-x+y=4$

$d+2x-y=4$ } (l,d) = (7,−3), (4,0), (1,3)

= (7,6), (4,0), (1,3)

(2,4), (4,7), (6,1), (8,4), (1,7), (3,1), (5,4), (7,7), (0,1)

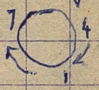
(1,1), (3,4), (5,7), (7,1), (0,4), (2,7), (4,1), (6,4), (8,7)

(0,7), (1,4), (2,1), (3,7), (4,4), (5,1), (6,7), (7,4), (8,1)

1, 7, 4

4, 1, 7

7, 4, 1.



$x=4$

$-3x+8=d$ $-3x+2y=2, 5, 8$

$3x=12$

d = 8, 5, 2,

$2y=14, 17, 20$

$y=7, 4, 1.$

$2y-3x = 2, 2, -16, -16, 11, -7, -7, -7, 2 \equiv 2$

$= -1, -1, -1, -19, 8, 8, -10, -10, -10 \equiv 8$

$\equiv 14,$ - - $\equiv 5$

(0,1), (1,7) (2,4)

(3,1), (4,7), (5,4)

(6,1), (7,7) (8,4)

$-3+29$

$-3x+2y=2.$

(0,1), (1,7), (2,4)

$n=9$ $\begin{pmatrix} 1 , & 2 \\ 4 , & -5 \end{pmatrix}$ u.d is $-3x+7y=c$

K.p.165 l.D is $5x-3y=d$

26, 52, 78, 104, 130, 156, 182, 208

$-3x+28=c, \rightarrow c = 28, 25, 22, 19, 16, 13, 10, 7, 4 \equiv 1, 7, 4, 1, 7, 4, 1, 7, 4.$

(14)

$+20-3y=d \rightarrow d = 20, 17, 14, 11, 8, 5, 2, -1, -4 \equiv 2, 8, 5, 2, 8, 5, 2, 8, 5.$

$-21x+7y=c$

$-21, -14, -7, 0, 7, 14, 21, 28, 35$

$(x, y) = $

6, 4, 2, 0, 7, 5,

$$\left.\begin{aligned} -3x+7y &= 1, 4, 7 \\ 5x-3y &= 2, 5, 8 \end{aligned}\right\}$$

$-15x+35y = 5, 20, 35$

$15x-9y = 6, 15, 24$

$26y = 11, 20, 29, 26, 35, 44, 41, 50, 59$

$y \equiv 7, 7, 7, 1, 1, 1, 4, 4, 4$

$x =$

$-3x+7y=1$

$5x-3y=2$

$-15x+35y=5$

$15x-9y=6$

$26y=11$, $y=7$, $x=7$,

155
164
101
110 173
119 182.
128
137
146

1$\cancel{5}$6
1$\cancel{3}$6

1, 2, 4, 5, 7, 8.

Members are
$$\left.\begin{matrix} (1,1), (1,4), (1,7) \\ (4,1), (4,4), (4,7) \\ (7,1), (7,4), (7,7) \end{matrix}\right\}$$

$5x-3y+2=35$

$c=0$. $7y=3$ $7y=8$

$7y=3x+c$.

$(0,0)$, $(1,3)$, $(2,6)$, $(3,0)$

(27)

$n=15$,

K.p166 $\begin{pmatrix} 2 & 8 \\ 7 & 2 \end{pmatrix}$ u.d is $-5x+6y=c$

l.d is $9x+10y=d$

$, -5x+42=c, c = 42, 37, 32, 27, 22, 17$

$\equiv 12, 7, 2, 12, 7, 2, \ldots$

$-35+6y=c, c = -35, -29, -23, -17, -11, -5$

$\equiv 10, 1, 7, 13, 4, 10, 1, 7, 13, 4. \ldots$

$\boxed{c=7}$

$9x+70=d, d = 70, 79, 88, 97, 106, 115,$

$= 10, 4, 13, 7, 1, 10, 4, 13, 7, 1, \ldots$

$63+10y=d, d = 63, 73, 83, 93, 103, 113, \ldots$

$= 3, 13, 8, 3, 13, 8, \ldots$

$d=13$

$\boxed{6, 6}$ (28)

$-5x+6y=7$

$9x+10y=13$

$-25x+30y=35$

$27x+30y=39$

$-45x+54y=63$

$45x+50y=65$

$104y=128$.

$\boxed{y=7, x=7}$

128
312

728
128
000

$\boxed{-5x=-35}$

$2y-3x=d$.

$y = \frac{3x}{2} + \frac{d}{2}$

$-3x+2y=d$

$(x,y) = (4,0), \ldots (4,8)$, $d = 0, 1, \ldots 8$

$(x,y) = (0,4), (1,4), \ldots (8,4), d = 2, 5, 8$

14×1.5

105

1575
105
15
1635

113
2.15.226 4 1
1695

8, 5, 2, -1, -16

8, 5, 2, 8, 5, 2, 8, 5, 2

21	22	23	24	25
16	17	18	19	20
11	12	13	14	15
6	7	8	9	10
1	2	3	4	5

9	21	13	5	17
12	4	16	8	25
20	7	24	11	3
23	15	2	19	6
1	18	10	22	14

$\begin{pmatrix} 2, & -1 \\ 1, & 1 \end{pmatrix}$. l.d is $3x = c$, u.d is $x - 2y = d$ }

1
2
$x-2y=2$
$2y = x-2$.

$x - 4 = d$.
$(0,2), (1,2), \cdots (4,2)$.
$-4, -3, -2, -1, 0$.
$(2,0), (2,1), (2,2), (2,3), (2,4)$

$x = 2$
$x - 2y = 0, 1, \dots 4$.
$2 - 2y = 0, 1$, $2y = 2 - (0, 1, 2, 3, 4)$
$= 2, 1, 0, -1, -2$.
$y = 2, 6, 0, 4, 8$
$= 1, 3, 0, 2, 4$.

$n = 5$] $2x - 3y = c$
$2x - 6 = c$, $c = -6, -4, -2, 0, 2 \equiv 0, 1, 2, 3, 4$.
$(2,0), (2,1), (2,2), (2,3), (2,4)$ represents
$(0,2), (1,2), (2,2), (3,2), (4,2)$ " "
$(x_0, y_0), (x_1, y_1), \quad \dots \quad (x_4, y_4)$.
$(x_0 + x_1 + \dots + x_4) + 5(y_0 + y_1 + \dots + y_4) + 5 = 65$.
$\Sigma x = 10 = \Sigma y$.

4
3 — 1
2
1
0 1 2 3 4

12
9
6
3
0 2 4 6 8

$0, -3, -6, -9, -12 = 0, 2, 4, 1, 3$,
$2, -1, -4, -7, -10 =$
$4, 1, -2, \dots -8$

$2y = -3x + 2y$.

8
2
1
0 1 2 — 8

$5x = 2$ $3y = -2$
$5x = 3y + 2$
$5x - 3y = 2$, $3y = 5x - 2$ $5x = 8$
$5x = 3y + 2$. $(4,0), (1,1), (7,2)$,

(16)

$n = 9$, $\begin{pmatrix} 1 , & 2 \\ 4 , & -5 \end{pmatrix}$. inv (3, −1

$$\left.\begin{aligned} -3x + 7y &= c \\ 5x - 3y &= d \end{aligned}\right\}$$

31	39	3					15	23
17	25	33	41	5				
		10	27	35	43	7		
				12	20	28	45	9
38	2					14	22	30
24	32	40	4					16
	18	26	34	42	6			
			11	19	36	44	8	
1					13	21	29	37

68	76	3	11	19	36	44	52	60
54	62	70	78	5	13	21	29	37
31	39	47	55	72	80	7	15	23
17	25	33	41	49	57	65	73	9
75	2	10	27	35	43	51	59	67
61	69	77	4	12	20	28	45	53
38	46	63	71	79	6	14	22	30
24	32	40	48	56	64	81	8	16
1	18	26	34	42	50	58	66	74

$y = -c, 1-c, 2-c,$
$3-c, 4-c.$

$4 - 36 = 9c$
$c = 16 = 7$

81	10	29	48	67	5	24	43	62
71	9	~~28~~ 19	38	57	76	14	33	52
61	80	18	28	47	66	4	23	42
51	70	8	27	37	56	75	13	32
41	60	79	17	36	46	65	3	22
31	50	69	7	26	45	55	74	12
21	40	59	78	16	35	54	64	2
11	30	49	68	6	25	44	63	73
1	20	39	58	77	15	34	53	72

$-3x + 7y = c.$

$-3x + 28 = c.$

(0, 4), (1, 4) . . (8, 4)

37, 38, . . . , 45

c = 28, 25, 22, 19, 16, 13, 10, 7, 4

= 1, 7, 4, 1, 7, 4, 1, 7, 4.

$-12x + 7y = c$

c = −12, −5, . . = 0, 1, . . , 8.

$$\left.\begin{matrix} (0,4), (1,4), (2,4) \\ (3,4), (4,4), (5,4) \\ (6,4), (7,4), (8,4) \end{matrix}\right\}$$

(4, 7), (1, 1), (7, 4).
(7, 1), (4, 4), (1, 7)
(1, 4), (7, 7), (4, 1)

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

38
41
44
———
123

(2, 7)
(7, 1)

(x, my)

$x + 2y = k.$

$\begin{pmatrix} 1 , & -1 \\ 1 , & -2 \end{pmatrix}$ $x - y = c.$

$x - y = 3$

k = 0, 1, 2, 3, 4

(1, 3) = −2
(2, 4) = −2
(3, 0) = 3
(4, 1) = 3
(0, 2) = −2

$x - y = c$

$x - 2 = c$, $c = -2, -1, 0, 1, 2. = 3, 4, 0, 1, 2.$

$2 - y = c$, $c = 2, 1, 0, -1, -2$

(1, 2)	12
(2, 3)	18
(3, 4)	24
(4, 0)	5
(0, 1)	6
	✓ 65

$-3x + 2y = d.$ $d = 1$

-3 0	0	0	0, 4	21
-3	2	2	1, 0	2
-6	4	4	2, 1	8
-9	6	1	3, 2	14
-12	8	3	4, 3	20
				65

$-3x + 2y = d$

x	-3x	2y	
0	0	0	0
-6	3	2	2
-9	0	4	4
-12	6	6	6
-15	3	8	8
-18	0	10	1
-21	6	12	3
-24	3	14	5
		18	~~8~~

9	0	0	0	0	0	0
5	-3	6	2	2	8	8
4	-6	3	4	4	7	7
12	-9	0	6	6	6	6
11 1	-12	6	8	8	14	5
9	-15	3	1	10	4	4
8	-18	0	3	12	3	3
	-21	6	5	14	11	2
	-24	3	7	16	10	1

$-3x+2y=2$

0	2	19
6	8	79
3	5	49
0	2	
6	8	
3	5	
0	2	
6	8	
3	5	

147 × 41

0	1	10
6	7	70
3	4	40
		130

$-3x + 2y = 1$ ~~is not a magic series~~ $n = 9.$

$y = 0, 1, 2, \ldots 8$

~~$-3x$~~ $3x = 2y - 1 = -1, 1, 3, 5, 7, 9, 11, 13, 15.$

$3x \equiv 3, 9, 15$ $x = 1, 3, 5.$

~~$-3y$~~ $3x = 2y - c.$ $3x = 2y - 2$, $3x = -2, 0, 2, 4, 6, 8, 10, 12, 14$

$-3x + 4 = c$, $c = 4, 1, -2, -5, -8, -11, -14, -17, -20$

$= 4, 1, 7, 4, 1, 7, 4, 1, 7 //.$

$-3x + 2y = 2.$

~~$-3x = 2$~~ $+8$ $-36 = 9c$

$c = -4 \equiv 5 \pmod 3$, ~~$5c \equiv -10 \equiv$~~ ~~$c = -2 \equiv 3 //$~~

(5)

5+

$9c \equiv 0, 1, 8$ ~~-4~~ $\equiv 1$ $\begin{matrix} 1, 4, 7 \\ 2, 5, 8 \end{matrix}$ ~~$c = 0, 1, 2, 3, 4, 5 //$~~

$-2 \pmod 3$

1, 4, 7, −2, −5, −8

−2 3 −5

$n = 27$ with $\begin{pmatrix} -1 , & 1 \\ 2 , & -1 \end{pmatrix}$

$\frac{\overset{13}{\cancel{26}} \cdot 27}{2}$ 351

a·d is $-3x+2y = d$. $27d = -351 \pmod 3$. $d = -13$. $\equiv 14$

$14 \pmod 3 = 2, 5, 8, 11, 14, -1, -4, -7, -10, \ldots$

$-13 \pmod 3 = 2, 5, 8, 11, 14, 17, 20, 23, 26$ //

$-3x + 26 = d$ $d = 26, 23, 20, 17, 14, 11, 8, 5, 2.$

$-1, -4, -7, -10, -13, -16, -19, -22, -25.$

$-28, -31, -34, -37, -40, -43, -46, -49, -52$

$= (26, \ldots 2), (26, 23, 20, 17, 14, 11, 8, 5, 2)$

$(26, 23, 20, \ldots 2)$ //

Both methods give correct results.

~~35 with~~

33 with $\begin{pmatrix} 1 , & 2 \\ 4 , & -5 \end{pmatrix}$ $\left.\begin{array}{l} -3x+7y = c \\ 5x-3y = d \end{array}\right\}$

$31 \equiv 10 ($m

$4 \cdot 528 = 33c$

$33c = 64 = 31$

$31 \pmod 3 = 10, 7, 4, 1, -2, -5, -8$

$-11, -14, -17, -20. -$

$\equiv (10, 7, 4, 1, 31, 28, 25, 22, 19, 16, 13)$ //

$c = (1, 4, 7, 10, 13, \ldots 31)$

$d = 32. \pmod 3$

$\equiv 32, 29, 26, 23, 20, 17, 14, 11,$
$8, 5, 2$

$d = (2, 5, 8, 11, \ldots 32)$

$n = 9$

$\begin{pmatrix} 1 , & -1 \\ 1 , & -2 \end{pmatrix}$ $\left.\begin{array}{l} \text{u·d is } y = c \\ \text{l·d } 2x-3y = d \end{array}\right\}$

$d \equiv -4 \pmod 3$ //

$= 2, 5, 8, -1, -4, -7, -10, -13, -16$

$\equiv 2, 5, 8, 8, 5, 2, 8, 5, 2$ //

11	22	33	44	46	57	68	79	9
21	32	43	54	56	67	78	8	10
31	42	53	55	66	77	7	18	20
41	52	63	65	76	6	17	19	30
51	62	64	75	5	16	27	29	40
61	72	74	4	15	26	28	39	50
71	73	3	14	25	36	38	49	60
81	2	13	24	35	37	48	59	70
1	12	23	34	45	47	58	69	80

(~~4~~,0)

3(0,4)

37, 38, 45

$y = 4$

$2x - 3y = 2, 5$

$2x = 14 \equiv 1$ $x = 7$

$2x - 3y = 8$

$2x = 20 \equiv 2$

$x = 1.$

$2x = 17 \equiv 8.$ 2) 12

(1,4)

38

(1,4) 44,

$x = 4$

(4,4) (0,4)

41, 44, 38 //

n = 21

$d\begin{pmatrix}1 & 2\\ 4 & -5\end{pmatrix}$ $-3x+7y = c$

$5x-3y = d.$

$21c = 4\cdot210,\ c = 40 = 19$

19 (mod 3) = (1, 4, 7, ~~10~~, 10, 13, 16, 19), (   ), (   )

19 (mod 7) = 5, 12, 19, 26, 33, 40, 47, 54, 61, -

= (5, 12, 19) 5, 12, 19, 5, 12, 19. ∴ c = 19

$21d = 2\cdot210$ ~~= 20~~ $d = 20$

20 (mod 3) = 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, . .

= (2, 5, 8, 11, 14, 17, 20), (2, 5, 8, 7, 14, 17, 20), (2, 5, 8, 7, -)

20 (mod 5) = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

= 0, 5, 10, 15, 20, 4, 9, 14, 19, ~~23~~ 3, 8, 13, ~~18~~ 19 2, ~~22, 25~~ 7, 12, 17, 1, 6, 11, 19 ✓ All values.

---

n = 21 $\begin{pmatrix}-1 & 1\\ 2 & -1\end{pmatrix}$ ~~a·d~~ $-3x+2y = d.$

$21d = -210,\ d = -10 = 11.$

11 (mod 3) = 2, 5, 8, 11, 14, 17, 20; 23, 26, 29, 32, 35, 38, 41; 44, 47, 50, 53, 56, 59, 62

= 2, 5, 8, 11, 14, 17, 20; (2, 5, 8, 11, 14, 17, 20); (2, 5, .   )

11 (mod 2) = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41

= (1, 3, 5, 7, . . 19), (0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20) all no

c = 10, d = (2, 5, 8, . . 20)

$\begin{pmatrix}2 & -1\\ 1 & 1\end{pmatrix}, \begin{pmatrix}1 & 1\\ 2 & -1\end{pmatrix}$

11	22	33	44	46	57	68	79	9
21	32	43	54	56	67	78	8	10
31	42	53	55	66	77	7	18	20
41	52	63	65	76	6	17	19	30
51	62	64	75	5	16	27	29	40
61	72	74	4	15	26	28	39	50
71	73	3	14	25	36	38	49	60
81	2	13	24	35	37	48	59	70
1	12	23	34	45	47	58	69	80

n = 9

7r $\begin{pmatrix}1 & -1\\ 1 & -2\end{pmatrix}$. n = 9

(9·DM)

								1								
							10		2							
						19		11		3						
					28		20		12		4					
				37	78	29	70	21	62	13	54	5				
			46	6	38	79	30	71	22	63	14	46	6			
		55		47	7	39	80	31	72	23	55	15		7		
	64		56	16	48	8	40	81	32	64	24	56	16		8	
73		65		57	17	49	9	41	73	33	65	25		17		9
	74		66	26	58	18	50	1	42	74	34	66	26		18	
		75		67	27	59	10	51	2	43	75	35		27		
			76	36	68	19	60	11	52	3	44	76	36			
				77	28	69	20	61	12	53	4	45				
					78		70		62		54					
						79		71		63						
							80		72							
								81								

works allright, this is the case $\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$ ie 8 q $(0, -2)$
which works.

