Summary of Lecture.

(1) ~~18~~ Eighteen types of O.R. techniques mentioned in first lecture – Only four to be dealt with (1) L.P (2) Game theory (3) Queueing theory (4) Monte Carlo technique.

(2) Linear programming & Game theory are intimately connected – Military O.R uses a single or a combination of several of the 18 techniques of O.R – fundamental stress on theory of games & hence also on L.P – direct use by military O.R of L.P. techniques in logistics, specially transportation problems, personnel problems, Production, storage & distribution problems, ~~military operations problems~~ weapon selection and target analysis problems – Reference to Bibliography on L.P. compiled in 1957 by F. Virginia Rohde as summer work at the U.S. Naval Proving Ground – maximums & minimums – sword, gift & soft words –

(3) Simple illustration of L.P by (i) Vitamins problem ~~& (ii) $y-2x \leqq 2$, $x-2y \leqq 2$, $x+y \leqq 5$, to minimise~~ ~~$y-x$~~ using geometrical methods – Generalisation to higher dimensions – Limitation of geometric methods

(4) Example of production planning to illustrate another limitation of geometric method – factory producing so many units per month, but sales small during summer & very large in December. Soln (i) run factory overtime in fall but overtime costs are more than for normal output; Soln (ii) Produce more during ~~summer~~ spring & summer & store for winter rush, warehousing costs money in storage, handling charges & interest on money tied up ~~(iii)~~ Soln (iii) is to fail to meet orders in December which is a council of despair! This is an L.P. problem, variables being regular production each month & overtime prodn each month & excess of production over sales each month is warehouse. Boundary condns are limits on production normal & overtime, & requirements that total production from the first of the year shall never be less than the total sales from the first. Quantity to be minimised is the total cost, including overtime & warehousing charge – Straightforward L.P. if we can predict our sales throughout the coming year, but this is not possible, only we have probability forecasts, sides of bounding polygon fuzzy but not sharp – Dynamic programming

(5) Non-linear programming – (4) & (5) ~~as optimum~~ bounded optimisation problems

(6) Necessity for algebraic method – Simplex method illustrated by $y-2x \leqq 2$, $x-2y \leqq 2$, $x+y \leqq 5$, to minimise $y-x$ – both graphical and algebraical

(7) Degeneracy & other complications – Five cases

(8) Numerous methods (1) Simplex method for ∞-programmes, (2) Fictitious play methods (3) Relaxation methods (4) Parametric L.P (5) Stochastic L.P (6) L.P of errors (7) ~~Calculus~~ Variational L.P. (8) Double description method (9) Geometric projection methods (10) Duality principle (11) Uncertainty principle. (12) Computer techniques

(9) Applications of L.P or applied L.P – Ref. to Bibliography in (2)

(a) Mathematics – Matrix theory, theory of numbers, set theory, function spaces, combinatorial problems

(b) Transportation problem – to be put explicitly

(c) Production planning – "

~~Read out other items~~

(d) Computers – analogue & digital.

Notes of linear programming (10/11/54)

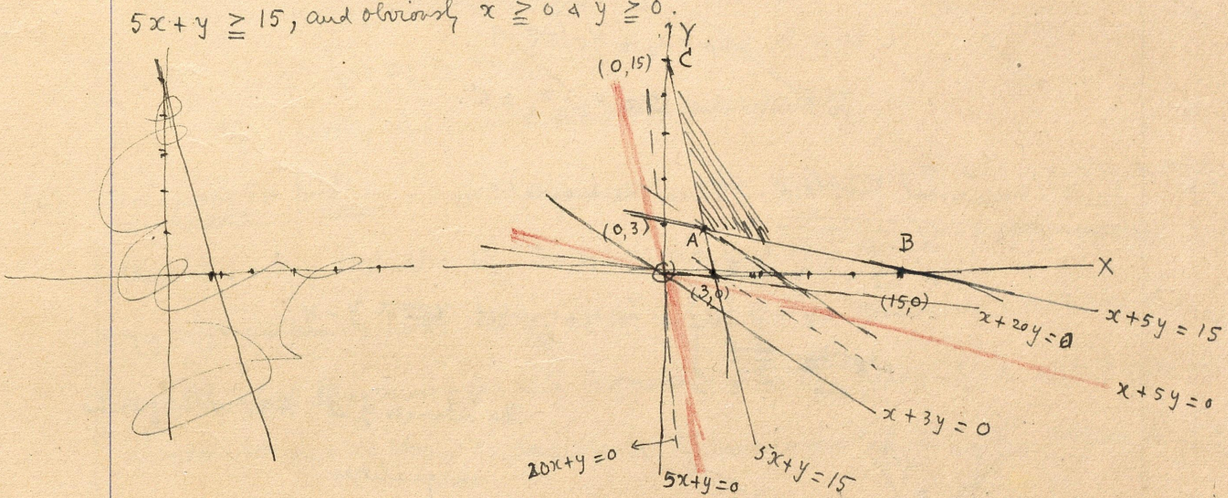
(1) P. M. Morse in 2nd AMS-OOR Symp. in Appl. Math – pp. 6-8.

(a) Problems of an oil company are linear programming problems – optimisation of crude-oil-cracking

(b) Linear programming problem visualised in geometrical terms.

(c) Production planning problem not a straightforward linear programming problem since exact prediction of sales throughout coming year is not possible – also case of function to be optimised not being linear – ~~Both these cases~~ Such general problems are called those of bounded optimisation – When these can be solved easily, problems of production planning, sales effort & of logistics will become easy.

(2) Illustration by the vitamins problem – A man has to take at least 15 units each of two vitamins v_1 & v_2 which cannot however be obtained pure, but only in two medicines. The first medicine contains 1 unit per oz of v_1 and 5 units per oz. of v_2. The second medicine contains 5 units per oz. of v_1 and one unit per oz. of v_2. The first medicine costs Re 1/- per oz., & the second Rs. 3/- per oz. How much of each medicine should he buy to spend the minimum amount? Let him buy x and y ozs. of the first & second medicines respectively. Problem is to minimise $x + 3y$ given $x + 5y \geq 15$, and $5x + y \geq 15$, and obviously $x \geq 0$ & $y \geq 0$.



Req'd x & y should fall in the shaded region. $x + 3y = \lambda$ for different values of λ gives a series of $\parallel^{le}$ lines & we have to consider all such $\parallel^{le}$ lines above $x + 3y = 0$ (right of) since λ is +ve & the least λ is given by that $\parallel^{le}$ passing thro' A. Now A is given by solving $x + 5y = 15$ and $5x + y = 15$ ie A is $(5/2, 5/2)$ and ~~xx~~ hence req'd x & y is given by $x = y = 5/2$ ie $2\frac{1}{2}$ ozs of each medicine & λ_{min} = Rs 10/-.

Same problem, except for the cost of medicines say Re 1/- & Rs 20/- per oz ~~x~~ resply ie to minimise $x + 20y = \lambda$. Here the set of $\parallel^{le}$ lines should pass through B ie $x = 15$, $y = 0$ & $\lambda_{min} = 15$

Same problem, except for cost of medicines say Rs 20/- & Re 1/- resply ie to minimise $20x + y = \lambda$. Here the set of $\parallel^{le}$ lines passes thro' C ie $x = 0$, $y = 15$ & $\lambda_{min} = 15$.

~~(3) Another numerical problem~~

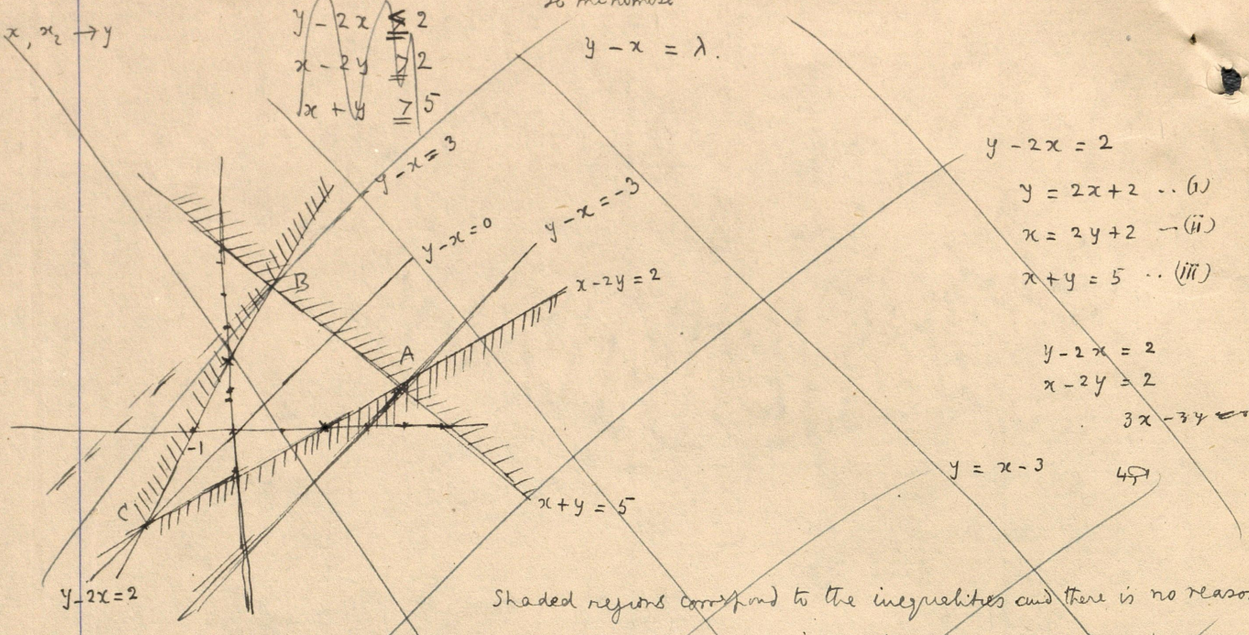
~~$x_2 - 2x_1 \geq 2$~~
~~$x_1 - 2x_2 \geq 2$~~
~~$x_1 + x_2 \geq 5$~~

~~To minimise $x_2 - x_1$~~

$x_1 \to x$, $x_2 \to y$

$y - 2x \leq 2$
$x - 2y \leq 2$
$x + y \geq 5$

To minimise $y - x = \lambda$.



$y - 2x = 2$

$y = 2x + 2$ .. (i)

$x = 2y + 2$ — (ii)

$x + y = 5$ .. (iii)

$y - 2x = 2$
$x - 2y = 2$
$3x - 3y$

$y = x - 3$ 45°

Shaded regions correspond to the inequalities and there is no reason enclosing all the shaded regions. So the minimum can be obtained only by taking $y - x = \lambda$ as passing through intersection of 2 shaded regions & lying in the third shaded region. In this case $y - x = \lambda$ must pass thro' A ie (4,1) giving $x_1 = 4$, $x_2 = 1$ and $\lambda_{min} = -3$.

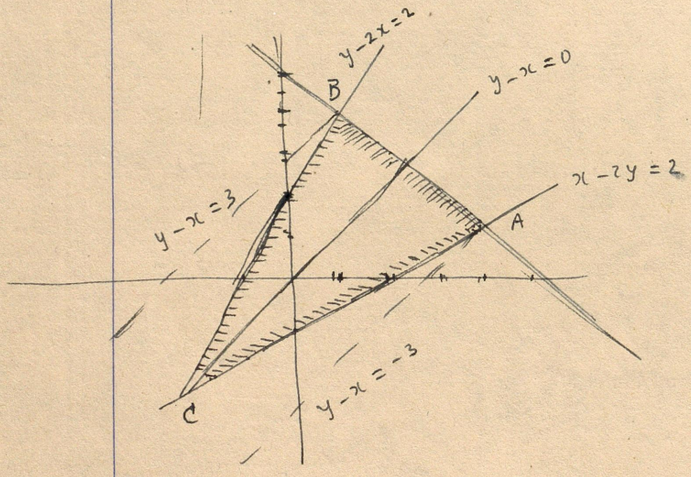
Algebraic simplex method for the same problem - using x_1, x_2 instead of x, y, the constraints are

$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= 2 \\ x_1 - 2x_2 + x_4 &= 2 \\ x_1 + x_2 + x_5 &= 5 \end{aligned}\right\} \quad x_3, x_4, x_5 \geq 0.$$

To minimise $x_2 - x_1 = C$.

(3) Another numerical problem: Given the inequalities

$$\left.\begin{aligned} y - 2x &\leq 2 \\ x - 2y &\leq 2 \\ x + y &\leq 5 \end{aligned}\right\}$$ To minimise $y - x$



The interior of $\triangle$ ABC satisfies the inequalities and the set of all lines $y - x = \lambda$ gives λ_{min} passes thro' A & gives $\lambda = -3$.

A is (4,1). ie $x = 4$, $y = 1$.

Algebraic simplex method for the same problem: using x_1, x_2 in place of x, y, we have the constraints

$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= 2 \\ x_1 - 2x_2 + x_4 &= 2 \\ x_1 + x_2 + x_5 &= 5 \end{aligned}\right\} \quad x_3, x_4, x_5 \geq 0$$

To minimise $x_2 - x_1$ with $x_1, x_2 \geq 0$.

