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ISTITUTO MATEMATICO
"ULISSE DINI"

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Dear Professor Varma,

thank you for your sending me the paper "Quantumlike theory etc", that I read with pleasure.

I think that your arguments about eq. (24) are absolutely appropriate and the appearance of second order eqs. like (29) is totally consistent. Indeed, consider an eq. of the following form

$$\partial f/\partial t + a(x)\,\partial f/\partial x + b(x)\,\partial f/\partial \phi = 0$$

and consider solutions f that are periodic in ϕ. Then the Fourier coefficients $f_n(x,t) = (1/2\pi)\int_{-\pi}^{\pi} e^{-in\phi} f(x,t,\phi)\,d\phi$ satisfy

$$\partial f_n/\partial t + a(x)\,\partial f_n/\partial x + i n b(x) f_n = 0 \qquad (n = 0, \pm 1, \pm 2, \dots)$$

an infinite system of first-order p.d.e. Suppose $a(x)$ & $b(x)$ are real-valued, and split f_n into real and imaginary part: $f_n = u_n + i v_n$. We have

$$\begin{cases} \partial u_n/\partial t + a(x)\,\partial u_n/\partial x - n\,b(x)\,v_n = 0 \\ \partial v_n/\partial t + a(x)\,\partial v_n/\partial x + n\,b(x)\,u_n = 0, \end{cases}$$

a coupled system for $u_n = \mathrm{Re}\, f_n$ and $v_n = \mathrm{Im}\, f_n$. The elimination of v_n (solve the first eq. for v_n and plug into the second eq.) gives:

$$\left\{\frac{\partial^2}{\partial t^2} + 2a(x)\frac{\partial^2}{\partial x\partial t} + \left(a(x)\frac{\partial}{\partial x}\right)^2\right\} u_n +$$

$$- a(x)\frac{b'(x)}{b(x)}\left\{\frac{\partial}{\partial t} + a(x)\frac{\partial}{\partial x}\right\} u_n + n^2 b(x)^2 u_n = 0,$$

just a second-order p.d.e.

I enjoyed very much my stay in India and remember with pleasure the conversations we had in Bangalore. Hoping to meet you again in future, I remain

Yours sincerely

Giorgio Talenti