		3		
	2	7	6	
1	9	5	1	9
	4	3	8	
		7		

		7		
	4	3	8	
1	9	5	1	9
	2	7	6	
		3		

		1		
	2	9	4	
3	7	5	3	7
	6	1	8	
		9		

		1		
	4	9	2	
7	3	5	7	3
	8	1	6	
		9		

(−1,0)
=(2,0)

(−1,0)=(2,0)

(~~−2,0~~)(0,1)
(0,−2)

(0,1)=(0,−2)

		7		
	8	3	4	
9	1	5	9	1
	6	7	2	
		3		

		3		
	6	7	2	
9	1	5	9	1
	8	3	4	
		7		

		9		
	8	1	6	
7	3	5	7	3
	4	9	2	
		1		

		9		
	6	1	8	
3	7	5	3	7
	2	9	4	
		1		

(1,0) = (−2,0)

(1,0)=(−2,0)

(~~−1,0~~)(0,−1) = (0,2)

(0,−1), =(0,2

(±1,0), (0,±2)

$\begin{pmatrix} 1, & -2 \\ 1, & -1 \end{pmatrix} \begin{pmatrix} 1, & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & 2 \end{pmatrix}, \begin{pmatrix} -1, & 2, \\ -1, & 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

		3		
	2	7	4	
1	9	5	1	9
	6	3	8	
		7		

	7	8	9	
	4	5	6	
	1	2	3	

X

$n = 7$ $\begin{pmatrix} 3, & -1 \\ 1, & 3 \end{pmatrix}$ & ~~$n = 9$ $\begin{pmatrix} 1, & 2 \\ 4, & -3 \end{pmatrix}$~~ wrong choice ∴ $b' = -3$ $ab' - a'b = (22)$

$i = (1-3+1)3 = -3$

$j = (1-1-3)3 = -9$

$(i, j) = (4,5)$

~~$i = (1-1-2)4 = -8$~~

~~$j = (1-4+3)4 = 0$~~

~~$(i, j) = (1,0)$~~

Choose $(5,0)$ −9

~~$\begin{pmatrix} 1, & 2 \\ 4, & -1 \end{pmatrix}$~~

$i = (1-1-2)4$

$j = (1-4+1)4$

$(3-1-3)3$

-9

$(4,5)$

46	34	15	3	40	28	9
2	39	27	8	46	33	21
14	44	32	20	1	38	26
19	7	37	25	13	43	31
24	12	49	30	18	6	36
29	17	5	42	23	11	48
41	22	10	47	35	16	4

associated but not magic

42	57	81	15	30	54	69	3	27
5	20	44	59	74	17	32	47	71
49	64	7	22	37	61	76	10	34
12	36	57	66	9	24	39	63	78
56	80	14	29	53	68	2	26	41
19	43	58	73	16	31	46	70	4
72	6	21	45	60	75	13	33	48
35	50	65	8	23	38	62	77	11
79	13	28	52	67	1	25	40	55

~~[illegible]~~

(4

77	61	45	20	4	69	53	28	12
3	68	52	36	11	76	60	44	19
10	75	59	43	27	2	67	51	35
26	1	66	50	34	18	74	58	42
33	17	73	57	41	25	9	65	49
40	24	8	64	48	32	16	81	56
47	31	15	80	55	39	23	7	72
63	38	22	6	71	46	30	14	79
70	54	29	13	78	62	37	21	5

✓

$(4,1)$

$(5,2)$

$\begin{pmatrix} 4, & -1 \\ 1, & 1 \end{pmatrix}$

$i = (1-4+1)4 = -8 \equiv 1$

$j = (1-1-1)4 = -4 \equiv 5$

$(1,5)$ 1 6 49

$9.5+3+1$ –

~~[illegible]~~

$1+4.$ $a(n+1)$?

10

21

11

$i + na + a' + (n-1)a + a + a'$.

$i + 2na + 2a'$. $2n+1$.

$i + 3na + 3a'$. $3n+1$.

$9n+1$

$i + xna + xa'$

$29+1$

19.

$x + yn + 1$.

$(2a', 2b')$.

10 (a', b')

$1 \cdot 9+1$.

$\begin{pmatrix}3 & -1\\ 1 & 3\end{pmatrix}, (x,y) = (4,5).$

2	17	32	47	13	28	36
46	12	27	42	1	16	31
41	7	15	30	45	11	26
29	44	10	25	40	6	21
24	39	5	20	35	43	9
19	34	49	8	23	38	4
14	22	37	3	18	33	48

$a\sum x + a'\sum y = nc.$

$a\cdot\frac{n(n-1)}{2} + a'\cdot\frac{n(n-1)}{2} = n\cdot c$

$(n-1)(a+a') = 2c.$

$\begin{pmatrix}7 & 4\\ 2 & 1\end{pmatrix}$

9

$\frac{n(n-1)}{2} + \frac{n^2(n-1)}{2} + n = \cancel{\frac{n(n-1)}{2}\{1+n(n-1)}+2\}$

$\frac{n}{2}\{(n-1)+n(n-1)+2\} = \frac{n}{2}\cdot(n^2+1)$

$(\cancel{1,1}), (0,0), (1,1), \ldots (\cancel{n,n})(n-1,n-1)/$

$(n-1,0), (n-2,1), \ldots (0,n-1).$

$n=15$

$(0,0), (1,1), \cancel{(m,m)}(2m,2m)$

$(2m,0), (2m-1,1), (m,m)(0,2m).$

$\begin{pmatrix}7 & 8\\ 2 & 1\end{pmatrix}$

$\cancel{2}m\cdot(n^2+1) + \left(\frac{n^2+1}{2}\right) = \frac{1}{2}(n^2+1)(2m+1)$

$= \frac{1}{2}n(n^2+1)//$

[illegible]

21	22	23	24	25
16	17	18	19	20
11	12	13	14	15
6	7	8	9	10
1	2	3	4	5

$m + m(2m+1) + 1.$

$2m^2+2m+1 = \frac{1}{2}\{(2m+1)^2+1\} = \frac{1}{2}(n^2+1)$

(4,8)	~~4~~. $4x-y \equiv 8$
(2,0)	-8
(0,1)	$-1 \equiv 8$
(7,2),	$26 = 8$
(5,3)	$17 = 8$
(3,4)	8
(1,5)	-1
(8,6)	$26 = 8$
(6,7)	$17 = 8$

~~$3\cdot36 \equiv 72$~~

~~$108 \equiv 72$~~

$a-b$

$a\pm b,\ a'\pm b'.$

$\cancel{a+}\ a-b=0,\ a'\cancel{\pm}+b'=0$ or $a'-b'=0$

$a+b=0,\ a'+b'=0$ or $a'-b'=0.$

$\begin{pmatrix}2 & a'\\ 1 & b'\end{pmatrix}$

$a+b=3$
$a-b=1$

$n(1+\cdots+n)$ $\quad 8-5=3;$

$(m,0), (m,1) \quad (m,n-1)$

$m+1,\ m+n+1, \quad m+n(n-1)+1$

$n(m+1)+\frac{1}{2}n^2(n-1)$

$= \frac{1}{2}n\{2m+2+2m(2m+1)\}$

$= \frac{1}{2}n(4m^2+4m+1) = \frac{1}{2}n(n^2+1)$

$\begin{pmatrix}1 & 2\\ 1 & 2\end{pmatrix}\begin{pmatrix}4 & 5\\ 1 & 2\end{pmatrix}$

$a-b=\lambda n$ ~~$a-b=$~~

$a'-b'=\mu n.$

~~$ab'-a'b=(a-b)(a'-b')$~~

$a=b+\lambda n$

$a'=b'+\mu n$

$b \quad \lambda m_1 b'$
$-\mu m_2 b.$

$ab'-a'b = (b+\lambda n)b' - (b'+\mu n)b \quad n(\lambda b'-\mu b)$

(a

(24)

$\begin{matrix} 2 \\ -3 \end{matrix}$ ~~$\sigma(a-b) \equiv 5x$~~ $a = b + 5n.$

~~[illegible]~~ $-3 + 5.$

$\begin{pmatrix} 2, & 2 \\ -3, & 1 \end{pmatrix}$ $5x + y \equiv c \pmod{n}$ & $y \equiv c \pmod{n}$.

⑭

$\left(\begin{matrix} 2, \\ -(12) \end{matrix}\right.$

(i) $a-b \not\equiv 0,\ a'-b' \equiv 0;\ a+b \not\equiv 0,\ a'+b' \not\equiv 0$

(ii) $a-b \not\equiv 0,\ a'-b' \equiv 0,\ a+b \equiv 0,\ a'+b' \not\equiv 0$ ✓

(iii) $a-b = 0,\ a'-b' \not\equiv 0,\ a+b \not\equiv 0,\ a'+b' \not\equiv 0$

(iv) $a-b = 0,\ a'-b' \not\equiv 0,\ a+b \not\equiv 0,\ a'+b' = 0$ ✓

(v) $a+b = 0,\ a'+b' \not\equiv 0,\ a-b \not\equiv 0,\ a'-b' \not\equiv 0,$

~~$a+b \not\equiv 0,\ a'+b' \not\equiv 0$~~,

(v) $a+b = 0,\ a'+b' \not\equiv 0,\ a-b \not\equiv 0,\ a'-b' \not\equiv 0.$

(vi) $a+b \not\equiv 0,\ a'+b' = 0,\ a-b \not\equiv 0,\ a'-b' \not\equiv 0.$

$a+b \not\equiv 0,\ a'+b' \not\equiv 0,\ a-b = 0,\ a'-b' \not\equiv 0.$

$a+b \not\equiv 0,\ a'+b' \not\equiv 0,\ a-b \not\equiv 0,\ a'-b' = 0$

$a+b = 0,\ a'-b' = 0,$

$a-b = 0,\ a'+b' = 0;$

$(-3, -2)$
$(-3, 1)$
$(3, 2)$
$(-3, -1)$
$(3, 1)$
$(-3, -1)$
$(-3, -2)$

$$mn^2 + \frac{n(n+1)}{2}.$$

$2m = n-1$

$$\tfrac{1}{2}n\{n(n-1) + (n+1)\} = \tfrac{1}{2}n(n^2+1)$$

33	44	46	78	62	64	15	26	1
14	25	3	32	43	48	77	61	66
76	63	65	13	27	2	31	45	47
28	42	53	73	60	71	10	24	8
12	23	7	30	41	52	75	59	70
74	58	72	11	22	9	29	40	54
35	37	51	80	55	69	17	19	6
16	21	5	34	39	50	79	57	68
81	56	67	18	20	4	36	38	49

Rouse-Ball.

(−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1), (3, 2).
(−3, 1), (−3, −2).

(1, 2), (−3, −2), (−3, 1), (3, 2), (−3, 1)
(−3, 1), (3, 2), (−3, 1), (−3, −2).

(4, 2), (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1)
(3, 2), (−3, 1), (−3, −2)

(−4, −1), (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1),
(3, 2), (−3, 1), (−3, −2)

(4, 2) (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1), (3, 2)
(−3, 1), (−3, −2)

(4, 2), (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1).
(3, 2), (−3, 1), (−3, −2)

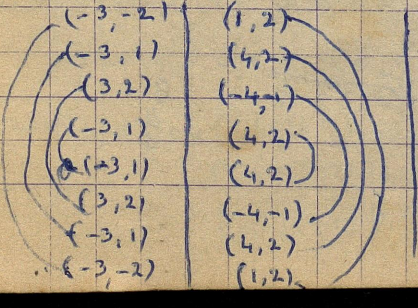
(−4, −1), (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1)
(3, 2), (−3, 1), (−3, −2)

(4, 2) (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1)
(3, 2), (−3, 1), (−3, −2).

(1, 2), (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1)
(3, 2), (−3, 1), (−3, −2).

Regular moves are (−3, −2), (−3, 1), (3, 2), (−3, 1), (−3, 1), (3, 2), (−3, 1), (−3, −2)

Break moves are ~~(4, 2)~~, (4, 2), (4, 2), (−4, −1), (4, 2), (4, 2), (−4, −1), (4, 2), (1, 2)

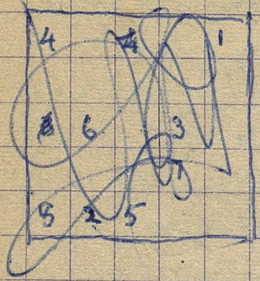


There appears to be some method in this madness

2 18
4 36
2 18
72

(26)

Take n = 5 Regular moves ~~[illegible]~~ (2,1), (-2, 2), (-2,2), (2,1)
break moves (-1,2), (2,-1), (2,+1), (-1,2) } what about (i,j) ?



4	11	14		1
			7	10
13	6	9	3	
8	2	5	12	15

(-1,-2)
(-1,1)
(-1,2)
(-1,1)
(-1,-2)
(-1,-2)

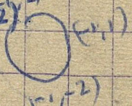
(4,4)
6 6
(1,0)
3 2

(a,b), (c,d), (c,d), (a,b)
(e,f), (g,h), (g,h), (e,f)

(i+a, j+b),

(~~i+2a+2c, j+2b+2d~~)(5)

(-1,1)
(-1,2)
(-1,1)
(-1,-2)



6	18	5	12	24
23	10	17	4	11
19	1	13	25	7
15	22	9	16	3
2	14	21	8	20

~~(-1,2)~~ (-1,1)
(-1,-2), (-1,1), (-1,2), (-1,1)
(-2,0), (-1,-2), (-1,-2), (-1,1), (-1,2)
(-2,0), (-1,1), (-1,-2), (-1,-2), (-1,1)
(-2,0), (-1,2), (-1,1), (-1,-2), (-1,-2)
(-2,0), (-1,1), (-1,2) (-1,1), (-1,-2)

(-1,-2), (-1,1), (-1,2), (-1,1)
(-1,-2), (-1,-2), (-1,1), (-1,2)
(-1,1), (-1,-2),

~~[illegible]~~

8
4
8

Chinese Loshu Skeleton

4	14	25	20	2
16	21	11	9	8
3	7	13	19	23
18	17	15	5	10
24	6	1	12	22

(2, −1), (1, −2), (0, 2), (−2, 2).

(−2, −1), (0, 2), (−2, 1), (−1, 0), (1, −2)

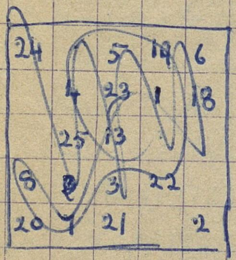
(−2, 2), (1, 2), (−1, 2) | (−1, 2), (1, 2), ~~(2~~

(−2, 2); (1, −2), (−1, 0), (−2, 1), (0, 2).

(−2, −1), (−2, 2), (0, 2), (1, −2), (2, −1)

(2, −1)
(1, −2)
(0, 2)
(−2, 2)
(0, 2)
(−2, 1)
(−1, 0)
(1, −2)
(1, 2)
(−1, 2)

X



(1, 2), (−2, 1), (−1, 2), (1, 1)

~~(2, 0)~~
(−1, 2)

(2, 1), ~~(1, −1)~~ (−1, 1), (−2, −1) (1, 2)

(0, −1) (−1, 1)

4		25		2
	21	20		
3		13		23
		6	5	
24	·	1		22

~~1, 5, 9~~, 4, 5, ~~9~~, 7, 8

~~9~~, 1, 2, 3,

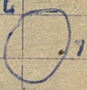
1, 4, 7

24	17	5	19	6
15	4	23	1	18
16	14	13	12	10
8	25	3	22	11
20	7	21	9	2

24	15	5	12	9
19	4	23	1	18
	20	13	6	
8	25	3	22	7
17	14	20	11	2

33 25

29



0, −1, −2, −3, −4, −5, −6, −7, −8

0, 3, 6, 9, 12, 15, 18, 21, 24

0, 2, 4, 6, 8, 10, 12, 14, 16

0, 1, 2, 3, 4, 5, 6, 7, 8

2y = 17

2

Kraitchik pg. 39 $\begin{pmatrix}-1, & 1\\ 2, & -1\end{pmatrix}_9$ l.d is $x = c$

u.d is $-3x+2y = d$

~~0, 3, 6, 0, 3, 6, 0, 3, 6.~~

$x = 4$

$-3x+2y = 2, 5, 8,$

$y = 7, 4, 1$

main u.d is (0, 0), (2, 3), (4, 6), (6, 0), (8, 3), (1, 6), (3, 0), (5, 3), (7, 6)

$-3x+2y = 0, 0, 0, 0, 0, 0, 0, 0, 0$ //

$-3 \cdot 36 + 2 \cdot 27$

$-108 + 54 = -54 \equiv 0$ // (0, 1), (1, 7),

magic u.d's are (2, 4), (4, 7) (6, 1), (8, 4), (1, 7), (3, 1), (5, 4), (7, 7), (0, 1)

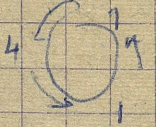
(4, 4), (6, 7), (8, 1), (1, 4), (3, 7), (5, 1), (7, 4), (0, 7), (2, 1)

(1, 1), (3, 4), (5, 7), (7, 1), (0, 4), (2, 7), (4, 1), (6, 4), (8, 7)