(i) A b.f.s is $x_1 = 0$, $x_2 = 0$, $x_3 = 2$, $x_4 = 2$, $x_5 = 5$ with x_3, x_4, x_5 b.v's & x_1, x_2 n.b.v's

Solving for former in terms of latter

$$\left.\begin{aligned} x_3 &= 2 + 2x_1 - x_2 \\ x_4 &= 2 - x_1 + 2x_2 \\ x_5 &= 5 - x_1 - x_2 \\ C &= x_2 - x_1 \end{aligned}\right\}$$

(ii) To decrease C (ie minimise) x_1 should increase & can increase upto $x_1 = 2$ and will keeps x_3, x_5 positive, so that we can take $x_2 = 0$, $x_4 = 0$ as n.b.v's and x_1, x_3, x_5 as b.v's. Solving for latter in terms of former

$$\left.\begin{aligned} x_1 &= 2 + 2x_2 - x_4 \\ x_3 &= 6 + 3x_2 - 2x_4 \\ x_5 &= 3 - 3x_2 + x_4 \\ C &= -2 - x_2 + x_4 \end{aligned}\right\}$$

(iii) Increase of x_2 decreases C, but can increase upto $x_2 = 1$ so as to make $x_5 = 0$ with $x_4 = 0$ & increase of x_2 will not make x_1, x_3 negative. Hence we can take $x_4 = 0$, $x_5 = 0$ as n.b.v's and get

$$\begin{aligned} x_1 &= 4 - \frac{x_4}{3} - \frac{2x_5}{3} \\ x_2 &= 1 + \frac{x_4}{3} - \frac{x_5}{3} \\ x_3 &= 9 - x_4 - x_5. \\ C &= -3 + \frac{2x_4}{3} + \frac{x_5}{3} \end{aligned}$$

(iv) No increase of x_4 or x_5 would further decrease C. Hence the minimum has been reached with $x_4 = 0$, $x_5 = 0$, $x_1 = 4$, $x_2 = 1$, $x_3 = 9$ and $C = -3$

This exactly corresponds to the geometrical method.

(4) Transportation problem – Suppose there are a_s ships in the ports P_s and we wish to move b_t of them to ports of destination Q_t. The cost of moving a ship from one port to the other is different for different pairs of ports. Let y_{st} be the number of ships to be moved eventually from P_s to Q_t and let cost of thus moving one ship is known and is C_{st}. The problem is to make the total removal process as cheap as possible ie to minimise $\sum_s \sum_t C_{st} y_{st}$ subject to $\sum_s y_{st} = b_t$ ~~for~~ for all t, and $\sum_t y_{st} = a_s$ for all s and all $y_{st} \geq 0$ – further complications: (i) cost of transportation between ports of the same set, (ii) also to and from ports which are neither P nor Q ie considering possibilities of transhipment or calling at intermediate ports (iii) cost of storage and docking to be taken into consideration

(5) Production planning – Suppose a firm can use various production processes P_j and the materials required are R_i. When intensity of process P_j is unity, let amount of R_i needed be a_{ij}, so that when P_j is used at intensity level y_j the requirement for $R_i = \sum_j a_{ij} y_j$. If the profit of the product of process P_j at unit level is p_j, the problem is to maximise $\sum_j p_j y_j$ subject to the fact that not more than r_i of R_i is available ie $\sum_j a_{ij} y_i \leq r_i$ – Case of prediction of sales not being exact.

(6) O.R. techniques – 18 mentioned in previous lecture – only to be briefly illustrates (1) Linear programming (2) Theory of games (3) Waiting line theory (4) Monte Carlo techniques.

(7) Fields of military O.R – 45 items mentioned in previous lecture – Each field uses a singly or a ~~combination~~

Combination of several of the 18 techniques.

(8) Fields of military O.R. where linear programming is applicable :-

(a) Military logistics – Provision, procurement, storage, transportation, distribution, maintenance and evacuation of personnel, supplies and equipment, with induction, classification, assignment, welfare and separation of personnel, and with facilities required for the support of military establishment including construction and operation thereof. It comprises both planning & implementation

(b) Automatic data processing system (ADPS) – Problems of logistics automatically handled by digital computers – L.P. ~~programme~~ problems to be suitably fed to the machine – Eg. use in processing orbits of satellites.

(8) Preliminary remarks to (8)

(i) L.P used wherever questions of maximum & minimum are involved – 'maximits' & 'minimits'

(ii) most efficiently manage military service in peace and most effective combat force in war.

(iii) Plan searching for attainment of the maximum benefits in the diplomatic, strategic & tactical arts – a sword in the right hand, a gift in the left hand & soft words in the mouth.

(iv) Evaluation of equipment and weapons.

(9) Degeneracy and other complications in L.P

(1) No b.f.s. to start off with – none may exist.

(2) Rules for carrying on fail, although final solution is not reached.

(3) Choice of a n.b.v may not be unique.

(4) Some of the b.v's may be zero. (5) no finite optimal f.s. exists

~~Example (9):~~ Examples of degeneracy.

(a)
$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= -2 \\ x_1 - 2x_2 + x_4 &= 2 \\ x_1 + x_2 + x_5 &= 5 \end{aligned}\right\} \text{ Minimise } x_2 - x_1$$

This comes under (1).

(b)
$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= 2 \\ -x_1 + 2x_2 - x_4 &= 8 \\ x_1 + x_2 + x_3 &= 5 \end{aligned}\right\} \text{ Minimise } x_2 - x_1$$

This also comes under (1) since there is no f.s. anywhere in the plane.

(c)
$$\left.\begin{aligned} -2x_1 + x_2 + x_3 &= 2 \\ x_1 - 2x_2 + x_4 &= 2 \\ x_1 + x_2 - x_5 &= 5 \end{aligned}\right\} \text{ Minimise } x_2 - x_1$$

This comes under (5)

(d)
$$\left.\begin{aligned} 2x_1 - x_2 + x_3 &= 4 \\ x_1 - 2x_2 + x_4 &= 2 \\ x_1 + x_2 + x_5 &= 5 \end{aligned}\right\} \text{ Minimise } x_2 - x_1$$

This comes under (4)

(e) $-2x_1 + x_2 + x_3 = 2$
$x_1 - x_2 + x_4 = 2$
$x_1 + x_2 + x_5 = 5$
} Minimise $x_2 - x_1$

This comes under (2)

(10) Principle of duality — Dual sets given by

$\sum_{i=1}^{n} a_{ij} x_i \geq b_j$ | $\sum_{j=1}^{m} a_{ij} y_{n+j} \leq c_i$

Minimise $C = \Sigma c_i x_i$ | Maximise $B = \sum_j b_j y_{n+j}$

Fundamental duality theorem is :- Given two dual sets, and provided that a finite minimum (maximum) value of C (B) exists, a maximum (minimum) value of B (C) exists as well, and the two optima are equal.

(11) Alternative ~~geom~~ graphical representation of L.P.

(12) Other notes — L.P & Theory of games

(A) J.O.R.S.A for 1957 -

(i) Bibliography on linear programming - p. 45 - 266 entries - done by F. Virginia Rohde as summer work at the United States Naval Proving Ground

(a) Refers to Charles, Cooper & Henderson as a standard & the only one real "text-book" - main source is Koopman's Activity analysis of production & allocation — Proceedings of the 1st & 2nd Symposia on linear programming as valuable sources (1951 & 1955).

(b) Refers to following topics under L.P :-

(1) Basic mathematics (2) Simplex method (3) Do extended to ∞ - programmes (4) Degeneracy cases (5) Coding techniques (6) Fictitious play methods (7) Relaxation methods ~~or SEAC~~ (8) Use of electronic analog computer techniques (SEAC) (9) Linear inequalities (10) Parametric linear programming (11) Dynamic programming - linear & non-linear (12) Stochastic linear programming (13) Programming of errors inherent in observations & modified simplex method for control of round-off error (14) Transportation problems adaptable to high speed digital computers (15) Combinatorial problems (16) Problems involving uncertainty (16) Travelling salesman problem (17) Assignment problem (18) Bid evaluations problem (19) Problem of distribution of effort (20) Assembly-line balancing program (21) Production scheduling (22) Diet problem (23) Maximal flows in net works (24) Personnel problems of executive (25) Theory of the firm (26) Production planning & smoothing (27) Transportation theory to scheduling a military tanker fleet (28) L.P. & structural design (29) Caterer problem (30) Capital budgeting (31) L.P of an air lift (32) Production distribution (33) Production patterns (34) Problems of calculus of variations (35) Weapon selection & target analysis (36) ~~Dist~~ Optimum distribution of effort (37) Non-linear programming (38) Economic analysis of industrial relationships (39) O.R. for Management (40) Petroleum refining industry

(41) Efficiency in operating a system of dams (42) Accuracy of economic observations (43) Computing machines (44) Problems of matrices (45) Manufacture of newsprint paper: trim problem (46) Double description method (47) Number theory - partitions (48) Price equilibrium & L.P. (49) Editing problems, (50) Projection methods (51) Convex polyhedra (52) Set theory.

Further notes on O.R. for Delhi Conf. on 18-20th March, 1959

28/2/59.

(1) Military Review, August 1958 — p. 75 — Article on Operations Research by Major J. Barbier in "Revue de Défense Nationale" (France) 7 Oct, 1957, translated and digested.

(a) Introduction = Applying scientific research not only to the creation of the weapons of war but also to the operations themselves — Intervention of Archimedes when Syracuse was besieged using mirrors to set fire to enemy ships — Trojan horse an example of O.R — Clausewitz analysing battles of Napoleon and coming to the conclusion that even the best general imaginable, even if he were as great as Napoleon, could not win a battle if his forces were inferior to half those of his adversary ie the differ. between a mediocre person general & a genius could be expressed numerically by the factor two — Work of British Air Ministry Section (1937-39) on radar installations brought the term O.R. into use.

(b) Fundamental factors = Efforts for creating new weapons and efforts towards realising the best possible use of these weapons — Necessity to improvise decisions which cannot but lag behind the development of operations; taking speeds into account & considering the interception of bombers or enemy missiles by interceptor aircraft or missiles, it is indispensable, to a greater degree than ever, to think before acting — "Speed" is important makes O.R. important, but "complexity" plays no less an important role.

(c) Complexity = The majority of military problems is so complex that a correct decision is not immediately obvious & several decisions may appear equally good — Extrapolation from experience may lead to errors — Growing complexity is an important factor in establishing O.R & its rapid growth — But O.R. is not merely an instrument to make scientific decisions.

(d) Definitions = Charles Kittel of Bell Tel. Labs: " O.R. is a scientific method for providing responsible leaders with mathematically established bases for their decisions" — Cf. with defn of Ellis. H. Johnson, director of O.R. office at John Hopkins Univ viz: " O.R. is the prediction & the comparison of the value, the efficacy, and the cost of a series of possible specific actions putting into play systems composed of men and machines with a view to the attainment of given objectives. To this end it makes use of an action model which has been determined by logical analysis and, when possible, by mathematical methods." This defn indicates that O.R. can exist without the necessity of an immediate decision — Defn of Ventura viz: It aims, first of all, at the facts relative to a situation with the views of afterward deducing from them rational principles of action which take the place of empirical recipes" — Studies of P. Massé confirm the idea that O.R. is a wider field than that of taking decisions — First studies confined themselves to solving practical problems & the methods adopted veiled the true nature of O.R, like trees hiding the forest.

(e) Errors in approach = methods of O.R. viz expectation theory, Monte Carlo, games & linear programming

are clearly defined, but there may be other methods & we cannot fix limits of O.R. – Appears to be an experimental science whose aim is to ensure the active direction of future operations & techniques and methods are only its instruments – B.O. Koopman's four "Fallacies in O.R." viz (i) linearitis is ~~affirmation~~ over-evaluation of linear programming; (ii) maximitis is the assurance that the most probable will happen; since the mean is often confounded with the most probable value, this leads to the belief in the virtual certainty of the mean; (iii) mechanitis is the state of mind of one so impressed by modern calculating machines that he believes that any problem will find a solution when he has at command a costly machine. Of this error, Dr. H. Thomas (of IBM) states "Proof of the correctness of certain methods of machine calculation requires such a high degree of knowledge that a direct mathematical attack would be easier". Monte Carloism & war gameism are other examples of this error; (iv) Authoritis is the belief that the responsible chief who must make use of the result can find the missing links of the solution ie that he can couple the theoretical problem with the practical realities.