(28)

$$\left.\begin{array}{l} a\quad (0,1),(1,7),(2,4),(3,1),(\underline{\underline{4,7}}),(5,4),(6,1),(7,7),(8,4) \\ (0,7),(1,4),(2,1),(3,7),(\underline{\underline{4,4}}),(5,1),(6,7),(7,4),(8,1) \\ (0,4),(1,1),(2,7),(3,4),(\underline{\underline{4,1}}),(5,7),(6,4),(7,1),(8,7) \end{array}\right\}$$

$(0,4),(1,4)$



$3y \equiv 12$

$2x - 3y \equiv 2, 5, 8$

$x = 7,\ \cancel{7}\ 4, 1$

$2x - 3y = 5$

$2x = 17$

$7\quad 2x = 11$

$2x - 3y = 2\}5$

$2x - 12 = 2, 5, 8$

$x = 7, 4, 1$

$2x = 17 \equiv$ 2v

$(\cancel{7,0}),$

$(1,0), (7,1), (4,2), (1,3), (7,1), (4,2)$

$(7,0), (4,1), (1,2), (7,3), (4,4), (1,5)$

$(7,6), (4,7), (1,8)$

$y = 0,\ 2x = 2$

$y = 1,\ 2x = 5$

$y = 2,\ 2x = 8$

$y = 3,\ 2x = 11 = 2$

$\begin{pmatrix}1\\7\\4\end{pmatrix}$

$2x - 3y = 5$

$8 + 14 + 20 + 35 + 41 + 47 + 62 + 68 + 74 = 369$

$7, 4, 1$

$i + 1 - 4 = 4,\ j + 1 - 8 = 4 \quad (i,j) = (7,2)$

$i + 4 - 4 = 4,\ j + 4 - 8 = 4 \quad (i,j) = (4,8)$

$i + 7 - 4 = 4,\ j + 7 - 8 = 4 \quad (i,j) = (1,5)$

$\dfrac{21 \times 20}{2} = 210$

$n = 21,\ \begin{pmatrix}1, & 2\\ 4, & -5\end{pmatrix}$

$-3x + 7y \equiv c$

$5x - 3y \equiv d$

$\cancel{8}4 \cdot 210 = 21c,\ c \equiv 40 \equiv 19$

$19 \pmod 3 \equiv 1, 4, 7, 10, 13, 16, \textcircled{19}$

$19 \pmod 7 \equiv 5, 12, \textcircled{19}$

$\underline{\underline{c = 19.}}$

$2 \cdot 210 = 21d,\ d \equiv 20 \quad 20 \pmod 3 = 2, 5, 8, 11, 14, 17, 20.$

145
19
64

$-3x+7y=19.$

$x=0,\ 7y=19,$

$x=1,\ 4y=19.$

$x=0,1 \ldots 20\big\}\ -3x=0,-3,-6, \ldots -60$

$y=0,1 \ldots 20\big\}\ 7y=0,7,14, \ldots 133, 140.$

$x=0,\ y=19$ ✓ (0,19) (19,0)

$x=1,\ y=14$ ✓ (1,14) (15,1)

$x=2,\ y=9$ ✓ (2,9) (11,2)

$x=3,\ y=4$ ✓ (3,4) (7,3)

$x=4,\ y=18$ ✓ (4,18) (3,4)

$x=5,\ y=13$ ✓ (5,13) (18,5)

$x=6,\ y=8$ ✓

$x=7,\ y=3$ ✓

$x=8,\ y=17$ ✓

$x=9,\ y=12$ ✓

$x=10,\ y=7$ ✓

$x=11,\ y=2$ ✓

$x=12,\ y=16$ ✓

$x=13,\ y=11$ ✓

$x=14,\ y=6$ ✓

$x=15,\ y=1$ ✓

$x=16,\ y=15$ ✓

$x=17,\ y=10$ ✓

$x=18,\ y=5$ ✓

$x=19,\ y=0$ ✓

$x=20,\ y=14$

$4\cdot 210 = 19\cdot 21$

$40 \equiv 19$ ✓

0	0
-3	7
-6	14
-9	21
-12	28
-15	35
-18	42
-21	49
-24	56
-27	63
-30	70
-33	77
-36	84
-39	91
-42	98
-45	105
-48	112
-51	119
-54	126
-57	133
-60	140

80
83
86
89
92
95.

140
-45

-33
14
19

108
104 111
114.

.8

85

.2

119
24
95

54
35
-60
98

6

.9

98
60
38

21
147.

$n = 15$ $\begin{pmatrix} 2 & 8 \\ 7 & 2 \end{pmatrix}$ u.d is $-5x+6y=c$
l.d is $9x+10y=d$.

$7 \pmod 6 = 1, ~~4~~ 7, 13, 19, 25$ 31
1, 7, 13, 4, 10.

$15c = 105 \Rightarrow c = 7$

$7 \pmod 3 = 1, 4, 7, 10, 13,$
$7 \pmod 5 = 2, 7, 12$ $9 \pmod 9$

19'

$19 \cdot 105 = 15d \Rightarrow d \equiv 133 \equiv 13.$

$13 \pmod 3 = 1, 4, 7, 10, 13$
$13 \pmod 5 = 3, 8, 13$

magic u.d is $-5x+6y=7$
" l.d is $9x+10y=13$

$13 \pmod 9 = 4, 13, 22, 31, 40, 49$
$= 4, 13, 7, 1, 10, 4$
$13 \pmod{10} = 3, 13, 23, 33$
4),

187

What about the actual (x, y) for these cases?

32, 110, 188, 41, 119, 182, 35, 113, 191, 44, 107, 185, 38, 116, 194.

(1,2), (4,7), (7,12), (10,2), (13,7), (1,12), (4,2), (7,7), (10,12), (13,2), (1,7), (4,12)
(7,2), (10,7), (13,12)

u.d members.

(1, 2), (4,2), (7,2), (10,2), (13,2)
(1,7), (4,7), (7,7), (10,7), (13,7)
(1,12), (4,12), (7,12), (10,12), (13,12)

l.d elements

(1,3), (4,3), (7,3), (10,3), (13,3)
(1,8), (4,8), (7,8), (10,8), (13,8)
(1,13), (4,13), (7,13), (10,13), (13,13)

(2)

l.d
~~47, 50, 53,~~
32, 35, 38, 41, 44
107, 110, 113, 116, 119
182, 185, 188, 191, 194

~~47, 50, 53, 56, 59~~
~~122, 125, 128, 131, 134~~

148, 158, 118, 63, 23
208, 153, 113, 73, 18
203, 163, 108, 68, 28

(2,13), (7,10), (12,7), (2,4), (7,1)
(12,13), (2,10), (7,7), (12,4), (2,1)
(7,13), (12,10), (2,7), (7,4), (12,1).

$\begin{pmatrix} 5 & 2 \\ 2 & -5 \end{pmatrix}$

$\left[\begin{array}{l} 3x+7y=c \\ 7x-3y=d \end{array}\right]$

(2,1), (2,4), (2,7), (2,10), (2,13)
(7,1), (7,4), (7,7), (7,10), (7,13)
(12,1), (12,4), (12,7), (12,10), (12,13)

0 (2,1), (3,4),

37
7

a−b, a′−b′.

$9x+10y = 28, 58, 88, 118, 148$
$\equiv 13, 13, 13, 13, 13$ ✓

$\begin{pmatrix} a & a' \\ b & b' \end{pmatrix}$

$a-b = 3k$
$a'-b' = 7l.$
$a+b = 7m$
$a'+b' = 3n$

$m=1, k=1$
$\left. a=5, b=2 \right]$

$\left.\begin{array}{l} 2a = 7m+3k \\ 2b = 7m-3k \end{array}\right\}$ $\left.\begin{array}{l} 2a' = 7k+3n \\ 2b' = 3n-7k \end{array}\right\}$

$k=1, n=-1$ $a' = 2$
$b' = -5$

~~$-5x+90=c$, $c=90, 85, 80, 75$~~

~~$=0, 10, 5, 0,$~~

$-5x+62=c$, $c=62, 57, 52, 47$

$=2, 12, 7, 2,$

$-35+6y=c$, $c=-35, -29, -23, -17, -11, -5$

$=10, 1, 7, 13, 4, 10$

$9x+70=d$, $d=70, 79, 88, 97, 106, 115.$

$=10, 4, 13, 7, 1, 10.$

$63+10y=d$, $d=63, 73, 83, 93, 103,$

$=3, 13, 8, 3, 13, 8,$

$i=-63\equiv 12$

$j=-56.\equiv 4$

226

$i=(1-a-a')7$

$=(1-2-8)7=\underline{\underline{-63}}$

$-5x+6y=7.$

$9x+10y=13.$

$9x=13-10y$

$9x=3$

$9x=-27$

$9x=-57$

$y=1, 4, 7, 10, 13.$

$x=2, 7, 12.$

27 13
63 78
90

100
13

$9x=-87$

$3x=-29.$

$-5x+6y=7.$

$-5x+12$

$-20+12=-8\equiv 7$

$-35+12=-23\equiv 7$

$-50+12=-38\equiv 7$

$-65+12=-53\equiv 7$

$18+10\equiv 13$

$18+40=58\equiv 13$

$18+70=88\equiv 13$

$18+100=118\equiv 13$

$18+130=148\equiv 13$

$63+10=73\equiv 13$

$63+40=103\equiv 13$

$108+10=118\equiv 13$

$\boxed{n=15}$

$\begin{pmatrix} 1, & -1 \\ 1, & -2 \end{pmatrix}$

$y=c=7.$

$2x-3y=d.$

$-105=15d$, $d\equiv -7\equiv 8.$

$\boxed{1, 4, 7, 10, 13}$

$-7\ (\mathrm{mod}\ 3) = 2, 5, 8, 11, 14.$

$i+x-y=7$

$j+x-2y=7$

~~$(i,j)=12$~~, 8

$2x-21=2$, $2x\equiv 23\equiv 8$, $x=4.$

$2x-21=5$, $2x=26\equiv 11$, $x=13$

$2x-21=8$, $2x=29$, $x=7$

$2x=32$, $x=1.$

$2x=35$, $x=10$

$(x,y)=(1,7), (4,7), (7,7), (10,7), (13,7)$?

$(i,j)=(13,5), (10,2), (7,14), (4,11), (1,8)$

$\boxed{n = 15}$ $\begin{pmatrix} 1, & -1 \\ -2, & 4 \end{pmatrix}$ u.d is $3x - 5y = c$ $-210 = 15c$

l.d is $-x + 3y = d.$ $c = -14$

$210 = 15d$, $d = 14$

$-14 \pmod 3 = 1, 4, 7, 10, 13$

$14 \pmod 3 = 2, 5, 8, 11, 14$

$-14 \pmod 5 = 1, 6, 11,$

48
133

$\boxed{n = 21}$ $\begin{pmatrix} 5 & , 2 \\ 2 & , -5 \end{pmatrix}$ u.d is $3x + 7y = c$ l.d is $~~2~~7x - 3y = d$ $\Big\}$

$3x + 7y = 16$

$7x - 3y = 19$

10
~~20~~ × 21
$\boxed{210}$

$10 \cdot 210 = 21c$, $c \equiv 100 \equiv 16.$

~~9~~ $58x = 181$

$16 \pmod 3 = 1, 4, 7, 10, 13, 16, 19$

$16 \pmod 7 = 2, 9, 16,$

$58 \overline{)181} (3$
174
7
4

232
181
51

290
181
~~109~~

5

348
181
167

476
181
295

112
57
55

$4 \cdot 210 = 21d$, $d = 40 \equiv 19$

$19 \pmod 3 = 1, 4, 7, 10, 13, 16, 19$

$19 \pmod 7 = 5, 12, 19.$

$58y = 55$

$y = 10$

116
55
61

76
21
97
21

118
21
139
21

160
21

181
21
202

223
21
244

265
21
286
42
328

328
42
370
42

412
42
454
42

496
42
538
42

$\boxed{580}$

19
30

$7x = 49$

$x = 7$

$580 \equiv 55$
55
525 (25
21) 42
105

1478

35
26

$i + 5x + 2y = 10$ $i + 55 = 10$

$j + 2x - 5y = 10.$ $j ~~+~~ - 36 = 10$

$i = -45$, $j = 46.$
63

$i = 18$, $j = 4$

14
-50

13

$\boxed{n=25}$ $\begin{pmatrix} 2, & 8 \\ 7, & 2 \end{pmatrix}_{25}$ $-5x+6y=c$ $25c=300$, $c=12 \pmod 5 = 2, 7, 12, 17, 22$

$9x+10y=d$ $19 \cdot 300 = 25d$, $d = 228 \equiv 3.$

$3 \pmod 5 = 9, 13, 18, 23$

$\frac{25}{2}$

$\begin{pmatrix} -1, & 1 \\ 4, & -1 \end{pmatrix}$

l.d is $3x=d$; $d=36$ giving $x=12$ //

u.d is $-5x+2y=c$. $-3 \cdot 300 = 25c$. $c=-36 \equiv 14$

14 ~~36~~ $\pmod 5) \equiv 4, 9, 14, 19, 24$

4

$\boxed{n=27}$ $\begin{pmatrix} 1, & -1 \\ 1, & -2 \end{pmatrix}$ $y=c$

$2x-3y=d$

$-351 = 27d$ $d=-13 \equiv (14)$

$\frac{\overset{13}{\cancel{26}} \cdot 27}{\cancel{2}} = \underline{351}$!

$14 \pmod 3 = $ ~~$0, 3, 6, 9, 12, 15, 18, 21, 24, 27$~~

$= 2, 5, 8, 11, 14, 17, 20, 23, 26,$

9 -15

$27d =$

$\lambda x - \mu y = 27d$

$d=-15$

$= 27(-15).$

$\overset{20}{\cancel{300}}(\lambda-\mu)$

$20(\lambda-\mu) = -27. \equiv 0$

$\lambda = \mu y$ //

$\lambda = a-b$

$\mu = b'-a'$

$a-b = b'-a'$

$a+a' = b+b'$ //

$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$

$(b+b'-a)$

$3-3$

a a'

b b'

$ab'-a'b = ab'-b(b+b'-a)$

$-x+y=c$ //

$3x+7y=d$

$3x-7y=d$

$=27d$

$10 \cdot 35 = 27 \cdot d$

$130 = d.$

$10 \cdot 30$

135

108

$d = 22$

$d = 22$ //

$\lambda x - \mu y = 27d.$ $(\lambda-\mu)351 = 27d$

$d = 13(\lambda-\mu) \equiv 12, = 39.$ $\lambda-\mu=3.$

(21)

$a-b-b'+a' = 3$

$a+a' = b+b'+3$ //

$\begin{pmatrix} 1 & 4 \\ -1 & 3 \end{pmatrix}$

$\left.\begin{matrix} 3x-2y=c \\ 4x-3y=d \end{matrix}\right\}$

$a-b=3$, $a'-b'=5$

$3+4$

$a+b=6$, $a'+b'=8$

$\boxed{a-b}$ //

Possible cases for $n = p_1 p_2$. Take $n = 5 \times 7 = 35$.

(i) $2x + 3y = c$

$5x + 4y = d$. ~~9.2~~

$9 \cdot 17 \cdot 35 = 35d$. 7×35 = cases

$\overset{17}{\cancel{34}} \cdot 35$ / 2

$d = 17 \cdot 9 = 153 \equiv 13$.

$13 \pmod 5 = 3, 8, 13, 18, 23, 28, 33$

(ii) $2x + 3y = c$

$7x + 4y = d$

$11 \cdot 17 \cdot 35 = 35d$

$d = 187 \equiv 15$

5×35 cases

$15 \pmod 7 = 1, 8, 15, 22, 29$

(iii) $\left.\begin{array}{l} 5x + 3y = c \\ 7x + 4y = d \end{array}\right\}$

$8 \cdot 17 \cdot 35 = 35c$ $c = 136 \equiv 21$

$21 \pmod 5 = 1, 6, 11, 16, 21, 26, 31$

7×5 cases:

(iv) $5x + 3y = c$

$5x + 8y = d$

$8 \cdot 17 \cdot 35 = 35c$, 7 cases

$c = 136$, 9

$13 \cdot 17 \cdot 35 = 35d$ $d = 221 \equiv 11$

$11 \pmod 5 = 1, 6, \ldots$

$7 \times 7 = 49$

7

175

(v) $2x + 3y = c$

$5x + 7y = d$.

$12 \cdot 17 \cdot 35 = 35d$, $d = 204 \equiv 29$

$29 \pmod 5 = 4, 9, 14, 19, 24, \boxed{29}$.

$29 \pmod 7 = 1, 8, 15, 22, \boxed{29}$

one case

315 4

7 cases:

$5x + 14y = d$

$19 \cdot 17 \cdot 35 = 35 \cdot d$; $d = 323 \equiv 12$

$12 \pmod 5 = 2, 7, \boxed{12}, 17, 22, 27, 32$.

$12 \pmod 7 = 5, \boxed{12}, 19, 26, 33$,

175

177

525

(vi) $5x + 3y = c$

$10x + 21y = d$

(c ≡ 7)

gives one case

1:

$31 \cdot 17 \cdot 35 = 35d$

$d = 527 \equiv 2$.

$2 \pmod 5 = \boxed{2}, 7, 12, 17, 22, 27, 32$

$2 \pmod 7 = \boxed{2}, 9, 16, 23, 30$

$\left.\begin{array}{l} 7x + 3y = c \\ 10x + 21y = d \end{array}\right)$ $5 \times 1 = 5$ cases;

(35)

(1) u.d coeffs prime to $\underline{n}$.

(a) one coeff of t.d prime to p_1 & not prime to p_2 — $\underline{np_2}$ cases

" p_2 " p_1 — $\underline{np_1}$ cases

(b) both coeffs g.l.d not prime to n —— $n \times 1 = n$ cases.

(2) one coeff of u.d prime to p_1 & not to p_2

(a) ~~n~~ p_2^2 ~~[illegible] n $p_1 p_2$~~ or ~~n~~ $p_1 p_2$ (b) p_2.

one coeff of u.d prime to p_2 & not to p_1

(a) ~~n~~ $p_1 p_2$ or ~~(b)~~ ~~n~~ p_1^2 (b) p_1.

(3) Both coeffs of u.d not prime to n.

(a) p_2 or p_1 (b) $\underline{\underline{1}}$.

	u.d	t.d
(1) np_2 or np_1 or n.	both coeffs prime to n	both prime
(2) p_1^2, p_2^2, p_1p_2 ; p_1, p_2	only one coeff prime to n	only one not prime
(3) p_1, p_2, 1	both coeffs not prime to n	both not prime

✓✓	
✓×	
××	

	✓✓	✓×	××
✓✓	n^2	np_1 or np_2	n
✓×	np_1 or np_2	p_1^2, p_2^2 or p_1p_2	p_1 or p_2
××	n	p_1 or p_2 ✓	1

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$\underline{n = 35}$

(1) $2x - y = c$ $\quad$ $a - b = 2$ $\quad$ $a + b = 3$ $\quad$ $a = 2, b = 1$

$4x + 3y = d$ $\quad$ $a' - b' = -1$ $\quad$ $a' + b' = 3$ $\quad$ $a' = 1, b' = 2$.

(2)

BM = (1,−2), (1,−2), (−2,0), (2,−1)

not regular.

(a,b's

(2,−2), (2,−2), (−1,1)(−2,2)

2	1	4	5	3
1	4	5	3	2
4	5	3	2	1
5	3	2	1	4
3	2	1	4	5

(4,1)

3 10_3	5_2	0_1	15_4	20_5
20_5	10_3	5_2	0_1	15_4
15_4	20_5	10_3	5_2	0_1
0_1	15_4	20_5	10_3	5_2
5_2	0_1	15_4	20_5	10_3

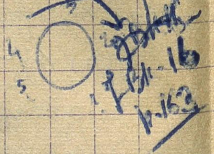
(1,4)

12	6	4	20	23
21	14	10	3	17
19	25	13	7	1
5	18	22	11	9
8	2	16	24	15

(nonassociative)

little

Nelson.



5	1	2	3	4
1	2	3	4	5
2	3	4	5	1
3	4	5	1	2
4	5	1	2	3

5	10	15	20	0
0	5	10	15	20
20	0	5	10	15
15	20	0	5	10
10	15	20	0	5

10	11	17	23	4
1	7	13	19	25
22	3	9	15	16
18	24	5	6	12
14	20	21	2	8

semi-magic

3	1	4	2	5
5	3	1	4	2
2	5	3	1	4
4	2	5	3	1
1	4	2	5	3

10	0	15	5	20
20	10	0	15	5
5	20	10	0	15
15	5	20	10	0
0	15	5	20	10

13	1	14	7	25

X

3	1	4	2	5
5	3	1	4	2
2	5	3	1	4
4	2	5	3	1
1	4	2	5	3

15	20	0	5	10
20	0	5	10	15
0	5	10	15	20
5	10	15	20	0
20	15	20	0	5
10	15	20	0	5
5	10	15	20	0
0	5	10	15	20
20	0	5	10	15
15	20	0	5	10

[illegible]

R·M = (1,1)

BM = (−1,−2)

18	21	4	7	15
25	3	6	14	17
2	10	13	16	24
9	12	20	23	1
11	19	22	5	8
13	16	24	2	10

X Rfl

1	2	3	4	5

4	5	1	2	3
5	1	2	3	4
1	2	3	4	5
2	3	4	5	1
3	4	5	1	2

10	5	0	20	15
15	10	5	0	20
20	15	10	5	0
0	20	15	10	5
5	0	20	15	10

✓ associated

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

(−2,2) is R M
(0,−1) is B M.

2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

(1,2,3,4,5) mixed

+

15	20	0	5	10
20	0	5	10	15
0	5	10	15	20
5	10	15	20	0
10	15	20	0	5

(0,5,10,15,25) in mixed
& ↗

=

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Going backwards
from the S D M
to
N − method

3	1	4	2	5
5	3	1	4	2
2	5	3	1	4
4	2	5	3	1
1	4	2	5	3

+

10	5	0	20	15
15	10	5	0	20
20	15	10	5	0
0	20	15	10	5
5	0	20	15	10

=

12	9	1	23	20
18	15	7	4	21
24	16	13	10	2
5	22	19	11	8
6	3	25	17	14

(2,−2) in R. M
(0,−1) is B. M

+ ↓ magic square

(0,5,10,15,20) in mixed
& ↘

Every regular 5×5 square can be ~~[illegible]~~ by N − method (?) Defect [illegible] square

3	16	9	22	15
20	8	21	14	2
7	25	13	1	19
24	12	5	18	6
11	4	17	10	23

=

3	1	4	2	5
5	3	1	4	2
2	5	3	1	4
4	2	5	3	1
1	4	2	5	3

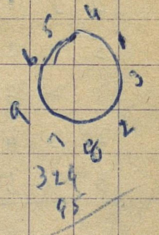
+

0	15	5	20	10
15	5	20	10	0
5	20	10	0	15
20	10	0	15	5
10	0	15	5	20

Andrews Fig 41, p. 17

Take a ~~[illegible]~~ bordered (5, 5) & go backwards

$$\begin{array}{|ccccc|}19 & 2 & 20 & 1 & 23\\ 4 & 16 & 9 & 14 & 22\\ 18 & 11 & 13 & 15 & 8\\ 21 & 12 & 17 & 10 & 5\\ 3 & 24 & 6 & 25 & 7\end{array} = \begin{array}{|ccccc|}4 & 2 & 5 & 1 & 3\\ 4 & 1 & 4 & 4 & 2\\ 3 & 1 & 3 & 5 & 3\\ 1 & 2 & 2 & 5 & 5\\ 3 & 4 & 1 & 5 & 2\end{array} + \begin{array}{|ccccc|}15 & 0 & 15 & 0 & 20\\ 0 & 15 & 5 & 10 & 20\\ 15 & 10 & 10 & 10 & 5\\ 20 & 10 & 15 & 5 & 0\\ 0 & 20 & 5 & 20 & 5\end{array}$$



Bordered square not obtainable by the strict N-method

Take the (9 × 9) Fig 61 of p. 78 of main book & go backwards, which is nasik & associated

(1, 2, 3, 4, 5, 6, 7, 8, 9) + (0, 9, 18, 27, 36, 45, 54, 63, 72)

6	8	1	6	8	1	6	8	1
5	7	3	5	7	3	5	7	3
4	9	2	4	9	2	4	9	2
1	6	8	1	6	8	1	6	8
3	5	7	3	5	7	3	5	7
2	4	9	2	4	9	2	4	9
8	1	6	8	1	6	8	1	6
7	3	5	7	3	5	7	3	5
9	2	4	9	2	4	9	2	4

~~not strict N-method~~ (A)

2	3	1	4	5	6	9	7	8
27	36	45	72	54	63	9	18	0
9	18	0	27	36	45	72	54	63
72	54	63	9	18	0	27	36	45
27	36	45	72	54	63	9	18	0
9	18	0	27	36	45	72	54	63
72	54	63	9	18	0	27	36	45
27	36	45	72	54	63	9	18	0
9	18	0	27	36	45	72	54	63
72	54	63	9	18	0	27	36	45

(B)

234678

1, 6, 8
3, 5, 7
2, 4, 9.

1 5 9
2 6 7
3 4 8

1894
273
10815

Take a 5×5 nasik & go backwards – Fig. 6, p. 30 of main book

$$\begin{array}{|ccccc|}20 & 22 & 4 & 6 & 13\\ 9 & 11 & 18 & 25 & 2\\ 23 & 5 & 7 & 14 & 16\\ 12 & 19 & 21 & 3 & 10\\ 1 & 8 & 15 & 17 & 24\end{array} = \begin{array}{|ccccc|}5 & 2 & 4 & 1 & 3\\ 4 & 1 & 3 & 5 & 2\\ 3 & 5 & 2 & 4 & 1\\ 2 & 4 & 1 & 3 & 5\\ 1 & 3 & 5 & 2 & 4\end{array} + \begin{array}{|ccccc|}15 & 20 & 0 & 5 & 10\\ 5 & 10 & 15 & 20 & 0\\ 20 & 0 & 5 & 10 & 15\\ 10 & 15 & 20 & 0 & 5\\ 0 & 5 & 10 & 15 & 20\end{array}$$

possibly N-method

Upward diagonals correspond 1, 4, 2, 5, 3
0, 15, 5, 20, 10)

To form a nasik 9 × 9.

	3	2	1	4	5	6	7	8	9
	9	8	5	6	7	4	3	1	2
	5	6	7	9	2	1	8	4	3
	8	3	4	5	6	7	9	2	1
267159348	4	5	6	7	2	8	1	3	9
.....5.....							5	6	8
		0	2	4	5	9	8	9	
	4	7	1	2	5	8	9	3	6

2 8
3 7
4 6

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3
0	5	10	15	20
15	20	0	5	10
5	10	15	20	0
20	0	5	10	15
10	15	20	0	5

~~a+a' = b+b' = c+c' = d+d'~~ = 10

~~a+d+c' = b+5+b' = c+d'+a'~~ = 15

a+d = c+5   c = 1, a = 2, d = 4

27	36	45	72	54	63	9	18	0
63	9	18	0	27	36	45	72	54
36	45	72	54	63	9	18	0	27
9	18	0	27	36	45	72	54	63
45	72	54	63	9	18	0	27	36
18	0	27	36	45	72	54	63	9
72	54	63	9	18	0	27	36	45
0	27	36	45	72	54	63	9	18
54	63	9	18	0	27	36	45	72
28	38	48	76	59	69	16	26	9
68	15	25	8	36	37	47	75	58
1	2	3	4	5	6	7	8	9
7	8	9	1	2	3	4	5	6
4	5	6	7	8	9	1	2	3
1	2	3	4	5	6	7	8	9
7	8	9	1	2	3	4	5	6

X

1	2	3	4	5	6	7	8	9
5	6	7	8	9	1	2	3	4
9	1	2	3	4	5	6	7	8
4	5	6	7	8	9	1	2	3
8	9	1	2	3	4	5	6	7
3	4	5	6	7	8	9	1	2
7	8	9	1	2	3	4	5	6
2	3	4	5	6	7	8	9	1
6	7	8	9	1	2	3	4	5

X repetition u.d.

1	2	3	4	5	6	7	8	9
8	9	1	2	3	4	5	6	7
6	7	8	9	1	2	3	4	5
4	5	6	7	8	9	1	2	3
2	3	4	5	6	7	8	9	1
9	1	2	3	4	5	6	7	8
7	8	9	1	2	3	4	5	6
5	6	7	8	9	1	2	3	4
3	4	5	6	7	8	9	1	2

a b c d 5 d' c' b' a'

d c b a 5 a' b' c' d'

a b c d 5. 10-d, 10-c, 10-b, 10-a

a+d+10-c = 5

b+5+10-b = 15

c+10-d+10-a = 15

c = a+d-5

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

X

1	2	3	4	5	6
5	6	1	2	3	4
3	4	5	6	1	2
1	2	3	4	5	6

X

1	2	3	4	5	6
4	5	6	1	2	3
1	2	3	4	5	6

X

1	2	3	4	5	6
3	4	5	6	1	2
1	2	3	4	5	6

X

1	2	3	4	5	6
2	3	4	5	6	1
3	4	5	6	1	2
4	5	6	1	2	3
5	6	1	2	3	4
6	1	2	3	4	5

X

1	4	5	6	3	2
3	5	1	2	4	6
4	2	6	5	1	3
2	1	4	3	6	5
5	6	3	4	2	4
6	3	2	4	5	4

X

1.	2	3	4	5	6	7
5	6	7	1	2	3	4
2	3	4	5	6	7	1
6	7	1	2	3	4	5
3	4	5	6	7	1	2
7	1	2	3	4	5	6
4	5	6	7	1	2	3

✓

The condition that numbers are not repeated in cols or rows does not hold for composite numbers ⊛. But this is not necessary – See for eg. Andrews' const^n for squares of 6, 12 etc by the N-method.

⊛ need not hold even for prime. See p 118

$0, n, 2n, \ldots n(n-1)$

$$\frac{1}{2}n(n+1) + \frac{1}{2}n^2(n-1) = \frac{1}{2}n\{n+1+n^2-n\} = \frac{1}{2}n(n^2+1) = S.$$

1	14	15	4
8	6	7	13
12	10	11	

4+4+4+12

2+2+2+4

0, 4, 8, 12

4	8	8	14
4	12	12	8
4	0	12	4.
12	4	0	0

0	12	12	0
4	4	4	12
8	8	8	0
12	0	0	12

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4.

1	1(0)	1
2	[illegible](1)	[illegible]
3	N(8)	[illegible]
n.	n(1n)	n^2

1, n+2, 2n

1	2	3	4
3	4		2
1	2	3	4
2	4		

a	g	g	a
b	c	c	b
c	b	b	c
g	a	a	g

x	y	t	v
v	t	y	x
v	t	y	x
x	y	t	v

$$\begin{array}{|cccc|} 7 & 12 & 1 & 14 \\ 2 & 13 & 8 & 11 \\ 16 & 3 & 10 & 5 \\ 9 & 6 & 15 & 4 \end{array} = \begin{array}{|cccc|} 3 & 4 & 1 & 2 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{array} + \begin{array}{|cccc|} 4 & 8 & 0 & 12 \\ 0 & 12 & 4 & 8 \\ 12 & 0 & 8 & 4 \\ 8 & 4 & 12 & 0 \end{array}$$

Khajuraho split by N-method

1, 2, 3, 4

0, 4, 8, 12

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

1	2	3	4
2	3	4	1
4	1	2	3
3	4	1	2

1	2	3	4
3	4	1	2
4	1	2	3
2	3	4	1

The condition that numbers shall not be repeated in cols or rows is not necessary for the N-method to hold. These condns are that in the summed up square the nos 1 to n^2 should not be repeated. For this two condns (vide Andrews p. 14) are needed. Call the primary squares A & B, where B leads to the ~~(a)~~ root square by replacement by of 0, n, 2n, .. (n−1)n by 1, 2, .. n.

(i) Every number 1 to n must appear once & only once in corresponding cells of A & B.

(ii) Two different numbers in 1 to n, must also appear once and only once in a pair of corresponding cells of A & B.

These conditions ensure that all the numbers 1 to n^2 appear once only once in the final sum-square.

Subject to these conditions, squares A & B ~~can be~~ are written [illegible] by using different artifices varying from author to author.

1	2	3	4
4	2	1	3
2	3	4	1
4	1	2	3

1	3	4	2
3	2	4	1
4	1	3	2
3	4	1	3

0	8	12	4
8	4	12	0
12	0	8	4
8	12	0	8

1	10	15	8
12	6	5	4
14	3	7	

1	2	3	4
0	4	8	12

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

1	2	3	4
4	2	3	1
4	2	1	3
3	4	1	2

0	4	8	12
12	4	8	0
12	4	0	8
8	12	0	4

1	6	11	16
15	8	9	2
14	7	4	9

1	2	3	4
3	4	1	2
2	3	4	1
4	1	2	3

4	1	2	3
2	3	1	4
3	4	1	2
1	2	3	4

1	3	4	2
4	4	1	1
4	2	3	3
1	3	2	4

0	12	12	0
0	12	12	0
4	8	8	4
4	8	8	4

1	13	14	2
4	16	15	3
8	12	11	7
5	9	10	6

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

1	2	3	4
4	3	2	1
4	3	2	1
1	2	3	4

✓

0	12	12	0
4	8	8	4
8	4	4	8
12	0	0	12

✓

1	14	15	4
8	11	10	3
12	7	6	9
13	2	3	16

✓

4

1	4	3	2
4	1	2	3
4	1	2	3
1	4	3	2

0	12	12	0
12	0	0	12
8	4	4	8
4	8	8	4

1	16	15	2
16	1	2	15

A_1	B_4	C_2	D_3
D_2	C_3	B_1	A_4
B_3	A_2	D_4	C_1
C_4	D_1	A_3	B_2

(43)

1	8	10	15
12	13	3	6
7	2	16	9
14	11	5	4

1	4	2	3
4	1	3	2
3	2	4	1
2	3	1	4

1	2	3	4
3	4	1	2
2	1	4	3
4	3	2	1

0	4	8	12
8	12	0	4
4	0	12	8
12	8	4	0

1	4	2	3
2	3	1	4
3	2	4	1
4	1	3	2

A	B	C	D
D	C	B	A
B	A	D	C
C	D	A	B

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

0	4	8	12
12	8	4	0
4	0	12	8
8	12	0	4

1	8	10	15
14	11	5	4
7	2	16	9
12	13	3	6

0	12	4	8
4	8	0	12
8	4	12	0
12	0	8	4

1	14	7	12
8	11	2	13
10	5	16	3
15	4	9	6

(2, 1), (1, -1) (-2, -1)
(2, -1), (-1, 1) (2, -1)

(0, 1)

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

$$\begin{array}{|ccc|}\hline 2&1&3\\3&2&1\\1&3&2\\\hline\end{array} + \begin{array}{|ccc|}\hline 6&0&3\\0&3&6\\3&6&0\\\hline\end{array} = \begin{array}{|ccc|}\hline 8&1&6\\3&5&7\\4&9&2\\\hline\end{array}$$

$$\begin{array}{|ccc|}\hline 1&3&2\\3&2&1\\2&1&3\\\hline\end{array} + \begin{array}{|ccc|}\hline 3&0&6\\6&3&0\\0&6&3\\\hline\end{array} = \begin{array}{|ccc|}\hline 4&3&8\\9&5&1\\2&7&6\\\hline\end{array}$$

$$\begin{array}{|ccc|}\hline 2&1&3\\3&2&1\\1&3&2\\\hline\end{array} + \begin{array}{|ccc|}\hline 0&6&3\\6&3&0\\3&0&6\\\hline\end{array} = \begin{array}{|ccc|}\hline 2&7&6\\9&5&1\\4&3&8\\\hline\end{array}$$