(f) Variables = One of the basic difficulties of O.R. is the choice of the variables to be studied or the choice of proper criteria even when the problem is well formulated – the success or failure of a study & its termination in a useful solution depends on this choice — Koopman distinguishes four phases in every complete development of O.R. viz. (i) choice of "figures of merit" (basic data, typical parameters), (ii) choice of the "indices of effort" (variables indicative of efficacy), (iii) establishment of a more or less precise model, (iv) search for the optimum. According to him & to Charles Hitch of the Rand Corporation, the first two phases are the most difficult since there is no theoretical bases for guidance in the choices

(g) Criteria = It is necessary to seek the means for choosing criteria and to learn what characteristics they must possess to be of value, for according to Hitch "we do not know how to choose the best criteria but we know enough, in many cases, to distinguish between good and bad ones" – Hitch further says "O.R. will be improved enormously if we give more attention to criteria than to models or techniques of calculation. In the end we shall choose them rationally" – the same idea is expressed by Mr Gwilliam as follows: "Everything that has succeeded must be gathered together (also failures even more instructive) to be analysed patiently & put together again in an ad hoc processing out of which a scientific doctrine as intelligible as possible will emerge – in order to achieve this scientific doctrine which must be the distant objective two ~~errors most~~ tendencies must be guarded against viz. (i) Limiting O.R. to being only an instrument of decision (ii) contrary tendency to see in O.R. nothing but intellectual speculation whose practised results are entirely questionable – in a scientific ensemble there is both theory & its practical applies for eg. pure maths opening to the engineer numerous paths leading to practical realisations like strength of materials, aero- & hydro-dynamics.

(h) Command = Command interested directly in practical applies & their results – O.R. as the study of the "dynamism" of situations, procedures, organisations or of methods must have at its disposal a broad & varied scientific apparatus — O.R. provides the chief aid in making

decisions only & makes no pretence of taking the place of this judgment. In all cases decision rests with the commander. Close collaboration is therefore indispensable — In the vast field of military activities O.R. must give effective assistance to command in (i) determining efficacy of weapon & equipment, (ii) analysing results of operations, (iii) studying reciprocal influence of weapons & ~~also~~ tactics & (iv) establishing comparative value of diff. organisations or methods — O.R. cannot always determine easily & rapidly a solution to every problem — first difficulty, well illustrated by the words of Foch "De quoi s'agit-il ?", is to formulate the problem well & to find criteria — Solving this next is to ~~not~~ resort to the most powerful techniques of mathematics (statistical tables & theory of probabilities) imparting to O.R. much of its power & fertility — Lastly one should translate the results of a scientific task into elements of decision to be utilised by the command — The last war illustrates great value of the applⁿ of scientific method to military matter.

(i) Conditions of success = Two conds to obtain practical results viz (i) distant objective of building up a well-balanced doctrinal structure by rational choice of criteria. All study must furnish the particular practical solⁿ & also build a theoretical doctrine, (ii) Trustful exchange of ideas between researchers and ~~executives~~ executors; the giving of all data & information ~~by the latter~~ and showing of patience in case of failure by the latter, and labouring without fear or favour by the former — Such fertile collaboration enables solving problems in their ensemble — O.R. is sufficiently rich in promises ~~to~~ for no doubts to exist concerning its future.

———o———

(2) Military Review, ~~May, 1955~~ August, 1955, p. 13 — Probability & atomic support by Lt. Col. Malcolm M. Jameson, Infantry Instructor, Command & General Staff College.

An understanding of the role of probability in atomic support planning is essential to the proper utilisation of these powerful weapons. The impact of probability considerations on tactical planning & fire support co-ordination should be appreciated fully by the commander & his staff.

Further notes on O.R

For conference at Delhi on 18th – 20th March 59

24/2/59 – Vajda's book on Theory of Games & Linear Programming.

Chap I – Outline of the theory of games – Problems of theory of games can be shown to ~~be~~ form a special case of those of linear programming – computational methods – geometric representation – Formal treatment also – two person game – zero-sum game – game & moves & play – game of matching pennies – Blotto game – strategy; for games with only one move, strategy is identical with a move – game of bluffing; chance move – normalising is reduction of a game's structure to strategies – games in extensive form & pattern of information – non-zero sum game with two players can be thought of zero-sum game with three players – n-person games may be zero-sum or non-zero sum – This book deals with only 2-person games – cooperative & non-cooperative n-person games & in the former players form coalitions (Vide Neumann-Morgenstern) & in the latter each player looks exclusively after himself; zero-sum two-person games are non-cooperative – Example with pay-off table (or matrix)

B chooses t =

A chooses s =	-1	0	1
-1	2	-1	-2
0	1	0	1
1	-2	-1	2.

Elements in cell are amounts that A wins [A is called first or maximising player] & B is called second or minimising player]. A tries to gain as large an amount as possible & B tries to lose as small an amount as possible. In this table there is an entry in the pay-off matrix which is the smallest in its row & at the same time the largest in its column viz 0 & when such an element exists it is called the saddle point (N & M use 'specially strictly determined game') ~~Also if minimum of column maximum = maximum of column minimum, then saddle point exists & equals this element~~ – Not every zero-sum two-person game has a saddle point eg. Matching pennies with matrix $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

First Lecture.

~~Operational Research as it is called in U.K or Operations Research, with a military bias, as it called in U.S. or O.R. merely O.R is a young and vigorous subject about 20 years old. The most difficult thing about the subject is to define it properly~~

(1) (1) Def^{n} & nature of O.R. (2) origin of O.R (3) O.R. techniques (4) Fields in which O.R. is used (5) Military O.R (6) Status of O.R.

First Lecture (4/11/59).

OR (U.K & U.S) a young subject about 20 years old, very vigorous & growing – been ~~teaching~~ lecturing for last 8 years & noticing changes in approach to problems, specially in this nuclear age – OR of great importance to military officers since OR was born as a military necessity during last war & Chief sc. feature of last war – Blackett father of O.R – Two examples (i) First O.R. problem about setting time-fuze of a bomb solved by Blackett (ii) Diversion of heavy bombers from bombing of Germany to U-boat hunting & about optimum size of convoys. This won the battle of the Atlantic – Blackett's Circus of physiologists, mathm, math. physicists, astrophysicist, army officer & an ex-surveyor – Trivial examples: (i) never fire a long burst from a machine gun, disproved by O.R. worker (ii) Anti-tank guns more effectively managed by 3 men instead of 6, (iii) 25-pounder gun problem – Effect of irregularities in ground height – greater activity when U.S & U.K. cooperation started – Military O.R. in U.S. started with ratio n : a : air = 1 : 5 : 10; exchange of reports & pooling; frequent meetings; Tizard, Cockcroft, Fowler, Blackett, ~~Conant~~, Hill, Darwin & Conant, Bush, Morse, Compton; Formation of ORG of NRDC, of ASWORG, of OFS; Morse's Circus of 44 men (6 maths, 14 actuaries, 18 physicists, 3 chemists, 2 biologists & an architect) & men diff. labs given specific work; Compton as Chief of OFS, collaboration of OFS with military; work of OFS, ~~initial dislike of OFS by military, later appreciation, Mc Arthur's welcome to Compton~~; 464 men at end of war, engineers, maths, marine biologists & ornithologists with great versatility – differences in approach of ~~air~~ force, navy & army to O.R. during war, romanticism idea of army, Hitler losing the war; when a scientist enters war he does so as a civilian & war in future will be conducted more & more in a civilian spirit – Relations between military & OR in wartime, initial dislike, then a crescendo of praise, Mc Arthur & Compton, "These boys won't even touch a soldering iron until – – Pacific war", Compton to be treated as Col. if captured, their lives well guarded eg of Moseley, H.P. Robertson's courses on OR, 1600 reports, 15000 items, called "combat team without guns", tribute by many generals – Selbert in Bangalore to repaint planes & make them invisible due to specular reflection, imaginary meeting with him & discussion about adding errors (root mean square)

Blackett a former naval officer – ATS girls

Given history of OR, but what is O.R? Definition is the most difficult one – 7 definitions (1) Dynamics (2) applns of sc. method (3) optimisation of performance (4) appln of common sense (5) Case history defn (6) Research at operation level (7) Providing sc. methods to executive Depts, none however satisfactory — Nature of O.R, an ind. Science using its own methods; academic research & OR; an applied Science not branch of maths; exptl science; experience of research; O.R. mentality in th. phys, biologists & military men & less in mathems, engineers & economists; emphasis on collaboration, also with military officers; characteristic attributes of sound O.R, four; O.R. worker not to make decision, but leave it to the executives, useful when these have authority, otherwise O.R. is philosophy; understanding between ~~workers~~ & OR workers & ~~executives~~ administrators in all fields, military & otherwise; analogy of doctors discussing patients; ~~Hen~~ Ref. from Henry VII; Recent trend towards non-military O.R – Poem about OR

O.R. techniques – (1) Experimental design (2) Queueing theory or waiting line problems – Eg. of line up of soldiers

soldiers wasting their kit, self-aggravating problem, converse example (3) Linear programming - used very frequently (4) ~~Information theory (5)~~ non-linear programming (5) information theory (6) Decision theory (7) Probability & statistical techniques (8) Computational techniques - eg. of inverting 20 x 20 matrix taking 108 years (9) Stochastic processes (10) Optimum distribution of effort (11) Inventory problems - eg. storage in hydroelectric system & use of Calculus of variations & digital computers (12) measure & counter measure - propaganda of several types (13) Cybernetics - not to be easily dismissed of automation (14) Monte Carlo techniques - ADPS - Handling uncontrolled data - random walk of drunkard, inability of mathes to solve problems, modern trend, (15) Theory of games - great use in military OR (16) Symbolic logic - Lewis Carroll's example modified - only dealing with (3), (14) & (15).

Fields other than military where O.R. is used - (1) O.R. & accounting, inspection, budgetting, salary scales, pension schemes, (2) Road research - Laboratories - Highway traffic, Accidents, time lost, risk taken in crossing a road & need of pts of controller crossing, shockways on high way, theory of traffic flow using kinetic theory of gases (3) Hydroelectric schemes - criteria for selecting water power projects (4) Heavy industries - Factory production & sales - optimisation of crude-oil cracking in petroleum companies - production scheduling & planning - spare machinery - ~~travelling salesman problem~~ - machine repair - travelling salesman (5) Railways & naval companies - transportation problems - pattern of shipments to make cost of transportation minimum - Mechanising ticket printing issue - Development plans (6) Municipal administration - City housing & municipal facilities - work against disease & deterioration - house heating, coal, oil or electric heating in next war ? (7) Psychiatric research - criminology & prediction - Borstal research project - why more criminals come back after release (8) Educational planning - 10 years forecast of school populations - of setting up Universities & Tech. Institutes.

O.R. Dwns in U.S. Rys

Hospital planning

Military O.R - Examples already given from war effort - Four classes of work in military O.R.

(1) Logistics - not logic Quartermaster - Automation & psychology - transportation aspects, storage in motion - Military traffic, movement & maintenance - Military Govt. logistics in U.S. its contents, eg. of Napoleon losing Waterloo - Logistics in future atomic war, keep it small & keep it moving - Logic in logistics (2) older military OR - Principles of war, Napoleon & Clausewitz (115 → 7) - Nelson's ten principles - Theorem about failure if $a < \frac{1}{2} b$ by Clausewitz an example of O.R (3) enemy greatest general - ratio of military genius to ordinary commander = 2 : 1 & not 100 : 1 or 1000 : 1 (3) Enumeration of items undertaken during last war (4) Some recent articles from JORSA. (5) Enumeration from Bull. Amer. Math. Soc, Vol 7 of 1959 B.29 - B.33.

Status of O.R - (1) Doctorates (2) in Universities (3) Societies (4) Journals (5) teams in 100 companies (6) Salaries (7) Report of Committee on O.R. (8) Library services (9) Courses of lectures (10) Symposia & Conferences.

First Lecture.

Synopsis

Difficulty of covering vast scope in three lectures – Origin of OR – young subject – achievements in War II – Differentiation from regular scientific war effort – Tributes to work of ORG – Started by Blackett in U.K (1940) first with Radar work – ~~need for OR & importance~~ classes of work under OR – need for OR & its importance in war work – in Air, Navy & Army – Blackett's Circus – mixed teams – Morse's Circus – other examples – suitability for OR work – growth of OR in U.S. war work – OSRD, NRDC, AirORG, ORS, ASWORG, AMP, APP, OFS – Description of some work done in all branches – Chem, Psychiatry etc – organisation of ORG's & training of men – OR in the field – Relations with military – ~~Facts~~ Non-military OR – facts & tips.

Simple examples* – tubs – AA gun manning – Depth charge setting – convoys – MAD – Barrier patrol – Steinhardt's work – acoustic torpedo – Rinehart – other simple examples.

Definition of OR – Measures of effectiveness – Strategy: linear programming and queuing problems. Strategical kinematics – Tactics: Search theory & Game theory

——— o ———.

* (1) 2+2 tubs ⟶ 3+1 tub – Use the joke of a leaking tub – its characteristics – queueing

Same type without added equipment

(2) Time & motion study in the armed services – gundrills – AA guns more effectively manned by 3 people – report passed on to the gunners .. Burn the report... also holiday (2) the horses

(3) Big guns & small guns in Normandy coast fighting

(4) A.T.S. girls.

more serious ones: (1) Depth charges – Simple description from Crowther – involves detailed probability study – numerical analysis of the chance of success as a fn of the degree of submergence. (Simple probability).

(2) Size of Convoys – Simple description by Crowther – M & K, p. 46-47. (Measures of effectiveness)

(3) Putting AA-guns on merchant vessels – M & K, p. 52

(4) Barrier patrol for submarines – Game theory – M & K, p. 105

(5) Duel problem – Game theory – M. p. 9.

(6) Linear transportation problem – J.O.R.S.A. May 55, p. 211.