~~i + n + k + c = 34~~ ✓
~~n + c = 17, i + h = 17~~ ✓
~~m + b + g + l = 34.~~

m + l + b + g = 34
i + n + h + c = 34
e + j + o + d = 34
m + j + g + d = 34
a + n + k + h = 34
e + b + o + l = 34
i + f + c + p = 34
a + f + k + p = 34

a + p = 17
b + o = 17
c + n = 17
d + m = 17
e + l = 17
f + k = 17
g + j = 17
h + i = 17

$p+b+34-(p+b)=34$ (44)

a	b	c	d
e	f	g	h
17 −h	17−g	17−f	17−e
17 −d	17−c	17−b	17−a

$a+h+34-f-c=34$

$a+h=f+c$ $\quad a+d+h+e=b+f+c+g$

$d+e=b+g$

$a+e=h+d$

$b+f=g+c$, $b+g=e+d.$ $\quad$ ~~$b-c=g-f$~~

$=g-d.$

$b+g+34-(e+d)$

$=34$

$d+h=a+e$

$a+b+c+d=e+f+g+h$

$a+b+e+f=c+d+g+h.$

c d

b d c

a a

b

~~$34-(c+h)+(c+h)$~~

~~$=34$~~

$(a+b)+(c+d)=e+f+g+h$

~~$e+34-(b+g)+d=34$~~ $(a+b)-(c+d)=g+h-(e+f)$

~~$b+g=e+d$~~

$a+p+b+q=37$

$a+h=f+\overset{c}{e}$ $\big\}$ $a+e=d+h$

$d+e=b+g$ $\big\}$ $b+f=g+c$

$a+h=f+c$

$a-h=d-e$

$a+b=g+h$ $\quad|\quad a+d+e+h=b+c+f+g$

$c+d=e+f$ $\quad|\quad a+d+b+c=e+h+f+g.$

$a+e=d+h$

$b+f=g+c$

~~$a+h=g+c$~~

~~$a+h=f+c$~~

~~$b+g=c+a$~~

$a+d=f+g$

$b+c=e+h$

$a+b=g+h$

$c+d=e+f$

$a+d-(b+c)=(f+g)-(c+h)$

$2a+p+\cancel{s}=c+d+q+\cancel{s}=17+q$

$2b+q+\cancel{r}=c+d+\cancel{r}+p=11+p.$

$2a=17-p+q$

$2b=17+p-q.$ $\quad \cancel{2}(b-a)=\cancel{2}(p-q)$

$a+h=f+c$ $\big\}$ $\quad a+b+c+d=34$

$b+g=d+e$ $\big\}$ $\quad e+f+g+h=34$

$a+e+b+f=34$

$2(a+e)+2(b+f)=68$

$a+e+b+f=34$

$a+b+c+d=34.$

$c+d=e+f$

$a+b=g+h.$

~~$a+h-f=b+f+g$~~

$a+b+h+g=c+d+f+e$

$a+b=c+d.=17.$

$17=2d-(r-s)$

$17=2c+(r-s)$

~~2~~ $2c-2d+2(r-s)=0$ //

$c-d=s-r$ //

$a, 17-a, c, 17-c.$

$e, 17-e, f, 17-f$

$c-d=s-r.$

$c+d=s+r.$

$c=s$

$d=r$ $\Big\}$ [illegible]

~~$17+p+r$~~ $=17+p+s.$

$a+b+\cancel{p}+r=2d+\cancel{p}+s$

$a+b+q+s=2c+p+r$

$2A+\cancel{q}=\cancel{2}B+\cancel{p}$

a,	17-a,	c	17-c
e,	17-e,	f,	17-f
f	17-f	e	17-e
c	17-c	a	17-a

a	b	c	d
p (17-a)	q (17-b)	r (17-c)	s (17-d)
17-s	17-r	17-q	17-p
17-d	17-c	17-b	17-a

$a + \cancel{17} - c + \cancel{17} - q + s = \cancel{34}$

$c + q + 17 - s + \cancel{17} - a = 34$

$p + \cancel{17} - r + \cancel{17} - b + d = 34$

$$\left.\begin{array}{l} a+p = d+s \\ b+q = c+r \end{array}\right\} \quad a+b+c+d = 34$$
$$p+q+r+s = 34$$
$$\left.\begin{array}{l} a+s = c+q \\ b+r = d+p \end{array}\right\}$$

$a+b+p+q = c+d+r+s$ $\;2a+p+\cancel{s}$ $\;= 2c+d+q+\cancel{s}$

$a+b+r+s = c+d+p+q$ $\;2a+p = d+b+q$

$(p+q)-(r+s) = (r+s)-(p+q)$

$p+q = r+s = 17$

$a+b = c+d = 17$

$a + b + \cancel{q} + s = 2c + \cancel{q} + r$ $\quad a+p = b+q.$

$\cancel{a} + d + s = \cancel{a}c + r$

$\cancel{d = r - s}$

$\cancel{a = r - p}$

$\cancel{b = p - s}$

$\cancel{c = a}$ $\;\cancel{p+s}$

$c+r = d+a$

$c+r = a+s$

$\cancel{d = 17}$

$\boxed{a+b} + \cancel{p} + r - \cancel{d} \cancel{p} \quad \cancel{a = q - s} = r - p$

$c + \cancel{f}$

$r = s + 17.$

$a = s + 17 - p. = q + s.$

$a+p = r - \cancel{p} + \cancel{s}$

$b + \cancel{q} = \cancel{r} - s + p.$

$\cancel{2}(a+p) + \cancel{2}(b+q) = \cancel{68}\ 34$

$c+q \equiv r - p + s$

$c = (r+s) - (p+q) = \cancel{0}$

$$\left.\begin{array}{l} a+p = 17+s \\ b+q = r \\ a+s = q \\ b+r = 17+p. \end{array}\right\} \begin{array}{l} a-s = q. \quad a=q,\ s=0, \\ a+s = q \quad a=q,\ s=0 \end{array}$$

$P-Q = Q-P$

$P-Q = 0-P$

$b = p.,\ r = 17.$

$(a+b)+(p+q) = (c+d)+(r+s)$

$(a+b)+(r+s) = (c+d)+(p+q)$

$$\left.\begin{array}{l} A+P = B+Q \\ A+Q = B+P \\ A+B = P+Q = 34 \end{array}\right\} \left.\begin{array}{l} A = B = 17 \\ P = Q = 17 \end{array}\right\}$$

$(a+b)+(\cancel{q}+s) = 2c + \cancel{q} + r$ $\quad ,\cancel{a+q} + 2a + p + \cancel{p} = c + d + \cancel{p} + q.$ $\quad 17 + r = 2d + s$

$= 17 + q$

$17 + s = 2c + r$

$r + q = a + b.$

$a + b + \cancel{q} + r = 2d + \cancel{q} + s.$

(46)

$$\begin{array}{|cccc|} a & b & p & q \\ c & d & r & s \\ 17-s & 17-r & 17-d & 17-c \\ 17-q & 17-p & 17-b & 17-a \end{array}$$

$$\left.\begin{array}{l} a+c=q+s \\ b+d=p+r \end{array}\right\} \begin{array}{l} a+b+p+q=34 \\ c+d+r+s=34 \end{array}$$

$$\left.\begin{array}{l} a+s=p+d \\ b+r=q+c \end{array}\right\}$$

$$a+p=b+q$$
$$=c+r=d+s=17.$$

$$2a+b+e-\frac{s}{2}=s$$
$$s-(a+b+c)+\frac{s}{2}(a-c+e)=s$$
$$\frac{s}{2}-c+b+c-e=s.$$
$$\frac{s}{2}-a+e=s$$

$$\left\{\begin{array}{l} a=p+d-s \\ \quad =p+p+r-b-s. \end{array}\right.$$

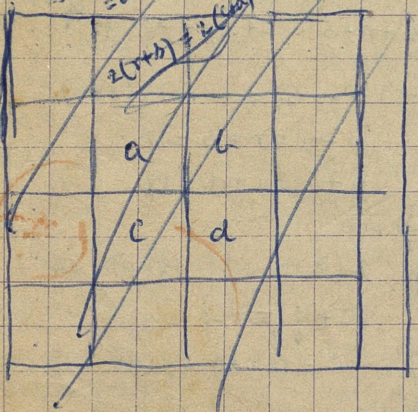
$a-b+c-r=s-c$

$a-b+s-d=d-r.$

$a-b+s-d.$

$s-c-c+r$

$=d-r-s+d.$



$$\begin{array}{|cccc|} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{array}$$

$$\begin{array}{|cccc|} a & b & c & d \\ p & q & r & s \\ 17-s & 17-r & 17-q & 17-p \\ 17-d & 17-c & 17-b & 17-a \end{array}$$

$$\begin{array}{l|l|} a-p=d-s & a-b=c-d \\ b-q=c-r & p-q=r-s \end{array}$$

$$\begin{array}{l|l} a+b=c+d & =p+q=r+s. \\ a-b=c-d & p-q=r-s \\ a=b=c=d & p=q=r=s \end{array}$$

$$(a-b)+(c-d)=(q-p)+(s-r).$$
$$(a-b)+(s-r)=(p-q)+(d-c).$$

$$(a-b)+(p-q)=(c-d)$$

$$A'+B'+C'+D'=0$$
$$A'+B'=C'+D'.$$

$$\left.\begin{array}{l} a-b=A' \\ c-d=B' \\ p-q=C' \\ r-s=D' \end{array}\right\}$$

$$A'+B'=C'+D'=0.$$

~~a+b+c~~

$$\boxed{\begin{array}{l} a-b+c-d=0 \\ a-b+c-d \\ a+b+c+d=34 \end{array}}$$

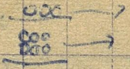
$a-b+c-d=0.$

$a+b+c+d=17$

$a+c=17$

$$\left.\begin{array}{l} a+b=17 \\ \quad =17 \end{array}\right\}$$

77
46



$b + c + f + g$

$j + k + n + p$
$= e + h + a + d$

$S - a - \cancel{b} - e + \cancel{b} + c + g$

$S + c + g - a - e$

$1, 2, \quad . \quad . \quad n$

$0, n, \quad n(n-1)$

$$\boxed{\begin{array}{l} p \to n + e^{+} + \bar{\nu}_e \\ n \to p + e^{-} + \nu_e \end{array}}$$

1	3	2	4
4	2	3	1
4	2	3	1
1	3	2	4

1	4	4	1
3	2	2	3
2	3	3	2
4	1	1	4

0	12	12	0
8	4	4	8
4	8	8	4
12	0	0	12

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

4	2	3	1
2	4	3	1
3	3		4
1	1	4	

1	2	3	4
3	4	1	2
1	2	3	4
3	4	1	2

1	2	3	4
4	1	2	3
3	4	1	2
2	3	4	1

1	2	3	4
2	3	4	1
3	4		2
4	1	2	3

1	15	14	4
5	11	10	8
12	6	7	9
16	2	3	13

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	1

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

1	24	17	15	8
14	7	5	23	16
22	20	13	6	4
10	3	21	19	12
18	11	9	2	25

20	22	4	6	13
9	11	18	25	2
23	5	7	14	16
12	19	21	3	10
1	8	15	17	24

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

23	1	2	20	19
22	12	11	16	4
5	17	13	9	21
8	10	15	14	18
7	25	24	6	3

1, 2, 3, 4, 5
0, 5, 10, 15, 20

a and n regular

(1)

1	24	17	15	8
14	7	5	23	16
22	20	13	6	4
10	3	21	19	12
18	11	9	2	25

1	4	2	5	3
4	2	5	3	1
2	5	3	1	4
5	3	1	4	2
3	1	4	2	5

(A) (4,1) 3

0	20	15	10	5
10	5	0	20	15
20	15	10	5	0
5	0	20	15	10
15	10	5	0	20

(2,3) ~~B~~(C)

1	5	4	3	2
3	2	1	5	4
5	4	3	2	1
2	1	5	4	3
4	3	2	1	5

(B) 2

regular. a not n

(2)

20	22	4	6	13
9	11	18	25	2
23	5	7	14	16
12	19	21	3	10
1	8	15	17	24

5	2	4	1	3
4	1	3	5	2
3	5	2	4	1
2	4	1	3	5
1	3	5	2	4

(A) (3,2)

15	20	0	5	10
5	10	15	20	0
20	0	5	10	15
10	15	20	0	5
0	5	10	15	20

(C) (2,3)

4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2
1	2	3	4	5

(B)

n not a regular.

(3)

10	18	1	14	22
11	24	7	20	3
17	5	13	21	9
23	6	19	2	15
4	12	25	8	16

5	3	1	4	2
1	4	2	5	3
2	5	3	1	4
3	1	4	2	5
4	2	5	3	1

(A) (3,2)

5	15	0	10	20
10	20	5	15	0
15	0	10	20	5
20	5	15	0	10
0	10	20	5	15

(C) (2,3)

2	4	1	3	5
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
1	3	5	2	4

(B)

a and n

not regular

(4)

23	1	2	20	19
22	12	11	16	4
5	17	13	9	21
8	10	15	14	18
7	25	24	6	3

3	1	2	5	4
2	2	1	1	4
5	2	3	4	1
3	5	5	4	3
2	5	4	1	3

(A)

20	0	0	15	15
20	10	10	15	0
0	15	10	5	20
5	5	10	10	15
5	20	20	5	0

(C)

5	1	1	4	4
5	3	3	4	1
1	4	3	2	5
2	2	3	3	4
2	5	5	2	1

(B)

(3,5), (1,1), (2,1)
(5,2), (4,4).
(2,5), (2,3), (1,3)
(1,4), (4,1)

neither a nor n (borders). (A) (C) (B)

RM Bq.

(1,0), (2,1), (0,3), (2,-1) | (1,0), (2,1), (0,-2), (1,-1) | (-2,-2), (1,-2), (1,2), (1,-2)

(2,0), (0,1), (-2,1), (-2,-1)

(-1,0), (1,-1), (1,-1), (-1,0)

(-2,-1) (-2,-1) (0,-2), (-1,0)

(-1,1), (0,1), (2,1) (-1,0)

~~(n-1){n(n+1)+1}~~

(n-1) R·m

(n-1) BM

No systematic variation either

in R M S or bachmann

Chinese 5×5 (This splitting is done on p. 137, Bk. 13

not regular

(49)

(5)

4	19	25	15	2
20	10	5	18	12
3	17	13	9	23
14	8	21	16	6
24	11	1	7	22

a not n.

4	4	5	5	2
5	5	5	3	2
3	2	3	4	3
4	3	1	1	1
4	1	1	2	2

(A)

0	15	20	10	0
15	5	0	15	10
0	15	10	5	20
10	5	20	15	5
20	10	0	5	20

(C)

1	4	5	3	1
4	2	1	4	3
1	4	3	2	5
3	2	5	4	2
5	3	1	2	5

(B)

Although associative like (1), (5) is more like (4) with

R.M's:-(2,-1), (1,-2), (0,2), (2,-1)
(-1,-1), (-2,1), (2,1), (-2,1)
(-2,-2), (-2,-1), (-2,-1), (-2,-2)
(-2,1), (2,1), (-2,1), (-1,-1).
(2,-1), (0,2), (1,-2), (2,-1)

B M's:- (2,-2), (0,2), (0,2), (2,-2).
↔ because associative.

R.M's like the 9×9 associative [illegible] Ball (not one single set of 4, but ~~4~~ 5 different sets same read backwards & forwards)

(6) Square on p. 36 worked out (neither a nor n — not bordered) — Single set of R.M's and 4 different B.M's.

~~[illegible] by the N method~~

3	2	1	4	5
5	3	2	1	4
4	5	3	2	1
1	4	5	3	2
2	1	4	5	3

(B)

(7) The N-method 5×5 in Eng. article due to Narayana.

4	5	1	2	3
5	1	2	3	4
1	2	3	4	5
2	3	4	5	1
3	4	5	1	2

(A) (4,1)

10	5	0	20	15
15	10	5	0	20
20	15	10	5	0
0	20	15	10	5
5	0	20	15	10

(C) (1,4)

3	2	1	5	4
4	3	2	1	5
5	4	3	2	1
1	5	4	3	2
2	1	5	4	3

(B)

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

$\begin{pmatrix} -2, 2 \\ 2, 2 \end{pmatrix}$ regular

a not n

Note: Even if centres have been interchanged, to get a magic square it needs 3 in the central cell of one & 1 in the other. But square is neither a nor n

Andrews Fig. 29, p. 15 [R.M: (2,1), B.M= (1,-1)

Regular

	1	15	24	8	17	1	5	4	3	2	0	10	20	5	25	1	3	5	2	4
(8)	23	7	16	5	14	3	2	1	5	4	20	5	15	0	10	5	2	4	1	3
	20	4	13	22	6	5	4	3	2	1	15	0	10	20	5	4	1	3	5	2
	12	21	10	19	3	2	1	5	4	3	10	20	5	15	0	3	5	2	4	1
	9	18	2	11	25	4	3	2	1	5	5	15	0	10	20	2	4	1	3	5
	a and n					(A) (2,3)					(C) (3,2)					(B)				

In all cases except (4) and (5) where there are repetitions, Reichmann's rules of writing rows by varying elements in the first row hold. I wonder if this be true in general for non-repetitive (A) & (B).

– All cases (1) – (8) expressible by the N-method. Essential condns ~~are~~ are that (1) all nos 1 to n^2 must appear in the final square and not to be repeated

(2) Sums of corresp. rows & corresponding cols & diagonals of (A) & (C) must sum to S.