(1) Intr. (2) origin (3) ~~no~~ O.R. not weapon technique & tributes (4) Hist of OR - Blackett & Difficulties

(5) OR in A, N & A (6) Negro OR (7) UK, US. coppr (8) U.S. effort (9) mines nature (10) R. Society

(11) work of OR (12) OFS (13) OFS (14) Conclus (15) ~~Stud~~ Growth of OR.

1st lecture (15/6/56)

When Squadron Leader Bambtrad asked me some time back to deliver a course of three Part III lectures on operational research, I readily agreed to do so without realising how difficult the task would be. In fact, to tell you in a general way what operational research is, one lecture is sufficient, but to tell you what all OR does three lectures are thoroughly inadequate. One would require a ~~complete~~ course of nearly 20 lectures, but that would take it completely out of the Part III scheme. I shall therefore content myself by telling you a little of what OR is, and something of what it does.

Operational research is a very young subject - called operations research in U.S. and operational research elsewhere, and fortunately both can be abbreviated to the same two letters OR, and I shall adopt this abbreviation throughout - having had its origin in World War II. However, it is not as if OR suddenly descended on the scientific world as a new technique; in fact, activities similar to modern OR must have been in existence in the past also. Reading military history, one is constantly struck by examples of military operations which bear a striking resemblance to recent OR. The same is also true of many industrial operations. But what appears to be new about OR is its importance, and the necessity for employing it in the military field in greater and greater measure comparable to the rapid rate of change of military equipment, of weapons and of situations.

This should not be taken to mean that OR is synonymous with the understanding and use of the modern extra-ordinary ~~use~~ development and evolution of weapons & strategy starting from radar, amphibious warfare, aerial combat, the proximity fuze and the atom bomb. In fact, many OR workers during the War found themselves being treated as scientific specialists in these armaments, although what was wanted by the Command was a group of field engineers or a panel of scientific consultants. Officers who, for example, had been channelling all their operational information on rockets to a OR worker naturally turned to the same man when something went wrong with the mechanism of the rocket like a fuze. But ~~if~~ it often

happened that the OR worker was either an actuary, or a biologist or a mathematics professor before he was recruited to war work. This confusion led in the early stages of the war to some misunderstanding ~~of the~~ between the administrator and the OR worker. Later on, however, the true functions of the OR worker were realised and immensely appreciated. It came to be commonly understood that the work done by the OR worker in the military field fell under the following four categories: (i) the determination of the operational effectiveness of weapons and equipments, (ii) the analysis of the results of operations to determine the effectiveness of tactics, the influence of weapons on tactics and tactics on weapons, (iii) the prediction of the results of future operations, (iv) the analysis of the efficiency of organisations or of methods. When once this was realised, and the right problems assigned to OR workers, the success achieved was amazing and their contribution to the war effort was outstanding. Thus Admiral Turner of the U.S. Navy wrote to Prof. Vannevar Bush who was the leader in the organisation of OR work "That your group ~~should~~ would contribute brilliant ideas and achievements to the war effort was expected, but that you would be so versatile, and that the OR scientists and the Navy should find themselves so adaptable to each others way of doing business was unexpected by many." Again Gen. Richardson in charge of the Pacific operations sent to Mr. Marshall who was in charge of what was called the "brain trust in Hawaii the following communication." Many of the individuals who have been connected with the operations research section are no longer in this area, and it will be impossible for me ~~to~~ express to each one personally my appreciation of his efforts. I should therefore like very much to have you consider this letter an expression of my heartfelt thanks to all who have participated in the splendid contribution made by the OR section to the success which has crowned our operations in the Pacific." The great help

rendered by these OR workers were was so much appreciated that in the later days of the war, it became a standing joke for several Commanding officers to demand for their services and that they be sent "the day before yesterday." Their methods of work also came to be well understood as is shown by the remark an eminent officer regarding a O.R. team "Those boys won't even use a soldering iron until they know the whole of the policy of the Pacific war."

Coming back to the history of OR with which I started, it is universally recognised that the U.K. is the home of OR, and its father [redacted] is Prof. Blackett who started in the early days of the war in all the three services. The actual name was coined A.P. Rowe, however. It started in the Air Force first centring centering round the problems of radar, then in the Navy centering round anti-submarine problem, and finally in the Army, first limited to anti-aircraft problem, and later again centred round radar. In the initial stages Blackett had great difficulty in persuading the authorities about the use of OR. There was in those days a considerable controversy about such things as the diversion of heavy bombers from the bombing of Germany to U-boat hunting, and the optimum size of convoys. It was only when the anti-submarine campaign culminated in the victories of 1943 that full credit to was given to the work of OR workers. There is always a psychological difficulty in mutual understanding between military officers and civilian scientists specially of the OR brand, and since this was not foreseen, a lot of confusion and inefficiency resulted during the last war. The executive thought there was really no worthwhile problem for the OR worker to solve, and that the use of unnecessary mathematical & statistical technique led to results which could after all be obtained by mere common sense often led to impatience and suspicion on the part of the commanding officer. Also a fear lurked in the minds of some officers that if the findings of the OR team were successful and varied from those obtained by the officers themselves, their

their reputations would be at stake. All this fear and suspicion vanished later when it was found that the OR worker was a purely scientific worker who only studied the operations, and made recommendations for the executive to take the final decision by combining the OR findings with other qualitative aspects. As Prof. Blackett has put it succintly " Many war operations involve considerations with which scientists are specially trained to compute, and in which serving officers are not generally in general trained. This is specially the case with all those aspects of operation into which probability considerations and theory of errors enter . . . .; the scientist can encourage numerical thinking on operational matters, and so can help avoid running the war by gusts of emotion . . . » to which can be added the so-called reliance on 'common sense' which often went wrong, and on the method of 'trial and error' which often resulted in 'trial and catastrophe'.

The advantages of associating OR workers with war effort was first recognised above, as mentioned above, by the Air Force. Air warfare is largely susceptible to calculation and human factors involved are merely courage and skill of individuals. Naval warfare is also susceptible to calculation to a lesser degree, while land warfare involves additional human factors and hence judgment and capacity to command, appearing to be master of situation, and at least giving the impression of his being able to control things, are specially important here. The air commander has no such difficulties. He can approach the problem with more objective calculated detachment and a less emotional attitude, and, if as mentioned by Group Captain Howell the other day, we succeed in complete automation in air warfare, it will be conducted in a purely civilian spirit manned only by OR workers. I hope that the course of lectures on OR that you are now attending will be helpful to you at least in that contingency when you will be metamorphosed from Air Officers to OR scientists.

In contrast to the great importance attached by Britain and America to operational research in the war, the Germans completely neglected it. This was to a great extent due to ~~Hitl~~ Hitler's romantic view of war and his contempt for a systematic scientific view. This is strikingly brought out by the fact that when a group of top-ranking German scientists including such great names as Heisenberg, Hahn, Schumann, Bothe and Geiger arranged a meeting to discuss the possible military applications of atomic energy and invited the Nazi military leaders to attend the meeting, the scientists got a polite brush-off when Gen. Keitel wished the nuclear scientists all success, but declined the invitation. Also the Nazis were of the opinion that operational research was a type of Jewish science, and this perfidious doctrine went so far that when a professor of the Kaiser Wilhelm Institute once ~~in the~~ used the word "relativity" in a lecture he was threatened with the loss of his job!

When the success of OR in ~~work~~ war work was well established by the brilliant work of Blackett and others in Britain, and when America joined the war, there ~~being~~ began a great cooperation between British and American scientists which was perhaps the one single factor that contributed greatly to the growth of OR. In the early days Britain gave more than what it received in this field. Consequent on the visit of A.V. Hill to Washington in 1940, U.S. scientists arrived in England with secret documents for exchange right in the middle of the German Blitz, and teams of American Engineers and scientists came to familiarise themselves with the operation of rockets, fuzes, radar etc, and, in particular America sent an Electronics Training group. A British Central Scientific Office headed by C.G. Darwin was established in Washington as a result of the report on scientific liaison between the two countries drawn up by Cockcroft & R.H. Fowler. Also the NDRC established a London Office due to Conant's efforts.

Then began in U.S. a search for suitable scientific man power for OR

and attempts to train proper men that have perhaps no parallel in the history of science. Each military command at the several centres, air, navy & army started building up its own OR groups. The Navy in particular had a very strong team styled the ASWORG. Besides there were other AirORG's and army ORS's. Since the existence of so many separate bodies would lead to confusion and duplication, the NRDC, with the concurrence of the OSRD the central body under which it worked, reorganised all the OR teams under 18 subdivisions. In particular the ASWORG was converted into a ~~more~~ ORG with branches consisting of a central AirORG, a PhitORG, another AAORG and a ~~Sea~~ SpecORG., and an ORC to deal with theoretical and analytical problems. Also the 18 divisions created by the NRDC included the AMP, and APP and an ASG directly under the OSRD. A rational roster of scientific men was made, and the long and difficult struggle for adequate scientific personnel, choice and allocation of this personnel, and devising overall plans was solved very satisfactorily. The cooperation of Universities was sought for and freely given. This the American-British Electronics Laboratory of Division 15 was placed in ~~charged a~~ charge of the Harvard University by an OSRD Contract, and this was a signal success. The Princeton University gave courses of lectures to OR men of six to eight weeks duration on probability, mathematics and statistics, mechanics, weapon analysis, ballistics, photographic interpretation and bomb damage analysis, in small groups of four to six men at a time. After this preliminary training at Princeton, these men were sent to the AMP to finish off their course before they were actually assigned to the several OR groups. By the summer of 43, the number of these OR men was in hundreds, but still there was great need and was met by transfer of men from several laboratories to the field of Universities like Columbia & others to the field.

A peculiarity of the several OR groups formed during the war was their mixed nature. When Blackett as head of the OR section at the

at the Coastal Command had a research group under him, it consisted of two physiologists, two mathematical physicists, one mathematician, an astro-physicist, an army Officer and an ex-surveyor, and was known in military circles as "Blackett's Circus". Similarly P. M. Morse who occupied in the US much the same position as Blackett in UK, and was the Navy's foremost expert and consultant to other services also had under him 44 men: 6 mathematicians, 14 actuaries and statisticians, 18 physicists, 3 Chemists, 2 biologists and one architect. In other groups there were marine biologists, ornithologists, Economists and Geologists. It was a common phenomenon to see biologists study the behaviour of submarines, Convoys and patrolling air-craft; Geologists became administrative officers, studied land mine countermeasures, and helped army officers to compile a manual on jungle warfare. We have also the example of a metallurgist from the Westinghouse Corporation who was once put the necessity of ruling a ~~town as~~ as a Major for three days a conquered town in Germany, and keeping the peace till the regular regiments arrived. The ages of these OR workers varied between large limits. There was an example of a young man aged 19 an expert on infra-red devices who deferred acceptance of a Navy Commission and sewed aboard a Pacific career. Another example was that of a dermatologist aged 62 who flew to New Guinea and stayed with the troops throughout the Philippine Campaign. In spite of this diversity, all the men had one common characteristic viz. ability in research, and this very diversity indicates that in OR work a special type of ability is required which these men had. Experience has shown that ~~in OR work~~ such ability and mentality is found in physicists and biologists, and less commonly in mathematicians, Engineers and Economists although there have been brilliant exceptions. I wish therefore you had asked my colleague Dr Patrick to deliver these lectures on OR to you.

It is perhaps not out of place here to give a few examples of in a the brilliant general way of the brilliant work done by some of these OR groups. Some of these groups did excellent jobs for war effort, and also contributed a lot for the development of operational research. In spite of the great diversity of personnel in the several groups there was the closest cooperation between them not merely in U.S alone, but as between U.S & U.K. too. In particular, the Navy Division ORG organised regular meetings for discussions at which British Admiralty delegations were invited. Papers presented at the meetings were rehearsed and monitored with meticulous care, and presented omitting some of the more abstruse mathematical considerations and concentrating on conclusions and results that would guide Navy doctrine. Prof. Blackett remarked after once attending one of these sessions that the quality of the presentations compared very well with those of the Royal Society in England. Four of these groups joined together and devised under the Chairmanship of Prof. H. P. Robertson a suitable course of 6 to 8 weeks' duration for all OR workers.

To mention even in a general way the ~~striking~~ achievements of each and every ~~group~~ OR group during the war would be a hopeless task since it would be tantamount to tracing the history of scientific effort in the war, and would take volumes. Moreover there are numerous books in which this effort is comprehensively described. Since, however, some of you are perhaps introduced to OR for the first time, I shall point a few striking cases. The classical results about the correct depth-setting for depth charges against submarines, and the optimum size for convoys which are the triumphs of the British OR School are classical, and I shall return to them a little later as illustrations of the OR method. The calculator devised by the AMP which could so determine the spacing of bombs in a train of bombs released from a single aircraft so as to maximise

the chance of hitting a target, and other similar studies made the bombing of Germany a great success. Operational research as applied to radar used the idea of measure and countermeasure equipe and involved some of the most extraordinary duels of wit in the history of war, and British superiority in radar OR technique did a great job for early victory. The search for German blockade runners successfully crossing the South Atlantic bringing strategic materials to Japan by means of radar-equipped planes by evolved by Dr. Steinhard a biochemist ~~for~~ came as a great surprise to the Navy and Air Force who were utterly amazed that effective patrol could be maintained across the South Atlantic with only a handful of planes instead of a whole Air Force. In 1943 the Germans had designed and successfully used against a British a torpedo equipped with acoustic gear which would listen to a ships screws and then direct the lethal charge to "home" on the hapless vessel much as a blood hound follows a scent. Dr. Uehling noted physicist of Washington University ~~of~~ devised a countermeasure based on operational technique which completely beat the menace, and the Germans were dumfounded at the speed in diagnosing their secret weapon. Another brilliant example of the success of the socalled search theory of OR technique is provided by the discovery of the weapon used by the Germans in sinking Allied submarines due to Dr. Rinehart able mathematician from the Case School of Applied Science. Security has prevented the disclosure of this work even to this day. The controversy between chemists of the correct ratio (HE/IB) to be used in high explosive bombs was settled in an operational way.