For (1) condns are (a) like numbers must appear once & only once in similarly placed cells of (A) & (B)

(b) pairs of unlike numbers are not repeated in the same order in any similarly placed cells in (A) & (B).

For eg. in (4) (p. 48), (a) number 4 appears in the corresp. cells (4,4) of both A & B, but it does not appear in any other corresp. cells of A & B

(b) Again 2 appears in cell (2,4) of A & 1 appears in cell (2,4) of B but 2 and 1 do not appear in ~~same~~ order in similar cells of (A) & (B) anywhere else, although 1 and 2 appear in (3,0) of (A) & (B).

Re Condn (2), note that in the cases 1, 2, 3, 6, 7, 8 all rows, cols & main diagonals of (A) sum to 15 & sim. all rows, cols & diagonals of (C) sum to 50 so that final square is magic – ~~On~~ Also (A), (B), (C) are themselves "magic" when final square is a or n or both a & n same is true of (A), (B), (C).

In case (4) where there is repetition; we find cols of (A) & (C) summing up to 15 & 50 respy & all rows except the 2nd & 4th sum to 15 & 50. For the 2nd row the sum in (A) is 10 but the sum in (C) is 55 so that total is 65

For 4th row " (A) is 20 " 45 " 65

Simy in case (5) rows 1 & 2 of (A) sum to 20 the corresp. rows of (C) sum to 45

& rows 4 & 5 of (A) 10 " (C) " 55

& col 1 of (A) sum to 20 & col of 1 of (C) to 45

col 5 of (A) " 10 " " to 55

1	4	3	5	2
5	2	1	4	3
4	3	5	2	1
2	1	4	3	5
3	5	2	1	4

1	3	2	5	4
2	5	4	1	3
4	1	3	2	5
3	2	5	4	1
5	4	1	3	2

0	10	5	20	15	1	14	8	25	17
5	20	15	0	10	10	22	16	4	13
15	0	10	5	20	19	3	15	7	21
10	5	20	15	0	12	6	24	18	5
20	15	0	10	5	23	20	2	11	9

~~1~~	~~3~~	~~2~~	~~5~~	~~4~~
~~1~~	~~5~~	~~4~~	~~1~~	~~3~~
	~~1~~	~~3~~	~~5~~	
~~3~~	~~4~~	~~2~~		~~4~~
~~2~~	~~1~~	~~4~~	~~3~~	~~5~~

~~1~~	~~3~~	~~2~~	~~5~~	~~4~~
~~4~~	~~5~~	~~1~~	~~3~~	~~2~~
~~3~~	~~2~~	~~3~~		
~~4~~	~~3~~	~~5~~	~~1~~	
~~2~~	~~1~~	~~4~~	~~3~~	~~4~~

n and q

~~1~~	~~3~~	~~4~~	~~5~~	~~2~~	2	4	1	3	5
~~3~~	~~2~~	~~5~~	~~1~~	~~4~~	3	5	2	4	1
~~5~~	~~4~~	~~3~~	~~2~~	~~1~~	4	1	3	5	2
~~2~~	~~5~~	~~1~~	~~4~~	~~3~~	5	2	4	1	3
~~4~~	~~1~~	~~2~~	~~3~~	~~5~~	1	3	5	2	4

n (A) q (B)

~~5~~	~~15~~	~~0~~	~~10~~	~~20~~
~~10~~	~~20~~	~~5~~	~~15~~	~~0~~
~~15~~	~~0~~	~~10~~	~~20~~	~~5~~
~~20~~	~~5~~	~~15~~	~~0~~	~~10~~
~~0~~	~~10~~	~~20~~	~~5~~	~~15~~

(C)

~~6~~	~~18~~	~~4~~	~~15~~	~~22~~
~~13~~				

waste of energy

	3	3	3	
				3
3		3		3
3				
		3	3	

4	2	5	3	6
6	4	2	5	3
3	6	4	2	5
5	3	6	4	2
2	5	3	6	4

1	0	4	3	2
0	4	3	2	1
4	3	2	1	0
3	2	1	0	4
2	1	0	4	3

Dakerian
& non-Dakerian
Rubbish

0, 5, 10, 15, 20

20 + 20 + 0 + 5 + 5 = 50//
10 + 0 + 0 + 15 + 15 = 50

In (4), p.48, pairing of el nos in corresp. cells of A·B are

(3,5), (1,1), (2,1), (5,4), (4,4)
(2,5), (2,3), (1,3), (1,4), (4,1)
(5,1), (2,4), (3,3), (4,2), (1,5)
(3,2), (5,2), (5,3), (4,3), (3,4)
(2,2), (5,5), (4,5), (1,2), (3,1).

Conditions (a) & (b) of p.50 are equivalent to the statement that all the 25 combinations $\begin{matrix}(A)\\(B)\end{matrix}\begin{pmatrix}1&2&3&4&5\\1&2&3&4&5\end{pmatrix}$ must appear once & once only.

In (3) (5,2), (3,4), (1,1), (4,3), (3,5).
(1,3), (4,5), (2,2), (5,4), (3,1)
(2,4), (5,1), (3,3), (1,5), (4,2)
(3,5), (1,2), (4,4), (2,1), (5,3).
(4,1), (2,3), (5,5), (3,2), (1,4)

(1,1), (1,2), (1,3), (1,4), (1,5)
(2,1), (2,2), (2,3), (2,4), (2,5)
(3,1), (3,2), (3,3), (3,4), (3,5)
(4,1), (4,2), (4,3), (4,4), (4,5)
(5,1), (5,2), (5,3), (5,4), (5,5)

In (5). (4,1), (4,4), (5,5), (5,3), (2,1)
(5,4), (5,2), (5,1), (3,4), (2,3).
(3,1), (2,4), (3,3), (4,2), (3,5)
(4,3), (3,2), (1,5), (1,4), (1,2)
(4,5), (1,3), (1,1), (2,2), (2,5)

(1,0), (1,5), (1,10), (1,15), (1,20)
(2,0), (2,5), (2,10), (2,15), (2,20)
(3,0), (3,5), (3,10), (3,15), (3,20)
(4,0), (4,5), (4,10), (4,15), (4,20)
(5,0), (5,5), (5,10), (5,15), (5,20)

Condition (2) i.e. sums of corresp. rows, cols & diagonals summing to 5 appear obvious if sums of A & B are each 15 in rows, cols & diagonals - This holds if all elements in every row, col & diag are different as in cases (1), (2), (3),

1, 6, 11, 16, 21 | 4, 9, 14, 19, 24
2, 7, 12, 17, 22 | 5, 10, 15, 20, 25
3, 8, 13, 18, 23 |

In addition to these eight we have considered, we have many more of [illegible] obtained by geometrical patterns & equilibrium designs (Andrews. Chap XI, p. 248) Take some of these which are all associated.

a not n

(2,3) + (4,1) + (3,2) + (4,1) + (2,3) || (3,2) + (1,4) + (2,3) + (1,4) + (3,2)

(9)

6	18	5	12	24	1	3	5	2	4	5	15	0	10	20	2	4	1	3	5
23	10	17	4	11	3	5	2	4	1	20	5	15	0	10	5	2	4	1	3
19	1	13	25	7	4	1	3	5	2	15	0	10	20	5	4	1	3	5	2
15	22	9	16	3	5	2	4	1	3	10	20	5	15	0	3	5	2	4	1
2	14	21	8	20	2	4	1	3	5	0	10	20	5	15	1	3	5	2	4

Non-regular (A) (C') (B)

R.M: (−1,−2), (−1,1), (−1,2), (−1,1). B.M (−2,0), (−2,0). (−2,0), (−2,0).

(−1,−2), (−1,−2), (−1,1), (−1,2).

(−1,1), (−1,−2), (−1,−2), (−1,1).

(−1,2), (−1,1), (−1,−2), (−1,−2).

(−1,1), (−1,2). (−1,1), (−1,−2)

(−1,−2), (−1,2), (−1,1).

Single B.M. two different R.M's symmetrically situated

(modifies Reichmann's rule).

(−1,−2) & (−2,0) → (a,b) = (−1,−2), (a',b') = (−2,2)

(−1,2) & (−2,0) → (a,b) = (−1,2), (a',b') = (0−1,−2)

(−1,1) & (−2,0) → (a,b) = (−1,1), (a',b') = (−1,−1)

} ab' − a'b ≠ 0 & prime to 5

(10)

14	10	1	22	18	4	5	1	2	3	10	5	0	20	15	3	2	1	5	4
2	23	19	15	6	2	3	4	5	1	0	20	15	10	5	1	5	4	3	2
21	17	13	9	5	1	2	3	4	5	20	15	10	5	0	5	4	3	2	1
20	11	7	3	24	5	1	2	3	4	15	10	5	0	20	4	3	2	1	5
8	4	25	16	12	3	4	5	1	2	5	0	20	15	10	2	1	5	4	3

a not n non-regular (A) (C') (B)

(1,4) + (2,3) + (4,1) + (2,3) + (1,4) || (4,1) + (3,2) + (1,4) + (3,2) + (4,1)

R.M: (−2,−1), (−2,−2), (−2,−1), (−2,2) : B.M: (0,1). (0,2), (0,2), (0,1).

(−2,−2), (−2,−1), (−2,2), (−2,2)

(−2,−1), (−2,2), (−2,2), (−2,−1).

(−2,2), (−2,2), (−2,−1), (−2,−2)

(−2,2), (−2,−1), (−2,−2), (−2,−1)

} 5 diff R.M's symmetrically situated

2 diff B.M's symmetrically situated

(~~modifies~~ Reichmann's method).
modifies

(9) & (10) are geometric patterns.

(52,) **a** but not **n** not regular.

(11)

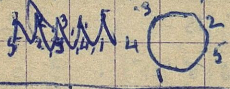
6	3	19	15	22	1	3	4	5	2	5	0	15	10	20	2	1	4	3	5
12	24	5	16	8	2	4	5	1	3	10	20	0	15	5	3	5	1	4	2
25	17	13	9	1	5	2	3	4	1	20	15	10	5	0	5	4	3	2	1
18	10	21	2	14	3	5	1	2	4	15	5	20	0	10	4	2	5	1	3
4	11	7	23	20	4	1	2	3	5	0	10	5	20	15	1	3	2	5	4

(A) (C') (B)

5 **R M's**, BM: (−2,1), (0,−1), (0,−1), (−2,1)

[~~[illegible]~~ Reichmann's
nothing to do with → method]

sym:



a not **n**

(12)

3	6	17	24	15	3	1	2	4	5	0	5	15	20	10	1	2	4	5	3
10	18	21	12	4	5	3	1	2	4	5	15	20	10	0	2	4	5	3	1
19	25	13	1	7	4	5	3	1	2	15	20	10	0	5	4	5	3	1	2
22	14	5	8	16	2	4	5	3	1	20	10	0	5	15	5	3	1	2	4
11	2	9	20	23	1	2	4	5	3	10	0	5	15	20	3	1	2	4	5

not regular. (A) (1,4) (C) (4,1) (B)

R.M: (−2,−2), (−1,−1), (−1,−1), (−2,2). **B.M**: (−1,−2), (0,2), (0,2), (−1,−2).
(−2,−2), (−1,−1), (−1,−1), (−2,−2) also sym

Single cut R.M. sym method

(Correct Reichmann scheme)

a not **n**

(13)

2	9	16	23	15	2	4	1	3	5	0	5	15	20	10	1	2	4	5	3
8	20	22	14	1	3	5	2	4	1	5	15	20	10	0	2	4	5	3	1
19	21	13	5	7	4	1	3	5	2	15	20	10	0	5	4	5	3	1	2
25	12	4	6	18	5	2	4	1	3	20	10	0	5	15	5	3	1	2	4
11	3	10	17	24	1	3	5	2	4	10	0	5	15	20	3	1	2	4	5

(A) (2,3) (C') (4,1) (B)

R.M: (1,1), (1,1), (1,1), (1,1). regular **BM**: (0,−1)

Identical ~~[illegible]~~ with D.M with (a,b) = (1,1), BM (0,−1).
& (i,j) = (4,3) (Correct Reichmann's scheme).

---

neither a nor n

Nelson's.

(55)

(14)

23	20	4	6	12
17	3	10	14	21
1	7	13	25	19
9	11	22	18	5
15	24	16	2	8

nonregular

3	5	4	1	2
2	3	5	4	1
1	2	3	5	4
4	1	2	3	5
5	4	1	2	3

(A) (14)

5	4	1	2	3
4	1	2	3	5
1	2	3	5	4
2	3	5	4	1
3	5	4	1	2

(B)

20	15	0	5	10
15	0	5	10	20
0	5	10	20	15
5	10	20	15	0
10	20	15	0	5

(C) (41)

R.M: (−2, −2), (−2, −2), (1,1), (2,2); B.M: (−1, −2), (−1, −2), (2,0), (−2, −1).

(−2, −2), (−2, 2), (1,1), (2,2),
(−2, −2), (−2, −2), (1,1), (2,2)
(−2, −2), (−2, −2), (1,1), (2,2)
(−2, −2), (−2, −2), (1,1), (2,2)

one set R.M & 4 B M's.

(Reichmann's scheme) ✓

3	5	1	2	4
5	1	2	4	3
1	2	4	3	5
2	4	3	5	1
4	3	5	1	2

+

[illegible]	10	5	25	15
15	20	10	5	25
25	15	20	10	5
[illegible]	25	15	20	10
10	[illegible]	25	10	20

15	5	0	20	10
10	15	5	0	20
20	10	15	5	0
0	20	10	15	5
5	0	20	10	15

18	10	1	22	14
15	16	7	4	23
21	12	19	8	5
2	24	13	20	6
9	3	25	11	17

X semi-magic

1	4	3	5	2
3	5	2	1	4
2	1	4	3	5
4	3	5	2	1
5	2	1	4	3

+

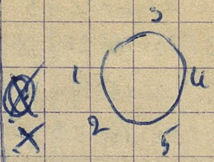
20	15	0	5	10
5	10	20	15	0
15	0	5	10	20
10	20	15	0	5
0	5	10	20	15

21	19	3	10	12
8	15	22	16	4
17	1	9	13	25
14	23	20	2	6
5	7	11	24	18

✓

4	3	1	5	2

4	5	2	1	3	5	0	15	10	20	9	5	17	11	23
2	1	3	4	5	0	15	10	20	5	2	16	13	24	10
3	4	5	2	1	15	10	20	5	0	18	14	25	7	1
5	2	1	3	4	10	20	5	0	15	15	22	6	3	19
1	3	4	5	2	20	5	0	15	10	21	8	4	20	12



4	5	2	1	3	9	5	17	11	23	10	20	5	0	15	14	25	7	1	18
3	4	5	2	1	3	19	15	22	6	0	15	10	20	5	2	16	13	24	10
1	3	4	5	2	16	13	24	10	2	20	5	0	15	10	23	9	5	17	11
2	1	3	4	5	12	21	8	4	20	15	10	20	5	0	20	12	21	8	4
5	2	1	3	4	25	7	1	18	14	5	0	15	10	20	6	3	19	15	22

✓

3	1	2	4	5	0	5	15	10	20	③	⑥	(17)	(14)	(24)	3	1	2	4	5
5	3	1	2	4	20	0	5	15	10	(24)	③	⑥	(17)	(14)	1	2	4	5	3
4	5	3	1	2	10	20	0	5	15						2	4	5	3	1
2	4	5	3	1	15	10	20	0	5			X			4	5	3	1	2
1	2	4	5	3	5	15	10	20	0						5	3	1	2	4

Rpb

3	1	2	4	5	0	15	20	10	5	3	16	22	14	10	3	6	17	14	25
5	3	1	2	4	15	20	10	5	0	20	23	11	7	4	21	2	9	20	13
4	5	3	1	2	20	10	5	0	15	24	15	8	1	17	12	24	5	8	16
2	4	5	3	1	10	5	0	15	20	12	9	5	18	21	19	15	23	1	7
1	2	4	5	3	5	0	15	20	10	6	2	19	25	13	10	18	11	22	4

5	4	3	1	2	5	0	10	15	20	10	4	13	16	22		3	5	1	4	2
4	3	1	2	5	20	5	0	10	15	24	8	1	12	20		2	3	5	1	4
3	1	2	5	4	15	20	5	0	10	18	21	7	5	14	X	4	2	3	5	1
1	2	5	4	3	10	15	20	5	0	11	17	25	9	3		1	4	2	3	5
2	5	4	3	1	0	10	15	20	5	2	15	19	23	6		5	1	4	2	3

(a)

(a) + (b) mettre [illegible]

23	5	6	19	12	5	20	15	10	0	10	24	18	11	2		20	0	5	15	10
2	8	20	11	24	0	5	20	15	10	4	8	21	17	15	X	0	5	15	10	20
9	17	13	25	1	10	0	5	20	15	13	21	7	25	19		5	15	10	20	0
16	14	22	3	10	15	10	0	5	20	16	12	5	9	23		15	10	20	0	5
15	21	4	7	18	20	15	10	0	5	22	20	14	3	6		10	20	0	5	15

(b)