I shall now turn to the very significant step in the development of American war effort brought about by the formation of OFS according to which OR workers from several groups joined this service, and were

actually sent to the several fronts to take part in field service as scientific consultants, and help the commanders in operational work. The great Newton once seems to have said "If, instead of sending the observations of seamen to able mathematicians at land, the land would send mathematicians to sea, it would signify much more to the improvement of navigation and safety of men's lives and estates upon that element." The innovation of World War II acting on the advice of Newton in sending scientific aid into the realm of operations is brought about in the formation of the OFS. This also avoided the mistake made in previous wars of recruiting able young men including even promising scientists and sending them to the front with the regular army. Thus wrote Bush to the Secretary of War "In 1914 Britain sent to the front line trenches one of the greatest modern physicists. He had revealed for the first time some of the secrets of the nucleus of the atom, and he was known throughout the world as one of the geniuses of his day. His name was Moseley. He was soon killed in action .... This example was repeated with many others of lesser calibre by Britain in 1914. In the present war Britain has learnt a lesson, and utilised its young scientists well." Wars can be lost if gifted and well-trained scientists are in the wrong place, in uniform at the front or wholly outside war-effort. Thus the scientists assigned to the OFS were inside the war-effort but not in full uniform as fighting soldiers.

The ~~The history of the~~ OFS an experiment in the organisation and mobilisation of scientists out of the laboratory and into the field in such a way that both could profit was due largely to the initiative and imagination of Prof. K.T. Compton, Head of the famous M.I.T., and

in the beginning gave rise to some difficulties consequent on the military prejudice against civilians, and the ~~consequent~~ resulting doubt that the relations between officers and scientists may not be harmonious. But the actual working showed however that these fears were groundless. It was clearly understood that the OR worker could only make recommendations but had very little power and responsibility and had no authority to issue orders. To make his work more useful he was allowed direct contact with several echelons of the military hierarchy, and could thus supplement the normal channels; otherwise it would be mere waste of technical talent. OR groups could also be assigned directly to the operating command and also make reports directly to the various echelons of the command. In view of all this, the civilian OR did not become a member of the armed service although he wore an army uniform with a shoulder patch inscribed "scientific consultant", and was accorded an assimilated rank which entitled him to the privileges of an officer as to messing, housing and transportation. In the case of Compton himself who also wore the uniform, he carried a card in his pocket mentioning him as consultant, and specifying that if he were captured by the enemy he should be treated as a Colonel in the regular army. These OR workers were permitted to move from theatre to theatre as necessity ~~as~~ arose and also to avoid dilution by administrative work. It is certainly correct to say that this contact of OR men with live research in the field was not a little responsible for success in the war effort.

I have so far spoken of OR as merely ~~as~~ a war effort, but ~~even~~ after the war OR has not remained static, but extended widely into ~~many~~ many fields other than military. Even in military science in peace time, OR has played an important role as an experimental subject, and is helping proper evaluation of armaments and their consequent improvement. In industry, business and accounting it has had manifold applications which appear to grow from day to day. In road

research specially it is playing a very important part, and is bound to be of great help to municipal and Governmental authorities. Many plans undertaken by Governments like river valley projects, Borstal projects related to criminology and prediction, queueing problems, operating Railway systems and several others appear amenable to treatment by O.R. methods. To understand the nature of these problems, one need only look through the contents of the issues of Journal of the Operations Research Society of America, and the Operational Research Quarterly of London for the last few years.

A recent survey has shown that OR is expanding rapidly everywhere and making spectacular headway in the United States, specially in US industry. Military OR which began in 1941-42 had 400 analysts by 1945 and in 1954 there were about 800 analysts & has perhaps now reached the peacetime ceiling of 1000. The ORSA had 870 members in 1954 and membership has been growing at the rate of 40 members per month. The number of OR members in industry is perhaps 1500 and its rate of increase is between 20 and 100 per month. About 100 companies use the techniques of OR mostly in the Aircraft, Electrical Machinery, Chemicals, Instruments, Machinery and Petroleum industries. Ultimate strength of OR workers in Industry may perhaps be 10,000. Twelve Universities give seminars in OR in which number of industries participate.

So far I have talked to you so much about the history of OR, its role in World War II and its later development in peace time, but I think I have ~~successfully~~ carefully succeeded in not telling you what ~~that~~ it exactly is. Even now I do not want to do this, for it is the most difficult thing is to define it, and I shall therefore come to it later on. In the meanwhile I shall consider a few simple examples, one or two of which I have merely mentioned earlier, to bring about some salient points about OR.

(1) I shall first of all consider a very trivial example involving the line-up of soldiers washing their mess kits after eating at a field mess station

(2) No. of men required to man an AA gun effectively - effective studies.

(3) Radar, Sense & A.T.S. girls

(4) Depth of depth charges

(5) Size of convoys: $k \cong 5\left(\frac{n}{c}\right)$; $l \cong \left(\frac{nc}{100}\right)$. $\frac{l}{k} \cong \left(\frac{c^2}{500}\right)$.

(6) BAA guns on merchant vessels

(7) Barrier patrol

(8) duel problem between fighter pilot and tail gunner of bomber.

(9) transportation problem.

$$\left.\begin{aligned} \min \quad & \sum_i \sum_j c_{ij} x_{ij} \\ \text{subject to} \quad & \sum_j x_{ij} = a_i \\ & \sum_i x_{ij} = b_j \\ \text{&} \quad & \Sigma a_i = \Sigma b_j \end{aligned}\right\}$$

2nd Lecture (20/6/56)

After the rambling talk I gave you in the first lecture, I could perhaps attempt now a definition of OR. In fact, this is the most difficult thing ~~to~~ to do about OR, and nobody has so far ~~succeeding~~ succeeded in doing it satisfactorily. The simplest definition due to P.M. Morse one of the foremost workers in the subject is to say that OR is research at the operational level, just as physics & chemistry would correspond to research at the basic or material level & the study of bridges and television sets, for example, to research at the engineering or applied level. But this definition does not really signify what exactly constitutes OR but

but merely avoids the issue by ~~distinguishing~~ by drawing a distinction between OR and some other disciplines, and is moreover a sort of begging the question. Another worker has defined OR as a systematic application of common sense, but this has the danger of implying that there are other branches of Science where common sense is either not applied or not systematically applied. Another definition which has found favour with many people is that given by Morse and Kimball at the very beginning of their book viz " OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control". Although it appears a little narrow in its significance, it ~~appears~~ is ~~more~~ perhaps more satisfactory than many others if it is taken with all its implications and generality. In the first place, it is a scientific method, and is certainly an applied science although it may be useful to develops many of the techniques used in OR in a pure scientific spirit with a view to later application. It is also an experimental science although the devising of experiments ~~especially~~ with a view to study operations is an extremely difficult thing, and it takes great ingenuity to devise suitable experiments which will yield ~~suc~~ data comparable to operations in actual practice. The phrase "basis for decisions" implies that quantitative aspects are not the whole story in executive decisions, and that many other factors which cannot be expressed in numbers will also have to be taken into account. It also implies that the prerogative of taking decisions is that of the executive departments, and that it is the duty of the OR worker to present the quantitative aspects in an intelligible form. In fact, it is very important for him not to be in charge of the execution at all. The last phrase " under their control " emphasises that OR is an applied science

One can recognise two schools of thought regarding the application of mathematics in OR. The Blackett School which may perhaps be taken to represent the British OR workers fights shy of mathematics, and would prefer it to

to be kept in the background. Blackett, of course, recognises that it is not possible to avoid it but he would prefer to not to have it exaggerated; only the right order of magnitude rather than the detailed figure should be aimed it. In the same way he looks on statistics; one should know it in order never to have to use it in its advanced forms. Thus, he never had occasion to use the χ^2-test and states that the correlation coefficient is never useful in OR. On the other hand, we have the Morse School representing American thought which would encourage mathematical techniques of the most varied and advanced types. Thus one finds in their work the use of techniques of statistical experimental design, waiting problems, linear programming, game theory, information theory, communication theory, cybernetics, decision theory and even symbolic logic. Both the schools are however agreed on the point that the limitations of the mathematical method should be clearly understood, and right emphasis laid on the conclusions derived therefrom (M & OR)

At the end of my first lecture, I mentioned four categories under which many or most of the problems in OR fall, but, in answer to the very pertinent question of Mr. Hegde, it might be said that they do not exhaust all the possibilities. In fact, if you look at the four categories I have mentioned you will notice that two of them relate to mathematical techniques used in OR viz. game theory and linear programming, and recent work has shown that, from the mathematical point of view, both these techniques are too closely interconnected. The other two categories viz. waiting line theory and search theory relate to the actual operations of waiting & searching and form part of methods of OR.

This tempts me to say a few words about methods of OR. Usually in OR problems there are only few numerical data available about phenomena of great complexity. In this respects OR are somewhat nearer, in general, to many problems in biology and economics rather than physics where usually many

numerical data are available even for simple phenomena. Of course, the most important ~~was~~ single mathematical tool for OR is the theory of probability & statistics. The actual method adopted is the usual scientific method. Past operations are studied to determine the facts, theories are elaborated to explain the facts, and finally the facts and theories are used to make predictions about future operations. Particularly important in OR problems is the choice of the proper operational parameter or parameters for theoretical elaboration and also for ~~ob~~ observational study. In fact, it is here the problem of finding the problem, and a correct choice of ~~these~~ measures of effectiveness of an operation, and their study in the are the first steps in any OR problem. For the sake of simplicity we might say that the study of a single measure of effectiveness, its variation ~~with~~ on the several operational parameters, and consequent results about the best choice of the measure may be said to constitute tactics. If, however, one studied various measurements of effectiveness, their inter-relationships and variations, and the results of this study on the operation as a whole, one might be said to have evolved a strategy. Thus the method of OR may be described in general as the devising measures of effectiveness, and deriving the right strategy and tactics based on operational data & theoretical elaboration.

In many cases also the complete theory will indicate that certain equipment involved in the operation can be improved. ~~These determine~~ This determination of the operational effectiveness of equipment may be called evaluation. I shall say a few words about these topics of measures of effectiveness, strategy & tactics, and evaluation a little later on.

Let me first consider the two important mathematical techniques of linear programming & game theory.

(a) Game Theory: (i) the game of matching pennies: rules - Each of A & B puts down a penny, head or tail up, without showing it to the opponent. The pennies are then uncovered

(a) Theory of games – (1) Games of chance & games of strategy – play and game – choice and move – 2 person & n person games – pay off matrix & rules of the game. – finite games (no of moves finite) – amount of information.

Ex 1. matching pennies

	1	2
1	1	-1
2	-1	1

Ex. 2. Bluffing in cards: A gets high card or low card. If he gets high card he must bid Rs 2/-. If he gets low card he may pay Re 1/- or bluff ie bid Rs 2/-. B may then merely pay Re 1/- or challenge the bid. If he wins the challenge ie if A had low card B receives Rs 2/- and if he loses he pays Rs 2/- to A.

(A_1) A pays Re 1/-. (B_1) B challenges

(A_2) A ~~bids~~ bluffs (B_2) B does not challenge

	B_1	B_2
A_1	$\frac{1}{2}$	0
A_2	0	1

($A_1 B_1$): Low card pays Re 1
high card bids Rs 2/- & gets it because B loses the challenge
∴ average payment to A $= (2-1)/2 = \frac{1}{2}$ Re

($A_1 B_2$): Low card: A pays Re 1/- ~~& B pays Re 1/-~~
High card: A bids & B pays Re 1/-
∴ average to A = 0

($A_2 B_1$): Low card: A bluffs & loses, A pays Rs 2/-
High card: A bids & wins, B pays R 2/-
∴ average to A = 0.

($A_2 B_2$): low card: A bluffs, B pays Re 1/-
High card A bids. B pays Re 1/-
∴ average to A = 1

Ex. 3. Two finger game:

	(1,1)	(1,2)	(2,1)	(2,2)
(1,1)	0	2	-3	0
(1,2)	-2	0	0	3
(2,1)	3	0	0	-4
(2,2)	0	-3	4	0

Ex. 4.

	1	2	3	4
1	2	1	10	11
2	0	-1	1	2
3	-3	-5	-1	1

Ex. 5.

$$\begin{pmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{pmatrix}$$

Saddle point at (2, 3)
$v = 4$.

P_1 gets at least 4
P_2 loses at most 4

Zero-sum games ie $\sum_{i=1}^{n} P_i = 0$ – Rectangular games – Question of optimal strategies for A and B – Minimax theorem – Saddle point – (Ex. 4 a special case).
– value of the game.

(2) Rectangular games without saddle point can be reduced to rect. games with saddle points by mixing strategies. $x = (x_1 \cdots x_m)$, $y = (y_1 - y_n)$

$$E(x,y) = \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij} x_i y_j = \text{expectation of } P_1$$

(i) $\max_{S_m} \min_{S_n} E(x,y)$ & $\min_{S_n} \max_{S_m} E(x,y)$ always exist and are equal $= v$

(ii) If (x^*, y^*) can be found such that

$$E(x, y^*) \le E(x^*, y^*) \le E(x^*, y)$$

then optimal strategy exists & $E(x^*, y^*) = v$ & (x^*, y^*) is the strategy.

Ex: (i) Penny matching:

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \qquad \begin{matrix} x_1, x_2 \to x, 1-x \\ y_1, y_2 \to y, 1-y. \end{matrix}$$

$$E(x,y) = xy + (-1)x(1-y) + (-1)(1-x)y + (1-x)(1-y)$$

$$= xy - x + xy - y + xy + 1 - x - y + xy$$

$$= 1 - 2x - 2y + 4xy = (1-2x)(1-2y).$$

& $E\left(x, \frac{1}{2}\right) = E\left(\frac{1}{2}, \frac{1}{2}\right) = E\left(\frac{1}{2}, y\right)$

∴ The optimal mixed strategy is $\left(\frac{1}{2}, \frac{1}{2}\right)$ for both A & B & value = 0.

ie mixed strategy would be to flip the coin & play.

(ii) To find the value optimal strategies & find value of the game $\begin{pmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{pmatrix}$

In cond^{n} of saddle point replace X by X_1, X_2, X_3 & Y by Y_1, Y_2, Y_3 then

ie $E(X Y^*) \le E(X^*, Y^*) \le E(X^*, Y)$

(i) $x_1 + (-1)x_2 + (-1)x_3 \ge v$, (i) $y_1 + (-1)y_2 + (-1)y_3 \le v$

with $\Sigma x_i = \Sigma y_i = 1$ & x_i, y_i lying between 0 & 1. ie we have to consider

2^6 cases corresponding to the six alternatives $\leq, \geq$. In this case taking all the six equalities we get the solution

$$x_1 = \frac{6}{13}, \; x_2 = \frac{3}{13}, \; x_3 = \frac{4}{13}$$

$$y_1 = \frac{6}{13}, \; y_2 = \frac{4}{13}, \; y_3 = \frac{3}{13}.$$

& $v = -\frac{1}{13}$.

(iii) Solution possible graphically for $2 \times n$ pay off matrices.

Ex.

	1	2	3
①	2	3	11
②	7	5	2

Let ① & ② strategies of P_1 correspond to mixture $x, 1-x$

from $E(x^*, Y^*) \leq E(X^*, Y)$

for all Y we may take the pure strategies for P_2 respectively & the least E for these choices will be $E(x^*, Y^*)$. These expected pays off are

$$2x + 7(1-x) = 7 - 5x$$

$$3x + 5(1-x) = 5 - 2x$$

$$11x + 2(1-x) = 2 + 9x$$

Plotting the lines $y = 7 - 5x$, $5 - 2x = y$, $y = 2 + 9x$, the reqd $\underline{x}$ is given as the max. of the minima of the three ordinates as large as possible ie max. ord. of the convex region bounded by these lines. We get $x = \frac{3}{11}$, $v = \frac{49}{11}$ & we can also find optimal strategies for P_2 as $\left(0, \frac{9}{11}, \frac{2}{11}\right)$.

(3) Thus every finite rect. game has always an optimal strategy but actual determ of soln is complicated. Two methods have been devised one accurate & useful for machine computation, and the approximate one in which labour increases linearly with m & n. In former cases labour is very great (100×100 takes years even with electronic digital computers. Also there is one soln or ∞ number for a rect. game. In latter case there is a last optimal strategy for P_1 given $E(X, j) \geq E(X', j)$ for every pure strategy j of P_2 (X being last strategy & X' any other). Approx^n method $X^{(n)} \to X^*$ if there be a unique optimal strategy.

(4) Games in extensive form (i) $n > 2$, (ii) moves involve choices (iii) more than one move (taking one move to begin & another one after the third step after P_2's move)

(iv) knowledge of what has happened before varying from move to move - In all cases of finite no. of choices we can normalise all games of extensive form - graphical representation by trees - complete & incomplete information, in former case normalisation with saddle point & latter case merely normalised - normalisation possible in case of chance depending moves & n-person games also - All finite games in extensive form can be normalised.

(5) Infinite games ie infinite strategies - Pure strategies - Pay off matrix being $M(x,y)$ ie paid to P_1 by if P_1 chooses x & P_2 chooses y [x, y in $(0,1)$] - continuous games - Here both max min $M(x,y)$ & min max $M(x,y)$ exist & if equal we have saddle point and there is a $M(x_0, y_0) = v = v_1 = v_2$.

For continuous games without saddle points, we use expectation fns $F(x)$ & $G(y)$ or dist^{n} functions such that if $G(x)$ = pay off to P_1 when he chooses x, his expectation is the Stieltjes integral $\int_0^1 G(x)\, dF(x)$ - Fundamental theorem in terms of $E(F,G) = \int_0^1 \int_0^1 M(x,y)\, dF(x)\, dG(y)$ viz

max min $E(F,G)$ & min max $E(F,G)$ exist & are equal & there is a (F_0, G_0) such that $E(F, G_0) \le E(F_0, G_0) \le E(F_0, G)$. & $\| F_0\ G_0 \|$ is saddlept.

ie by mixed strategy every Continuous game can be solved - Special cases of Continuous games called separable & with purely convex or concave pay off functions - In Convex case unique strategy for second player = min max $M(x,y)$ & in Concave case unique strategy for first player = max min $M(x,y)$ & in each case there are optimal strategies for the other players also.

pure strategies case

Ex(1) $M(x,y) = xy - \frac{1}{3}x - \frac{1}{2}y$

Here $M(x, \frac{1}{3}) \le M(\frac{1}{2}, \frac{1}{3}) \le M(\frac{1}{2}, y)$ for all x, y in $(0,1)$

Hence $(\frac{1}{2}, \frac{1}{3})$ is a saddle point & value of game $= -\frac{1}{6}$

Ex. (2) Col. Blotto's game

Exc.(3) – ~~The bomber fighter duel from M & K~~. The Bombr & Fighter duel and Barrage Patrol are continuous games with distribution & will be considered under Search theory

Case of n-person, non-zero sum games – non-rectangular.

(b) Linear programming – Statement of the problem with equalities or inequalities – Geometrical picture – There may be no solutions because given inequalities are inconsistant or because of unboundedness of the domain. It may have more than one in which case it has an infinite number.

Every finite rectangular can be reduced to a special type of linear programming problem.

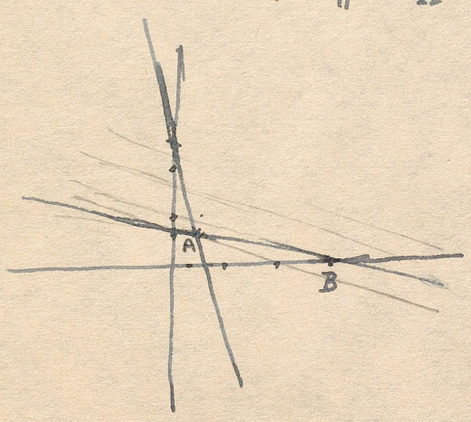
Exc(i) A person requires two vitamins in quantities at least b_1 and b_2, but cannot get them pure & has to purchase two medicins ~~which~~ C_1 & C_2 in which both are contained. C_1 contains a_{11} of V_1 & a_{12} of V_2. C_2 contains a_{21} of V_1 & a_{22} of V_2. costs of C_1 & C_2 are p_1 & p_2 per unit. How much of C_1 & C_2 shall he purchase to get req^{d} quantities & make cost a minimum? Say x_1, x_2. Then to

minimise $p_1 x_1 + p_2 x_2$ subject to

$$a_{11} x_1 + a_{21} x_2 \geq b_1$$

$$a_{12} x_1 + a_{22} x_2 \geq b_2$$

Case(i) $a_{11} = a_{22} = 1$; $a_{12} = a_{21} = 5$; $b_1 = b_2 = 15$.; $p_1 = 1$, $p_2 = 3$.



$$\left.\begin{aligned} x_1 + 5x_2 &= 15 \\ 5x_1 + x_2 &= 15. \\ x_1 + 3x_2 &= p. \end{aligned}\right\}$$

Pencil line $\parallel^{l}$ to $x_1 + 3x_2 = 0$ through P gives the req^{d} soln. A is $(5/2, 5/2)$

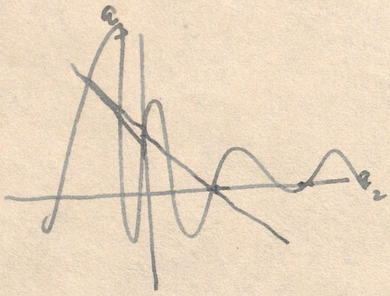
ie $x_1 = x_2 = 5/2$ & $p = 10$. & quantities obtained are 15 & 15 exactly

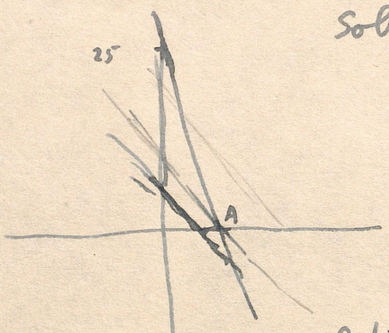
Case(ii) $p_1 = 1$, $p_2 = 20$. Here by drawing similar pencil lines through $\parallel^{l}$ to $x_1 + 20x_2 = 0$ we find the min. of p is given by line through B.

ie $x_2 = 0$, $x_1 = 15$, $p = 15$.

& quantities obtained are ~~15 & 75~~

Ex (ii) Housewife baking apple pie – Read from J.O.R.S.A – Here solution given by Mr OR is what is called the simplex method. This was thoroughly unnecessary. He might have used geometrical method. Take for simplicity the same cases without McDonald's ie put $q_1 = 0$. The problem is to maximise $q_2 + q_3$ subject to $q_2 + q_3 \geq 4$, $5q_2 + 2q_3 \leq 50$.





Soln reqd is pencil line through A.

ie $q_3 = 0$, $q_2 = 10$.

Similarly we can draw a three-dimensional figure for the given case.

This brings out another point re the cost of apples. Since these may vary ~~we have~~ between certain limits, the boundary lines of convex domain should be taken each with two alternatives. Also if subject to chance fluctuations they may be fuzzy. Such problems are difficult to solve. Similar problems arise in factory production to meet sales. Sales in summer small & large in winter. One soln is to run factory in full overtime but production cost rises. Another soln is to produce more in spring & summer & store it, but then storage costs rise. A third soln is to fail to meet orders but this is a counsel of despair. Such problems are those of L.P's with bounded optimisation & not yet completely investigated.

		B (t)	
		1	−1
A	0 1	1	−1
A (s)	−1	−1	1

$\frac{1}{2}\{s(t-s)+t(t+s)\}$

$s=1, t=1, 2$

$s=1, t=-1, -2$

$s=-1, t=1, -2$

$s=-1, t=-1, 2.$

$xy - x + xy - y + xy$

$+1 - x - y + xy$

$E(x,y) =$

$xy + \cancel{(-1)}x(1-y)$

$-(1-x)y + \cancel{(1-x)}(1-y).$

$1 - 2x - 2y + 4xy.$

$4(x-\frac{1}{2})(y-\frac{1}{2}) + 0$

$-3, -2, -4, -3$

max min (rows) ~~= min (max) rows~~. $-2, 0, -2 \to 0.$

= min max (cols). $2, 0, 2 \to 0.$

max min (rows) $-2, 0, -2 \to 0.$

= min max (rows) $2, 1, 2 \to 1.$

-2 ‖ $3, 2, 4, 3$ 2

Defns play & game: Choice & move – person (bridge & chess) – payments always – (general sense).

~~n-person~~ zero-sum $\sum p_i = 0$, p_i = payment to set P_i.

every play is zero-sum game is zero-sum – no. of moves – information

(1) rectangular games – minimax theorem – saddle points –

(2) rect. games without saddle point. – mixed strategy $(x, 1-x)$, $(y, 1-y)$.

$$E(x, y^*) \leq E(x^*, y^*) \leq E(x^*, y). \text{ – mixed}$$

Corresp. mini-max theorem.

Every finite rectangular game has a value or a player of a rectangular game has always an optimal strategy – generalised saddle point – To find the soln – extreme complication – ~~Geometrical~~ Graphical method for $2 \times n$ matrices – maximum ordinate of the convex set.