															2	4	5	3	1
5	4	3	1	2	15	10	0	5	20	5	4	3	1	2	5	15	20	10	0
			2		20	15	10	0	5	1	2	5	4	3	20	10	0	5	15
		2			5	20	15	10	0	4	3	1	2	5	0	5	15	20	10
	2				0	5	20	15	10	2	5	4	3	1	15	20	10	0	5
2					10	0	5	20	15	3	1	2	5	4.	10	0	5	15	20
										2	4	5	3	1					
18	19	23	11	2	5	4	3	1	2	5	15	20	10	0	10	19	23	11	2
21	12	5	9	18	3	1	2	5	4	10	0	5	15	20	13	1	7	20	24
4	8	16	22	15	2	5	4	3	1	15	20	10	0	5	17	25	14	3	6
17	25	14	3	6	4	3	1	2	5	0	5	15	20	10	4	8	16	22	15
13	1	7	20	24	1	2	5	4	3	20	10	0	5	15	21	12	5	9	18
					1	2	3	4	5										
1	2	3	4	5	0	5	10	15	20	1	7	13	19	25					
3	4	5	1	2	15	20	0	5	10	18	24	5	6	12					
5	1	2	3	4	5	10	15	20	0	10	11	17	23	4					
2	3	4	5	1	20	0	5	10	15	22	3	9	15	16					
4	5	1	2	3	10	15	20	0	5	14	20	21	2	8					
4	5	1	2	3	10	15	20	0	5	1	2	4	5	3	10	0	5	15	20
5	1	2	3	4	5	10	15	20	0	2	4	5	3	1	20	10	0	5	15
1	2	3	4	5	0	5	10	15	20	4	5	3	1	2	15	20	10	0	5
2	3	4	5	1	20	0	5	10	15	5	3	1	2	4	5	15	20	10	0
3	4	5	1	2	15	20	0	5	16	3	1	2	4	5	0	5	15	20	10.
3	4	5	1	2	4	5	1	2	3	15	20	0	5	10	18	24	5	6	12
2	3	4	5	1	5	1	2	3	4	20	0	5	10	15	22	3	9	15	16
1	2	3	4	5	1	2	3	4	5	0	5	10	15	20	1	7	13	19	25
5	1	2	3	4	2	3	4	5	1	5	10	15	20	0	10	11	17	23	4
4	5	1	2	3	3	4	5	1	2	10	15	20	0	5	14	20	21	2	8

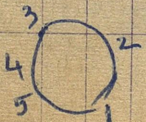
✓ n

✓ n

n

✓

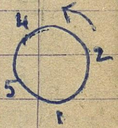
a
n√n



5	4	1	2	3	3	5	4	1	2	10	20	15	0	5	15				8
4	1	2	3	5	2	3	5	4	1	5	10	20	15	0					
1	2	3	5	4	1	2	3	5	4	0	5	10	20	15					
2	3	5	4	1	4	1	2	3	5	15	0	5	10	20					
3	5	4	1	2	5	4	1	2	3	20	15	0	5	10	23				12
3	2	1	5	4	5	0	20	15	10	3	1	2	5	4	0	5	20	15	10
4	3	2	1	5	0	20	15	10	5	4	3	1	2	5	5	20	15	10	0
5	4	3	2	1	20	15	10	5	0	5	4	3	1	2	20	15	10	0	5
1	5	4	3	2	15	10	5	0	20	2	5	4	3	1	15	10	0	5	20
2	1	5	4	3	10	5	0	20	15	1	2	5	4	3	10	0	5	20	15
5	4	2	1	3	a ~~10~~	20	15	5	0	3	4	2	1	5	15	5	0	20	10
4	2	1	3	5	0	10	20	15	5	5	3	4	2	1	5	0	20	10	15
2	1	3	5	4	5	0	10	20	15	1	5	3	4	2	0	20	10	15	5
1	3	5	4	2	15	5	0	10	20	2	1	5	3	4	20	10	15	5	0
3	5	4	2	1	20	15	5	0	10	4	2	1	5	3	10	15	5	0	20

not a.

not a



(a)

not a

1 2 3 4 5
1 4 3 2 5
2 5 3 1 4
2 1 3 5 4.

(*)

N-method & Nelson's method are the same. with the difference that if the mirror be any of these four or their reverses the sum-square is associate otherwise the sum-square is not associated. (call this Nelson's rule).

3	1	4	2	5	20	15	5	0	10
5	3	1	4	2	15	5	0	10	20
2	5	3	1	4	5	0	10	20	15
4	2	5	3	1	0	10	20	15	5
1	4	2	5	3	10	20	15	5	0

(associate)

using N-method with 25314 in primary
& 21354 in root square.
& taking elements diagonally upwards & downwards.
(generalisation of N-method giving associate squares)

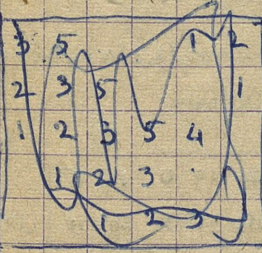
4	5	1	2	3	3	5	4	1	2	10	20	15	0	5	14	25	16	2	8
5	1	2	3	4	2	3	5	4	1	5	10	20	15	0	10	11	22	18	4
1	2	3	4	5	1	2	3	5	4	0	5	10	20	15	1	7	13	24	20
2	3	4	5	1	4	1	2	3	5	15	0	5	10	20	17	3	9	15	21
3	4	5	1	2	5	4	1	2	3	20	15	0	5	10	23	19	5	6	12
(A)																			

magic but not <u>a</u>

ie Combining 12345 in (A) with 12354 ie 0, 5, 10, 20, 15

ie generalisation of N-method giving non-associative square.

Also a generalisation of Nelson's method with 12345 & 12354 in centre row



3	5	2	1	4	20	15	0	5	10
4	3	5	2	1	15	0	5	10	20
1	4	3	5	2	0	5	10	20	15
2	1	4	3	5	5	10	20	15	0
5	2	1	4	3	10	20	15	0	5

generalisation Nelson's method with both rows not of the asymmetric type as in (*) p.58

What about <u>nasiks</u>?

(4)					3	4	5	1	2	3	4	5	1	2					
④	5	1	2	3	10	15	20	0	⑤	10	15	20	0	5	4	5	1	2	③
2	3	④	5	1	20	0	⑤	10	15	0	5	10	15	20	1	2	③	4	5
5	1	2	3	④	⑤	10	15	20	0	15	20	0	5	10	③	4	5	1	2
3	④	5	1	2	15	20	0	⑤	10	5	10	15	20	0	5	1	2	③	4
1	2	3	④	5	0	(15)	10	15	20	20	0	5	10	15	2	③	4	5	1
	<u>n</u>																		

					1	~~2~~	3 <u>n</u>	4	5	3	5	1	4	2	3	5	1	4	2
1	2	3	4	5	0	5	10	15	20	10	20	0	15	5	10	20	0	15	5
4	5	1	2	3	10	15	20	0	5	15	5	10	20	0	0	15	5	10	20
2	3	4	5	1	20	0	5	10	15	20	0	15	5	10	5	10	20	0	15
5	1	2	3	4	5	10	15	20	0	5	10	20	0	15	20	0	15	5	10
3	4	5	1	2	15	20	0	5	10	0	15	5	10	20	15	5	10	20	0
	<u>n</u>					<u>n</u>					<u>n</u>					<u>n</u>			

To get nasiks, take two first rows (same or different) & apply Reichmann's rules (~~same~~ or different) for each

To get nasiks starting from middle row, same rule applies [illegible]

					2	3	4	5	1	2	3	5	4	1	4	1	2	3	5
5	1	2	3	4	5	10	15	20	0	5	10	20	15	0	15	0	5	10	20
3	4	5	1	2	15	20	0	5	10	20	15	0	5	10	10	20	15	0	5
1	2	3	4	5	0	5	10	15	20	0	5	10	20	15	0	5	10	20	15
4	5	1	2	3	10	15	20	0	5	10	20	15	0	5	20	15	0	5	10
2	3	4	5	1	20	0	5	10	15	15	0	5	10	20	5	10	20	15	0

(1) n & a (2) n & a (3) n but not a (4) n but not a

(1) & (4) gives repetitions

Could we also generalise N-method or Nelson's method for Reichmann's (2,3) or (3,2) rule for associative squares? We have done it for the (4,1) or (1,4) rules

5	1	2	3	4	5	10	15	20	0	15	5	20	10	0	20	10	0	15	5
3	4	5	1	2	15	20	0	5	10	10	0	15	5	20	0	15	5	20	10
1	2	3	4	5	0	5	10	15	20	5	20	10	0	15	5	20	10	0	15
4	5	1	2	3	10	15	20	0	5	0	15	5	20	10	10	0	15	5	20
2	3	4	5	1	20	0	5	10	15	20	10	0	15	5	15	5	20	10	0

(1) n & a (2,3) (2) n & a (3,2) (3) n & a (2,3) (4) n & a. (3,2)

(1) & (3) no

~~yes~~

(2) & (4) no

Does the N-method for associative square hold with same (4,1) ~~or~~ or (1,4) for both?

4	5	1	2	3	20	15	5	0	10	15	5	0	10	20
5	1	2	3	4	15	5	0	10	20	10	20	15	5	0
1	2	3	4	5	5	0	10	20	15	5	0	10	20	15
2	3	4	5	1	0	10	20	15	5	20	15	5	0	10
3	4	5	1	2	10	20	15	5	0	0	10	20	15	5

a not n a not n a & n

no it does not ~~(4,1) & (1,4)~~ ∵ of repetitions

also (4,1) or (1,4) with (2,3) or (3,2) also gives associative square.

So we have after all arrived at some sensible rules for squares a but not n, n but not a and a & n both for 5×5 squares all starting from middle row. Obviously what holds for middle row holds for middle column also.

for n squares middle row or col or any row or col is same since rules are (2,3), (3,2).

What about ~~[illegible]~~ the other nasty ones like (4) and (5) on p. 49. (4) appears hopeless & we shall discard it. Since (5) is associated, let us see if we can make sense out of it & get others like it.

5.	1	5	1	3
4.	2	4	2	3
5	3	3	3	1
.	4		4	2
3	5		5	1

1	5	1	3	1
4	2	2	4	3
1	4	3	2	5
3	2	4	4	2
5	3	5	1	5

19	10	1	12	23
15	21	17	8	4
6	2	13	24	20
22	18	9	5	11
3	14	25	16	7

This is 12345 acted on by (1,1) and 21354 acted on by (2,3). gives associated square?

4		25		2
3		13		23
24	0	1		22

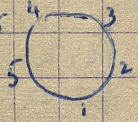
=

4	5	2
3	3	3
4	1	2

0	20	0
0	10	20
20	0	20

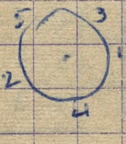
Seems very typical!

In (9), (10), (11) of Frierson (all a) we find Reichmann's rule modified. In ~~[illegible]~~ (10) with the central row 1,2,3,4,5



the other rows are in same cyclic order, but starting with 5,3,4,2 resp.

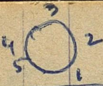
In (9) middle row is 41352



& other rows are cyclic order but starting with 5,2,1,3.

Suppose we start in some systematic order starting from 1,2,3,4,5 resp. we have analogous to (9)

(B 2)



4	5	1	2	3
5	1	2	3	4
1	2	3	4	5
2	3	4	5	1
3	4	5	1	2

1	2	4	5	3
2	4	5	3	1
4	5	3	1	2
5	3	1	2	4
3	1	2	4	5

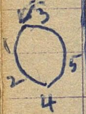
Same as N- method with
(4,1), [illegible]

5	1	2	3	4
1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
3	4	5	1	2

1	3	4	2	5
5	1	3	4	2
3	4	2	5	1
2	5	1	3	4
4	2	5	1	3

0	10	15	5	20
20	0	10	15	5
10	15	5	20	0
5	20	0	10	15
15	5	20	0	10

X



5	3	1	2	4
3	1	2	4	5
4	5	3	1	2
1	2	4	5	3
2	4	5	3	1

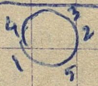
+

4	2	1	3	5
5	4	2	1	3
2	1	3	5	4
3	5	4	2	1
1	3	5	4	2

5	2	3	4	1
1	5	2	3	4
5	2	3	4	1
4	1	5	2	3
3	4	1	5	2

20	10	0	5	15
15	20	0	5	20
15	20	10	0	5
10	0	5	15	20
0	5	15	20	10

$(2,3) + (4,1) \overset{a}{+} (3,2) + (4,1).$ ✓ $\overset{a}{(3,2)} + (1,4) + (2,3) + (1,4)$



X

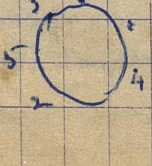
3	1	2	4	5
1	2	4	5	3
4	5	3	1	2
1	2	4	5	3
2	4	5	3	1

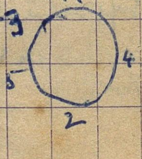
4	1	3	5	2
2	4	1	3	5
4	1	3	5	2
5	2	4	1	3
3	5	2	4	1

2	4	1	3	5
5	2	4	1	3
4	1	3	5	2
3	5	2	4	1
1	3	5	2	4

1	3	5	2	4
3	5	2	4	1
4	1	3	5	2
5	2	4	1	3
2	4	1	3	5

$(2,3) + \overset{*}{(4,1)} + (2,3) + (4,1)$ $(2,3) \overset{\times}{+} (1,4) + (2,3) + (1,4).$





5	2	4	1	3
3	5	2	4	1
4	1	3	5	2
1	3	5	2	4
2	4	1	3	5

(4,1) + (2,3) + (1,4) + (4,1)
X

1	3	5	2	4
3	5	2	4	1
4	1	3	5	2
3	5	2	4	1
5	2	4	1	3

(3,2) + (4,1) + (2,3) + (4,1)
x

2	4	1	3	5	15	10	5	0	20
5	2	4	1	3	10	5	0	20	15
4	1	3	5	2	20	15	10	5	0
3	5	2	4	1.	5	0	20	15	10
1	3	5	2	4	0	20	15	10	5

(3,2) + (1,4) + (2,3) + (1,4) ‖ (2,3) + (4,1) + (3,2) + (4,1)

1	3	5	2	4	15	10	5	0	20	0	10	20	5	15
3	5	2	4	1	10	5	0	20	15	10	20	5	15	0
4	1	3	5	2	20	15	10	5	0	15	0	10	20	5
5	2	4	1	3	5	0	20	15	10	20	5	15	0	10
2	4	1	3	5	0	20	15	10	5	5	15	0	10	20

(2,3) + (4,1) + (3,2) + (4,1), – Do – X † X

(9) & (10) of Frierson are explained by the series (2,3) + (4,1) + (3,2) + (4,1) and (3,2) + (1,4) + (2.3) + (1,4) combined (giving answer squares) ~~in same series with same middle row or diff middle row~~, or diff series with same or different middle rows to avoid repetition

Re (11)

1	3	4	5	2	5	10	15	0	20	6	13	14	5	22
3	2	5	1	4	15	20	0	5	10	18	22	5	6	14
5	4	3	2	1	20	5	10	15	0					
2	5	1	4	3	10	15	20	0	5					
4	1	2	3	5	0	20	5	10	15					

X
Repetitions

(11) of Frierson appears as tough as (4) & (5) but in (11) there are no repetitions.

Let us go to n = 7 & n = 9.

n = 7 : Reichmann's steps are (6,1), (5,2), (4,3), (3,4), (2,5), (1,6).

1, 2, 3, 4, 5, 6, 7

0, 7, 14, 21, 28, 35, 42

(1) Narayanas examples – [illegible]//

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(a)

4	3	2	7	1	6	5
5	4	3	2	7	1	6
6	5	4	3	2	7	1
1	6	5	4	3	2	7
7	1	6	5	4	3	2
2	7	1	6	5	4	3
3	2	7	1	6	5	4

14	35	0	42	7	28	21
35	0	42	7	28	21	14
0	42	7	28	21	14	35
42	7	28	21	14	35	0
7	28	21	14	35	0	42
28	21	14	35	0	42	7
21	14	35	0	42	7	28

18	38	2	49	8	34	26
40	4	45	~~14~~ 9	35	22	20
6	47	11	31	23	21	36
43	13	33	25	17	37	7
14	29	27	19	39	3	44
30	28	15	41	5	46	10
24	16	42	1	13	12	32

(1,6) ——— + ——— (6,1) (1,6) + (6,1). ✓ a not n

(b)

0	42	7	28	21	14	35
14	35	0	42	7	28	21
28	21	14	35	0	42	7
42	7	28	21	14	35	0
35	0	42	7	28	21	14
21	14	35	0	42	7	28
7	28	21	14	35	0	42

4	45	9	35	22	20	40
19	39	3	44	14	29	27
34	26	18	38	2	49	8
43	13	33	25	17	37	7
42	1	48	12	32	24	16
23	21	36	6	47	11	31
10	30	28	15	41	5	46

3	6	1	7	2	5	4
6	1	7	2	5	4	3
1	7	2	5	4	3	6
7	2	5	4	3	6	1
2	5	4	3	6	1	7
5	4	3	6	1	7	2
4	3	6	1	7	2	5

(2,5). (1,6) + (2,5) ✓ a (6,1)

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3	48	8	35	23	19	39
20	36	7	44	12	32	24
29	28	16	40	4	45	13
49	9	33	25	17	41	1
37	5	46	10	34	22	21.
26	18	38	6	43	14	30
11	31	27	15	42	2	47.

7	46	8	34	24	19	37
17	40	2	49	11	29	27
32	22	20	38	5	44	14
47	9	35	25	15	41	3
36	6	45	12	30	28	18
23	21	39	1	48	10	33
13	31	26	16	42	4	43

7	4	1	6	3	5	2
3	5	2	7	4	1	6
4	1	6	3	5	2	7
5	2	7	4	1	6	3
1	6	3	5	2	7	4
2	7	4	1	6	3	5
6	3	5	2	7	4	1

(6,1) + (2,5) ✓ a (2,5) + (3,4) ✓ a a n (3,4)

All these are a but n

(2) Nasiks

							5	6	1	7	4	3	2							
2	1	4	5	6	7	3	35	0	42	21	14	7	28	37	1	46	26	20	14	31
7	3	2	1	4	5	6	42	21	14	7	28	35	0	49	24	16	8	32	40	6
5	6	7	3	2	1	4	14	7	28	35	0	42	21	19	13	35	38	2	43	25
1	4	5	6	7	3	2	28	35	0	42	21	14	7	29	39	5	48	28	17	9
3	2	1	4	5	6	7	0	42	21	14	7	28	35	3	44	22	18	12	34	42
6	7	3	2	1	4	5	21	14	~~14~~ 7	28	35	0	42	27	21	10	30	36	4	47
4	5	6	7	3	2	1	7	28	35	0	42	21	14	11	33	41	7	45	23	15

(2,5) (5,2) (2,5) + (5,2) n not a.