(3) Solns – one or ∞ – ~~Extremal set K (X)~~ – method specially suited for computing machines. becomes with large m, n complicated for even modern electronic computers. For eg. 100×100 game would not be feasible for even these machines & would perhaps take some years to calculate the solutions. For many rect. games these ∞ by many strategies for one or both players. In such a case choose best strategy ie dominated. Existence of best strategy $E(X, j) \geq E(X', j)$ for every pure strategy j of P_2

(4) Approximate method of solving rect. games ie finding approx. value of game approximately & opt. strategy to any degree of accuracy – labour involved is linear fn of m & n ie better than (3)

ie If there be a unique opt. strategy for P_1, $X_0^{(1)}, X^{(2)}, \ldots \longrightarrow X$.

(5) Games in enterisive form (i) more than two players (ii) chance devices (iii) several moves instead of 1 – normalisation – Graphical repⁿ – Games with perfect information when normalised have saddle point – games with partial information also can be normalised – games with chances also can be normalised – Also n-person games can be normalised ie all games in enterisive form can be normalised if games be finite

(6) Games with ∞ly many strategies – $M(x,y)$. $(x \in A,\ y \in B)$ $[A \& B = (0,1)]$ – continuous
(7) games – Exs: Bl. Blotto game – If both

$$\max_{0\le x\le 1} \min_{0\le y\le 1} M(x,y) \text{ \& } \min_{0\le y\le 1} \max_{0\le x\le 1} M(x,y) \text{ are } = v_1, v_2$$

If $v_1 = v_2$, M has a saddle point (x_0, y_0) such that $M(x_0, y_0) = v_1 = v_2 = v$ (value of game)

If v_1 & v_2 do not exist instead of max, min take sup & inf ie least upper bound and greatest lower bound.

(8) Distribution fn – to deal with cases where v_1 & v_2 exist but $v_1 \neq v_2$ ie for continuous games without saddlepoints – $F(x)$ distn fn

(9) Stieltjes integral $\int_a^b G(x).dF(x)$ ~~pay off to~~.

where if $G(x)$ = pay off to P_1 when he chooses x & if he chooses x with expectation $F(x)$ then his expectation = $\int_0^1 G(x)\,dF(x)$

(10) Fundamental theorem for continuous games

- $E(F,G)$ = expectation of P_1 = $\int_0^1\int_0^1 M(x,y)\,dF(x)\,dG(y)$

$$= \int_0^1 \int_0^1 M(x,y)\,dG(y)\,dF(x).$$

If $v_1 = \max_{F\in D} \min_{G\in D} E(F,G)$ & $v_2 = \min_{G\in D} \max_{F\in D} E(F,G)$

& if $v_1 = v_2 = v$, v = value of game & there is a $E(F_0, G_0)$ such that

$$E(F, G_0) \le E(F_0, G_0) \le E(F_0, G)$$ game saddlepoint (F_0, G_0) & soln

ie v_1 & v_2 always exist and are equal ie F & G can be chosen such that.

ie mixed strategy for continuous game.

3rd Lecture (30/6/56).

(1) waiting line or queueing theory. Relationship between the mean length of waiting line and the degree of randomness of arrival and disposal. ~~Let P_n be the probability that at any given time, 0, 1, 2, .. or n units~~ Both arrival rate & servicing rate may be random. If the mean arrival rate is greater than the mean servicing rate the waiting line will increase indefinitely. But even if the mean service rate is larger than the mean arrival rate, the waiting line is not abolished unless both arrivals & service operations are regularised, not random. Let P_n be the probability that at any given time, zero, one, two or n units will be in the waiting line, and A & S the mean arrival and service rates. If $S > A$, the line can reach an average equilibrium and when this has happened, the P_n's will be constants and equations can be set up to represent the balance of arrivals and disposals. For the very simplest case of one service exit, and when both arrivals and service departures are pure random according to the Poisson distribution, we have simple solutions. In this case, the chance of having n units in waiting line turns out to be

$$P_n = \left(\frac{A}{S}\right)^n \left(1 - \frac{A}{S}\right) \qquad (S > A)$$

If $S = 2A$, $P_n = \frac{1}{2^{n+1}}$ & chance of finding a large waiting line is very small for large n. If $S \cong A$, then $P_n \cong 0$ ie diminishes very slowly with n. The mean length of the line L comes out to be, $L = A/(S-A)$ which $\to \infty$ as $A \to S$.

Consider the particular example of aircraft arriving at an airport and circling in the air in what is often called a stack waiting their turn to land. Aircraft arriving at an airport come at random unless special efforts are made to adjust their speeds during the half hour preceding their arrival. Let us find out what would be the effect ~~on~~ of careful scheduling of aircraft arrivals on the material reduction of the waiting line. Computing the effect ~~of~~ on the waiting line of changes of randomness in arrivals and landing is a difficult mathematical problem. If arrivals are completely

regular each a time ($1/A$) after the next previous but the servicing is still purely random, then we find $L = A/2(S-A)$ ie half of the value for the case of both arrival and servicing being random. But in actual practice such rigid scheduling arrivals and landings is impossible. However much one tries there always creeps in some element of randomness. Thus both arrivals and landings are partially random & it is difficult to deal with this case mathematically.

Use of Monte Carlo technique in such cases shows that a small reduction of queue length results from scheduling arrivals. We can then estimate the cost entailed by tightening of schedules against gain resulting in reduction of delay time. Elaboration where there are several service facilities or several connected bottlenecks the waiting lines being in series & hence with interconnected randomness can be dealt with either analytically or using Monte Carlo techniques.

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(2) Search theory
~~Measures of effectiveness~~. I have spoken to you so far of three of the mathematical techniques used in OR, and it only remains for me to say a few words about search theory. The general principles of the theory of search which is fundamental for tactical analysis are based on many advanced considerations in the theory of probability and statistics, and a proper presentation of these principles would itself require a series of lectures. I shall therefore illustrate the principles by discussing one important theorem called the "mean free path theorem" by its analogy with certain concepts in statistical mechanics.

The theorem is concerned with the probability of locating, or of damaging or of colliding with some object called a target placed at random within an area A. The search object may be a patrol plane, a torpedo, a bomb or a 16"-shell etc. Each search object has an effective range of action against the target. For example, in the case of a patrol plane this is ~~equal to~~ covered area is equal to twice the effective lateral range of vision of the plane, times the speed of the plane, times the length of time spent in searching the area. We assume the covered area is distributed at random inside the area A. The mean free path theorem

gives the probability of success as a function of the ratio between the covered area and the area A. Let $P(0,\alpha)$ be the value of the probability that the target is not yet discovered at the instant when the covered area is α. Increase the covered area by $d\alpha$. If this new covered area is placed at random inside A, then the chance that the target will be found in this new area is equal to the diminution in the probability of not finding it

$$d P(0,\alpha) = -\frac{d\alpha}{A} P(0,\alpha)$$

The solution of this differential equation with $P(0,\alpha) = 1$ when $\alpha = 0$ gives

$$P(0,\alpha) = e^{-\phi}, \text{ where } \phi = \left(\frac{\alpha}{A}\right) = \text{coverage factor}$$

$$\therefore \text{ Probability of hit } = P(>0) = 1 - e^{-\phi}$$

This form shows that we are here dealing with a Poisson distribution with ϕ as the parameter which is equal to the expected value of the number of hits. Hence using the results obtained for Poisson distributions, the probability that the target will have been hit m times when the covered area is α is given by

$$P(m,\phi) = \left(\frac{\phi^m}{m!}\right) e^{-\phi}, \quad \phi = \frac{\alpha}{A}.$$

and $$P = \sum_{m=1}^{\infty} P(m,\phi) = 1 - P(0,\phi) = 1 - e^{-\phi}.$$

We shall now give some examples of the application of the above theorem:

(a) Merchant vessel sinkings: Suppose a merchant vessel can make $2\frac{1}{2}$ trips on the average across the ocean before it is sighted by an enemy submarine, and that on the average 1 out of every 4 ships sighted by the submarine are sunk. Then the 'mean free path' of a ship before it gets hit would be 10 trips across the ocean & expected number of hits in n trips would be $n/10$ which is therefore equal to ϕ. Hence the probability of coming through unscathed in n trips is $P(0,\phi) = e^{-n/10}$.

(b) Effectiveness of mine fields. Suppose an influence mine has a range of action R for a given ship. If there are n mines in a given area A, the probability of the ship hitting at least one mine is given by

$$P = 1 - e^{-\phi} \quad \text{where} \quad \phi = \frac{2nRL}{A}$$

where L is the length of the ship's track through the mine field. The mean freepath of the ship in the field is therefore $L/\phi = A/2nR$.

(c) Ships sighted and sunk by submarines. If N submarines are in independent patrol in a shipping lane, F is the total traffic in ships per month, it can be shown that the average number of ships sunk per month H_i is given by

$$H_i = F\left(1 - e^{-2NPr/W}\right)$$

where W = width of shipping lane, P = the probability of sinking a merchant vessel once it is sighted, and r = effective range of vision of each submarine.

If on the other hand the N submarines act together as a group ie when any submarine sights a ship it will signal all the others which will home on the first one, and get a chance to sink the ship. In this case P has to be replaced by $1 - (1-P)^N$ and equal to P' say. We can now write H_g in place of H_i by changing P to P'. The graphs of $R = H_g/H_i$ show that for small W, there is an optimum size for the group while for large W, R continually increases. Taking into consideration the fact that not all submarines can home on the one which gave the signal & also other effects of group interference, it is safe to say that there is in general an optimum size for the group of submarines. In the Pacific War against Japanese ~~German~~ shipping it was found that yield per submarine in a group of three was

about 50% greater than the yield per independent submarine. Although German "wolf packs" ~~reached~~ hunted in dozens or more, they did not realise optimum yields.

Many other examples of the mean freepath theorem could be given in its applications to other tactical situations, but we shall now treat briefly a problem from the theory of search where measure and countermeasure are involved.

If $P(x)$ = probability of contact if barrier is at x and submarine is surfaced. We assume $P(x)$ given

$\phi(x)$ = probability density (∝ rel. frequency of barrier placed at x) for barrier at x. $\int \phi(x)\,dx = 1$

$\psi(x)$ = Probability that submarine is submerged at x., $\int \psi(x)\,dx = a$.

G = probability of contact $= \int P(x)\,\phi(x)\,\{1-\psi(x)\}\,dx$.

(a = distance that can be seen by submarine submerged)

Side A would like to have G as great as possible. If ψ be chosen so that (barrier) $P(x)\{1-\psi(x)\}$ is a maximum, then A will choose this value of x. Hence B, with which cannot make it zero, has to choose it such that it has no single maximum ie it has to choose $P(1-\psi) = H$ (constant) over as great a range as possible and H is given by $\int (1-\frac{H}{P})\,dx = a$ which makes H as small as possible. Now A would like to choose ϕ such that $P\phi$ is largest, but then B would submerge in these regions of max $P\phi$. Hence A has also to choose ϕ such that $P\phi$ has not any maximum ie

$\psi(x) = 1-\frac{H}{P}$ $(P>H)$
$= 0$, $(P<H)$
$G \leqq H$

$$\phi = \frac{h}{P}, \quad P > K$$
$$= 0, \quad P < K$$

$\int \frac{1}{h}\,dm = \int_{x_0}^{x_1} \frac{dm}{P}$ where in (x_0, x_1), $P > K$.

$\therefore G \leqq H\,(L-a)$ which is a max. for $K = H$.

It can be shown that safe tactics for both sides is when $H = h(L-a)$

where $\frac{1}{h} = \int \frac{dm}{P}$ and $L = \int dx$ = total range of x over which $P > H$.

The above is an example of infinite zero-sum two-person game mentioned in my last lecture, dealt with in an elementary way using ordinary single integrals in place of double integrals.

(2) Having so far dealt with mathematical techniques used in OR, let us now come to OR and illustrate some of the methods of OR confining ourselves mainly to military applications. The first essential is to first of all to choose a few broad, very approximate constants of the operation or what might be called

(3) measures of effectiveness. It is also important that the notion of approximateness is correctly realised. These constants are important because they are approximate, and we find it sufficient to locate the value of a constant to within a factor of 3 (or a hemibel - hemibel = $\sqrt{10}$, bel = 2 hemibels = 10, 30 = 3 hemibels $\sim 10^{3/2}$ etc) ie we get used to hemibel thinking. We give a few examples of the measures of effectiveness:

(a) Sweep rates - used in search theory is defined operationally as the number of contacts with an enemy unit per unit search time multiplied by the area searched over and divided by the estimated number of enemy units in the area ie

$$Q_{Op} = \left(\frac{CA}{NT}\right) \frac{\text{Sq. miles}}{\text{hour (or day)}}$$

$$= \frac{\text{no of contacts per hour } (C/T)}{\text{average density of enemy craft } (N/A)}.$$

This can be calculated theoretically & we have

$$Q_{th} = 2RV. \ \text{Sq. mls}/\text{hour (or day)}$$

where R = effective lateral range of detection in miles

v = average speed of search craft in miles per hour (or day)

Q_{op}/Q_{ik} is an estimate depends on effectiveness of search and the effectiveness of enemy in escaping detection. Comparison of the two search rates affords a powerful means of deciding the efficacy of search operations - (i) Example of the table for contacts on merchant vessels by submarines.