So the whole thing is correct for n = 7. ~~[illegible]~~ and there appears no need to go into Frierson types for n = 7. Perhaps we need not go into n = 9 either.

~~[illegible] squares~~:

n = 4

Summary of results for odd squares n = 5

(1) Narayana's original example -

middle of primary square (A) with 3 in the centre & middle row of root square (B) with 10 in the centre. Complete A by drawing ~~all~~ five lower diagonals (upper diagonals) through the elements of the middle row & ~~[illegible]~~ similarly complete B by drawing ~~all~~ five upper diagonals (lower diagonals) through the elements of the middle row. Add A & B to get square ~~C~~ magic square C - equivalence of these to obtaining rows in A by step (1,4) (or (4,1)) & in B by step (4,1) (or (1,4)).

If middle rows of A & B be symmetric about the centre elements 3 or 10 resp., C will be an associated magic square.

(2)

3	1	2	5	4	5	0	15	20	10	8	1	17	25	14
4	3	1	2	5	0	15	20	10	5	4	18	21	12	10
5	4	3	1	2	15	20	10	5	0	20	24	13	6	2
2	5	4	3	1	20	10	5	0	15	22	15	9	3	16
1	2	5	4	3	10	5	0	15	20	11	7	5	19	23

neither a nor n

(A) (B) (4,1) (C)

(1,4) (down) (up)

(66) Symmetric series in middle row in both A & B

(ii)

4	1	5	2	3	10	5	20	0	15	14	6	25	2	18
1	5	2	3	4	15	10	5	20	0	16	15	7	23	4
5	2	3	4	1	0	15	10	5	20	5	17	13	9	21
2	3	4	1	5	20	0	15	10	5	22	3	19	11	10
3	4	1	5	2	5	20	0	15	10	8	24	1	20	12

(4,1) (A) (up)   (1,4) (B) (down)   (C)

a but not n.

(2) other modifications.

(iii) In (i) with same middle rows, take (1,4) for A & step (2,3) for B

3	1	2	5	4	0	15	20	10	5	3	16	22	15	9
4	3	1	2	5	10	5	0	15	20	14	8	1	17	25
5	4	3	1	2	15	20	10	5	0	20	24	13	6	2
2	5	4	3	1	5	0	15	20	10	7	5	19	23	11
1	2	5	4	3	20	10	5	0	15	21	12	10	4	18

(1,4) (A)   (2,3) (B)   (C')

neither a nor n

(iv) In (ii) take (~~4,1~~ 1,4) for A & (2,3) for B ~~[illegible]~~ is

3	4	1	5	2	20	0	15	10	5	23	4	16	15	7
2	3	4	1	5	10	5	20	0	15	12	8	24	1	20
5	2	3	4	1	0	15	10	5	20	5	17	13	9	21
1	5	2	3	4	5	20	0	15	10	6	25	2	18	14
4	1	5	2	3	15	10	5	20	0	19	11	10	22	3

(1,4) (A)   (2,3) (B)   (C)

a but not n

(v) Symmetric series in A & B with (2,3) for A & (3,2) for B

5	1	2	3	4	20	10	0	5	15	25	11	2	8	19
3	4	5	1	2	0	5	15	20	10	3	9	20	21	12
1	2	3	4	5	15	20	10	0	5	16	22	13	4	10
4	5	1	2	3	10	0	5	15	20	14	5	6	17	23
2	3	4	5	1	5	15	20	10	0	7	18	24	15	1

(2,3)   (3,2)

a and n.

(vi) Any series in middle rows of A & B & steps (3,2), (2,3) or (2,3), (3,2).

1	2	4	5	3	5	15	20	0	10	6	17	24	5	13
5	3	1	2	4	20	0	10	5	15	25	3	11	7	19
2	4	5	3	1	10	5	15	20	0	12	9	20	23	1
3	1	2	4	5	15	20	0	10	5	18	21	2	14	10
4	5	3	1	2	0	10	5	15	20	4	15	8	16	22

(2,3) A   (3,4

n but not a

In all cases diff. steps must be taken for A & B otherwise there will be repetitions even though middle rows be different. Steps (1,4) or (4,1) do not lead to n squares.

This covers all cases for n = 5

~~For n = 7, [illegible]~~

For n = 7, steps are (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

(i) For symmetric series in middle row i.e. nos. equidistant from centre $\overset{4}{}$ adding to 8, any two steps combined give a-squares; if (1,6) or (6,1) be used squares won't be n: other steps used (not repeated) for A & B give nasik squares whether middle rows are symmetric or not. If symmetric a & n result.

For n = 9, it would be interesting to find if we can get a nasik square since we have no such regular nasiks of order 9. In fact Fig. 61, p. 78 of main book is such an associated nasik square (taken out from Ball, Fig XXII, p. 210).

Let us get (i) a nasik not associated
(ii) an associated nasik

For (i) take middle row of A & B as

6 7 8 3 2 1 9 4 5 and 18, 45, 36, 72, 63, 9, 27, 0, 54

and take steps (4,5) & (5,4)

For (ii) take ~~[illegible]~~ 2,6,1,7,5,3,9,4,8 for A.

and 0 27 54 9 36 63 18 45 72

& take steps (7,2) & (2,7)

8,1), (7,2), (6,3), (5,4)
4,5) (3,6), (2,7), (1,8)

(68)

$S = \frac{1}{2}9\cdot 82 = \underline{369}$

45 + 324

4	5	6	7	8	3	2	1	9
3	2	1	9	4	5	6	7	8
5	6	7	8	3	2	1	9	4
2	1	9	4	5	6	7	8	3
6	7	8	3	2	1	9	4	5
1	9	4	5	6	7	8	3	2
7	8	3	2	1	9	4	5	6
9	4	5	6	7	8	3	2	1
8	3	2	1	9	4	5	6	7

$(A)_1 (4,5)$

36	72	63	9	27	0	54	18	45
27	0	54	18	45	36	72	63	9
45	36	72	63	9	27	0	54	18
9	27	0	54	18	45	36	72	63
18	45	36	72	63	9	27	0	54
63	9	27	0	54	18	45	36	72
54	18	45	36	72	63	9	27	0
72	63	9	27	0	54	18	45	36
0	54	18	45	36	72	63	9	27

$(B)_1 (5,4)$

no magic with rows

$(C)_1$

fails ✗

Surprising

however magic

40	77	69	16	35	3	56	19	54
30	2	55	27	49	41	78	70	17
50	42	79	71	12	29	1	63	22
11	28	9	58	23	51	43	80	66
24	52	44	75	65	10	36	4	59
64	18	31	5	60	25	53	39	74
61	26	48	38	73	72	13	32	6
81	67	14	33	7	62	21	47	37
8	57	20	46	45	76	68	15	34

35	56	15	37	70	23	48	81	4
18	40	71	20	51	73	7	32	57
68	21	54	76	8	29	60	10	43
46	79	5	30	63	13	44	65	24
2	33	55	16	41	66	27	49	80
58	17	38	69	19	52	77	3	36
39	72	22	53	74	6	28	61	14
25	50	75	9	31	62	11	42	64
78	1	34	59	12	45	67	26	47

magic ✓

a ✓

$(C)_2$

not n

✗

So it looks as if the rule does not hold for n = 9

re-results

See Book-18 for clarification

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8	2	6	1	7	5	3	9	4
9	4	8	2	6	1	7	5	3
5	3	9	4	8	2	6	1	7
1	7	5	3	9	4	8	2	6
2	6	1	7	5	3	9	4	8
4	8	2	6	1	7	5	3	9
3	9	4	8	2	6	1	7	5
7	5	3	9	4	8	2	6	1
6	1	7	5	3	9	4	8	2

$(A)_2$ (2,7)

27	54	9	36	63	18	45	72	0
9	36	63	18	45	72	0	27	54
63	18	45	72	0	27	54	9	36
45	72	0	27	54	9	36	63	18
0	27	54	9	36	63	18	45	72
54	9	36	63	18	45	72	0	27
36	63	18	45	72	0	27	54	9
18	45	72	0	27	54	9	36	63
72	0	27	54	9	36	63	18	45

$(B)_2$ (7,2)

1 2 3 4 5 6 7 8 9

1 2 3 5 6 4 9 7 8

n = 5

7	8	9	1	2	3	4	5	6	27	36	45	54	63	72	0	9	18
1	2	3	4	5	6	7	8	9	0	9	18	27	36	45	54	63	72
4	5	6	7	8	9	1	2	3	54	63	72	0	9	18	27	36	45
7	8	9	1	2	3	4	5	6	27	36	45	54	63	72	0	9	18
1	2	3	4	5	6	7	8	9	0	9	18	27	36	45	54	63	72
4	5	6	7	8	9	1	2	3	54	63	72	0	9	18	27	36	45
7	8	9	1	2	3	4	5	6	27	36	45	54	63	72	0	9	18
1	2	3	4	5	6	7	8	9	0	9	18	27	36	45	54	63	72
4	5	6	7	8	9	1	2	3	54	63	72	0	9	18	27	36	45

A_3 (6,3) + B_3 (3,4) ≠ magic square

not magic in cols at all.

3	1	4	5	2
5	2	3	1	4
1	4	5	2	3
2	3	1	4	5
4	5	2	3	1

(2,3)

10	25	0	5	15
5	[illegible]	[illegible]	20	0
25	10	20	0	5
15	10	20	0	5
0	5	15	10	20
10	20	0	5	15
5	15	10	20	0
20	0	5	15	10

(2,3)

3	6	1	8	6	1	8	6	1	9	36	63	72	18	45	27	54	0	17	42	64	80	24	46	35	60	1
5	3	7	5	3	7	5	3	7	27	54	0	9	36	63	72	18	45	32	57	7	14	39	70	77	21	52
2	9	4	2	9	4	2	9	4	72	18	45	27	54	0	9	36	63	74	27	49	29	63	4	11	45	67
1	8	6	1	8	6	1	8	6	9	36	63	72	18	45	27	54	0	10	41	69	73	26	51	28	62	6
7	5	3	7	5	3	7	5	3	27	54	0	9	36	63	72	18	45	34	59	3	16	41	66	79	23	48
4	2	9	4	2	9	4	2	9	72	18	45	27	54	0	9	36	63	76	20	54	31	56	9	13	38	72
6	1	8	6	1	8	6	1	8	9	36	63	72	18	45	27	54	0	15	37	71	78	19	53	33	55	8
3	7	5	3	7	5	3	7	5	27	54	0	9	36	63	72	18	45	30	61	5	12	43	68	75	25	50
9	4	2	9	4	2	9	4	2	72	18	45	27	54	0	9	36	63	81	22	47	36	58	2	19	40	65

A_4 B_4 C_4 a and n

(4,2,9), (7,5,3), (1,8,6)

4 7 1 2 5 8 9 3 6

4 11 2 5 8 9 3 6

Different from the one in Rouse - Ball

(70)

3,8,4 || 1,5,9 || 2,7,6

$A_5(\underline{n})$

$B_5(n)$ $\therefore A_5 + B_5 = n$ and $\underline{a}$

1,2,3,5,6,4,9,7,8 ✓

(1,2,3,5,7,8,4,6,9)

1,5,9 ✓
1,6,8 ✓
2,4,9 ✓
2,5,8 ✓
2,6,7 ✓
3,4,8 ✓
3,5,7 ✓
4,2,9 x
4,3,8 x
4,5,6 ✓
5,1,9 x
5,2,8 x
5,3,7 x
5,4,6 x
6,1,8 x
6,2,7 x
6,4,5 x
7,2,6 x
7,3,5 x
8,1,6 x
8,2,5 x
8,3,4 x
9,1,5 x
9,2,4 x

4	8	3	4	8	3	4	8	3
6	7	2	6	7	2	6	7	2
5	9	1	5	9	1	5	9	1
3	4	8	3	4	8	3	4	8
2	6	7	2	6	7	2	6	7
1	5	9	1	5	9	1	5	9
8	3	4	8	3	4	8	3	4
7	2	6	7	2	6	7	2	6
9	1	5	9	1	5	9	1	5

36	45	27	72	54	63	0	9	18
72	54	63	0	9	18	36	45	27
0	9	18	36	45	27	72	54	63
36	45	27	72	54	63	0	9	18
72	54	63	0	9	18	36	45	27
0	9	18	36	45	27	72	54	63
36	45	27	72	54	63	0	9	18
72	54	63	0	9	18	36	45	27
0	9	18	36	45	27	72	54	63

5	7	8	4	6	9	1	2	3
1	2	3	5	7	8	4	6	9
4	6	9	1	2	3	5	7	8
5	7	8	4	6	9	1	2	3
1	2	3	5	7	8	4	6	9
4	6	9	1	2	3	5	7	8
5	7	8	4	6	9	1	2	3
1	2	3	5	7	8	4	6	9
4	6	9	1	2	3	5	7	8

9	7	8	1	2	3	5	6	4
1	2	3	5	6	4	9	7	8
5	6	4	9	7	8	1	2	3
9	7	8	1	2	3	5	6	4
1	2	3	5	6	4	9	7	8
5	6	4	9	7	8	1	2	3
9	7	8	1	2	3	5	6	4
1	2	3	5	6	4	9	7	8
5	6	4	9	7	8	1	2	3

3	5	6	4	9	7	8	1	2
9	7	8	1	2	3	5	6	4
2	3	5	6	4	9	7	8	1
4	9	7	8	1	2	3	5	6
1	2	3	5	6	4	9	7	8
6	4	9	7	8	1	2	3	5
8	1	2	3	5	6	4	9	7
5	6	4	9	7	8	1	2	3
7	8	1	2	3	5	6	4	9

diff

There are eight combinations giving sum = 15

(1,5,9), (1,6,8), (2,4,9) (2,5,8)

(2,6,7), (3,4,8), (3,5,7), (4,5,6)

Of these (1,5,9), (2,5,8), (3,5,7), (4,5,6)

have 5 in centre & remaining 2 digits

In Ex. in Ball: we have ~~taken~~

(i) (2,4,9), (3,5,7), (1,6,8)

1,7,3,4,
6,7,9

~~In Ex. in previous page we have taken~~

~~(ii) (2,6,7) (1,5,9) (3,4,8)~~

1,2,4,6,8,9

(2,5,8) x

(3,5,7) see (i)

1,2,3,7,8,9

~~4,5,8~~ x

~~(4,2,9), (7,5,3), (1,6,8)~~ (1,5,6)

$A_5'(\underline{n})^{(6,3)}$ $\therefore A_5' + B_5 = (\underline{n})$ $B_5'^{(5,4)}$

not $\underline{a}$

$A_5' + B_5'$ is also $\underline{n}$

449 3 456978 132654879

213654798

312456397 1 2 3 4 5 / 0 5 10 15 20

~~1236548.7~~

~~[illegible]~~

132659874 1 3 2 6 5 9 8 7 4

1 2 3 5 6 4 9 7 8

125643897

123564978

(258), 348159267

471258936

(456), (753), (159) (258).

~~(429) (753) (186)~~

~~(219) (357) (168)~~

~~(429) (357) (168)~~

492 5

(1) (2,6,7), (1,5,9), (3,4,8).

159

672159834

267159348

236951478

213654798

168249537 258613874

only 2 combinations with all digits different

(159), (267), (348)

and (168), (249), (357).

168357249

471258936

244 267159348 267951438 267159346

2 213654798. 294 / 65

2	5	8	6	9	3	7	1	4	2	3	1	4	5	6	9	7	8
7	9	4	2	1	3	6	5	8	2	1	3	6	5	4	7	9	8
									4	7	1	2	5	8	9	3	6
2	1	3	6	5	4	7	9	8	6	5	8	9	1	3	7	9	4
									7	9	9	2	5		6	1	

Contd in BR. 18.

2nd tst

(1) Bedi ✓ — (1)

(2) Gavaskar ✓ — (2)

(3) Viswanath ✓ — (3)

(4) Brijesh Patel ✓ — (11)

(5) S. Venkataraghavan ✓ — (8)

(6) E. A. S. Prasanna ✓

(7) Surinder Amarnath ✓

(8) Mohinder Amarnath ✓ — (4)

(9) S. Kirmani ✓ — (5)

(10) Bharath Reddy ✓

(11) K. Ghavri ✓

(12) Abid Ali (?) X

(13) B. S. Chandrasekhar ✓ — (6)

(14) Ashok Mankad. ✓ — (9).

(15) D. Vengsarkar ✓ — (7)

(16) P. Sarma (?) X

(17) Madanlal ✓ — (10).

(18) T. E. Srinivasan X

(29) Sivaramakrishnan X

(20) Chetan Chauhan (19) ✓

(21) Yash Pal. X

(22) Kapil Dev. X

1st tst

~~5 3 8 9 8~~

8 2 16 24 15

Bedi ✓ 5 18 22 18 4

Prasanna ✓ 19 25 13 7 1

Chandrasekhar ✓ 21 14 10 3 17

Kirmani ✓ 12 6 4 20 23

Chauhan X 3 2 1 4 5

Vengsarkar ✓ 5 3 2 1 4

Viswanath ✓ 4 5 3 2 1

Patel. ✓ 1 4. 5 3 2

Ghavri X 2 1 4 5 3

Mohinder ✓ 2 1 4 5

Mankad. ✓ 1 4 5

Gavaskar ✓ 4 5 3

Madanlal. 5 3 2 1

3 2 1 4 5

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 - u^2 = 2as$

$= 2gh$

Newton — Newton

Einsten

$\boxed{E = mc^2}$

$E = mc^4$ $E = m_0c^4$

1755.59
55.00
1700.59

16 people chosen as guessed by me