(ii) Example of anti-submarine flying in the Bay of Biscay - This is the problem dealt with under measure and counter-measure in search theory - tactics of U-Boats staying submerged good part of the day & coming to surface at night & hence low sweep rate — radar planes equipped for night flying - Germans equipping their submarines with radar receivers capable of hearing the L-band radar set on British planes - fitting British anti-submarine craft with S-band radar sets - Countermeasure by Germans using S-band receivers. This involved great delay - Showing how study of changes of Q_{op} is a study of warfare in action. Here Q_{op} is given by knowing the hours of anti-submarine flying in the Bay per month, the number of sightings of U-boats resulting, and estimated average no. of U-boats in the Bay area during the month & area of Bay searched over ($= 130,000$ sq. miles). - Case of tactics

(b) Exchange rates: ratio between enemy loss and own loss - If m & n be no. of forces and k & l losses, in general k & l will depend on m & n and nature of dependence is determined by the tactics adopted - use of exchange rate as correct measure of effectiveness has been already illustrated in the first lecture in the problem of convoys vs submarines viz $K \simeq 5\left(n/c\right)$; $l \simeq \left(nc/100\right)$

$\left(l/k\right) \simeq \left(c^2/500\right)$ independent of m.

when this finding was acted upon and size of convoys & escorts was increased, the Germans found it unprofitable to attack North Atlantic Convoys & the U-Boats went elsewhere. This defeat contributed to the turning point in the Battle of the Atlantic.

(c) Comparative effectiveness - Submarine vs Aircraft as antiship weapons - Merely taking sweep rates into consideration the conclusion drawn from data from Coastal Command publications is that a single submarine is equivalent as far sinking enemy shipping goes to a squadron of long range antishipping planes.

This cannot be taken as final since we have to compare the cost of outfitting a submarine as compared to cost of outfitting a squadron of planes, and also we have to compare exchange rates.

* [P. T.O.]

(d) Evaluation of equipment performance (i) antiaircraft guns for merchant vessels. Proper measure of effectiveness — antitorpedo nets, (ii) Aircraft antisubmarine depth bombs. Numerical analysis of the chances of success of the attack as a function of the degree of submergence indicated that a change in depth setting from 50 ft. to 25 ft. would at least triple the chance of success of the average attack. (iii) Effects of practice and training shown operationally in the case of long range strategic bombing of Japan.

(4) Strategical Kinematics - Use of measures of effectiveness, and comparison of such measures in answering strategical questions. Description of a combat or a battle can be set up using exchange rates in as differential equations, but these should be treated more as indications for directions of further investigation rather than as description of methods of proven utility.

* In connection with the question of comparative effectiveness, a strategic problem raised by Group Captain Howell the other day comes to my mind. He talked of ~~latest~~ types of submarines using atomic engines and the diesel technique, and also referred to the problem of submarine detection. He pointed out that when submarine parts like periscopes show up, aircraft equipped with radar of high search power should be able to detect them, and also that submarines can be detected by the noise of cavitation of submarine propellers. He concluded that surfacing ~~and~~ fighting with aircraft is sometimes better for submarines & that aircraft should be prepared to fight submarines on the surface. This raises a new type of OR problem altogether where one has to compare the exchange rates for this type of surface combat with the ~~exchange~~ sweep rates of the older type of warfare, and decide which type of fighting is more successful.

Let m = no. of combatants in Red force at any instant

n = " Blue " "

t to denote engagements made up of combats denoted by T.

on the understanding that only average values are used in the sense of probability theory, the number of Red forces lost per combat is equal to $-K/K+l$ since $K+l$ = total number of combats ie

$$\frac{dm}{dT} = -\frac{1}{1+E} \; ; \; \frac{dn}{dt} = -\frac{E}{1+E}$$

(E = exchange rate = $\frac{\text{Blue loss}}{\text{Red loss}} = l/K$).

$$m = m_0 - \frac{T}{1+E} \; , \; n = n_0 - \frac{TE}{1+E}$$

$$\frac{dn}{dm} = E, \quad n_0 - n = E(m_0 - m)$$

[~~Eqns for chemical reactions - also used in Biology~~]

These are Lanchester's equations for the linear law corresponding to ancient warfare. In terms of t

$$\frac{dm}{dt} = -\frac{1}{1+E} F(m,n,t) \qquad [F(m,n,t) = \text{no of Combats in the } t^{th} \text{ engagement}]$$

$$\frac{dn}{dt} = -\frac{E}{1+E} F(m,n,t)$$

giving $\frac{dn}{dm} = E$ as before.

In the case of modern warfare with extended fire power, we have cannot break up engagements into combats for each participant can fire at every opponent. Hence time variable is $\underline{t}$. Let in a tth engagement a single Red Combatant can put out of action $\left(E/1+E\right) G(t)$ Blue combatants on the average, G measuring the 'intensity of the combat' at time t, and an individual Blue Combatant can put out of ~~cond~~ action $\left(1/(1+E)\right) G(t)$ Red Combatants. The corresponding diff. eqns are

$$\frac{dm}{dt} = -\frac{n}{1+E} G(t), \quad \frac{dn}{dt} = -\frac{mE}{1+E} G(t)$$

$$\frac{dm}{dn} = \frac{n}{mE} \; ; \quad n_0^2 - n^2 = E\left(m_0^2 - m^2\right)$$

Δ this is Lanchester's square law

The above eqns become identical when $E = n^2/m^2$ ie opposing forces are equally matched then. Hence it is more profitable to increase the number of combatants than it is to increase E (by increasing effectiveness of individual weapons). This shows that a strategical use of concentration may counterbalance any moderate advantage in weapon efficiency - Case of 1000 Reds meeting 500 blue, leaving 866 Red & 500 Blue - leaving 707 Red & all Blue lost - advantage of concentration - U.S. fighting planes in groups of 2 or 3 & Japanese singly.

Putting $\tau = \frac{\sqrt{E}}{1+E}\int G(t)\,dt$, Koopman reduces Lanchesters sq. law eqns to $\frac{dm}{d\tau} = -\frac{n}{\sqrt{E}}$; $\frac{dn}{d\tau} = -m\sqrt{E}$

$$m = m_0 \cosh\tau - (n_0/\sqrt{E}) \sinh\tau$$

$$n = n_0 \cosh\tau - (m_0\sqrt{E}) \sinh\tau$$

Hyperbolic orbits show property of acceleration towards the end. The last half of weaker force is annihilated in a shorter time than the first half. Examples of German collapse in Tunisia & Allied-vs-German air struggle in 1943-44.

It can be shown that a suitable probability analysis may be made of Lanchester's equations, and these equations if not pressed too hard represent war conditions sometimes quite closely.

Lanchester's equations can be generalised to the overall trend of a war, and can be written in the form

$$\frac{dm}{dt} = P - an - cm,$$

$$\frac{dn}{dt} = Q - bm - dn$$

where P & Q are production rates. Diffl eqns of this sort appear in biological problems relating to struggle between animal species and also in chemical kinetics.

- division into tactical & strategic forces - Expsn for P & Q in terms of these - Putting $a = b = c = d = \alpha$, and finding expsn for

$$\frac{dm}{dt} - \frac{dn}{dt} = L(m_t, n_t)$$

minimax principle of making Red loss least & Blue loss greatest - Red general must assume that Blue General is trying to minimise L & Blue general assumes that Red general is trying to make L maximum - optimum solution existing.

(5) Gunnery and Bombardment problems. Evaluation of weapons like guns, bombs etc and analysis of the best methods for their use. Effectiveness measured by the ratio of number of targets destroyed to the number of shells etc employed. The number of shells etc. reqd to destroy a target depends on (i) destructiveness of weapon ie probability that target is destroyed if weapon hits it (ii) accuracy of the weapon ie probability that weapon will hit the target. In addition if weapons are used in groups result depends on the pattern used. Methods of calculating probabilities of destroying targets and determining patterns creating maximum destruction.

(6) OR in cold war and atomic war - In the former psychological & in the latter of a very complicated type.

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(11) Special cases of continuous games - separable games - can be solved

(12) Games with convex pay off functions or concave pay off functions:

In first case unique strategy for second player is $v = \min_y \max_x M(x,y)$

In second case " first is $v = \max_x \min_y M(x,y)$.

& it can be proved that if in each case there are optimal strategies for the other players also.

(13) Applic to statistical inference

(14) Linear programming - Connection with game theory - To pick out from the n-tuples $(y_1 \cdots y_n)$ satisfying $\sum_{i=1}^{n} a_{1i} y_i \geq b_1, \cdots \sum_{i=1}^{n} a_{mi} y_i \geq b_m$ an n-tuple which will minimise $\sum_{i=1}^{n} b_i y_i$.

(i) need not necessarily have a soln

(ii) Can have more than one soln - If it has one has only many.

Solving an arbitrary rectangular game can be reduced to the solving of a special linear programming problem - Example of medicine.

(15) Zero-sum n-person games - theory not satisfactory

(16) Do - solutions

(17) Games without zero-sum restriction.

Urgent need is to evolve satisfactory theory for (15) & (17)

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Definition of O.R. & Nature of O.R.

(1) military, surgical or op. calculus? (2) A branch of P.M. or App. M.? Branch of Probability & Statistics? of Psychology, Philosophy, Logic? (3) An independent Science (4) What is O.R., still unanswered (5) operational & operations research in U.K. & U.S., the countries separated by a common language (6) O.R. is perhaps not new, what is new is its importance since we cannot afford to go by trial & error otherwise trial & catastrophe (7). A study of the dynamics of situations, of processes, of organisation or of methods (8) appln of scientific method to determine how best to run a war, run a business, or in general to conduct any operation or group of operations (9) Concerned with optimising the performance of a given system (10) O.R. is systematic appl. of common sense (11) Is a type of philosophy.

(12) Case history definition (13) Research at operational level just as Physics & Chemistry correspond to research at the material level, and Engineering to research at the applied level like building bridges & television sets (14) O.R. both experimental and analytical (15) Name O.R. coined by A.P. Rowe in 1937-39 to distinguish it from normal research in radar at Air Ministry Research Station at Bawdsey

(16) Characteristic attributes of sound O.R.: (a) ability to see the matter as a functional whole & to see what wants analysing ie statement of a problem and its importance in a real context (b) Construction of a cogent quantitative hypothesis or model supported by experimental results (c) doing the necessary functional, dynamical or statistical analysis, using an amount of mathematical sophistication appropriate to the problem, (d) Assessment of the effectiveness of the implemented conclusions of the study, and to present them in a straightforward understandable manner

(17) Prejudice against maths among O.R. workers wrong (18) Analogy between academic research & fishing. Academic research workers appear more brilliant because he merely catches any fish that comes his way while an O.R. worker has to catch one particular fish within a prescribed time, (19) Knowledge of latest mathematical gadgets not to be badge of office for O.R. worker. He should be a hard headed empl. scientist with a very broad vision, who trusts only what he can measure (20) O.R. is a scientific method of providing executive depths with a quantitative basis for decisions regarding the operations under their control (21) O.R. is an applied science, not a branch of Maths (22) O.R. worker does not & should not make the decision (22) most impt mathematical tool is prob. & statistics (23) few data as in biology or economics, not like physics, must use its own methods (24) O.R. men are not gadgeteers; improvement in equipment to

be done by scientists (25) OR worker may be physicist, mathematician or [illegible] considerable experience in research (26) Particular O.R. mentally found in physicists, [illegible] soldiers (?) & less commonly in mathems, engineers & economists with [illegible] exceptions (27) Unsatisfactory between OR worker & administrator (28) fruitful only when O.R. should operations & partnership is with an administrator who has authority, otherwise it is a philosophy, not a science (29) Development of OR a chief scientific feature of ~~[illegible]~~ Blackett's Circus consisting of physiologist, math. physicists, an astro-physicist, an army officer & an ex-surveyor — Blackett a former Naval officer (29) mats of wire netting on the ground round radar receiver, grass growing under them luxuriantly, Stir caused by introducing geese to keep the grass down & asking 12 ATS girls to look after the geese

(30) Remark of an eminent Officer re. O.R. team "Those boys won't even use a soldering iron until they know the whole of the policy of the Pacific war"

(31) Major military calculus (32) whole of p. 36 of "Notes on O.R." (33) Blackett's [illegible] of using Maths in O.R., (~~[illegible]~~) Example of an App. Math man using complicated maths to impress an Air Vice-Marshall, Blackett says he should know statistics in order never to have to use it (34) Studies in OR to be done in collaboration with uniformed colleagues

(34) OR techniques & non-professional men — analogy of Doctors discussing patient — Tarkowski's ref. to Henry VII

(35) Influence of non-military OR noticeable recently as shown by Case History (p. 3 of notes of 3/11).

~~Commercial 2~~

(36) Poem on O.R. by O.R.D. of Sept 57

Origin of O.R. – Young created by Blackett in Training Command, Research Classic – Young 41 years, 25 years old – Battle of Atlantic won by O.R. – Began as military OR in 1941–42 with retros in U.S. as Navy : Army : Air Force = 1 : 5 : 10 – Army O.R. set up in the Far East in 1952/53 & O.R. Sections in Korea Malaya & Singapore – Early problems treated by Blackett : (a) Operational use of new radar sets, (b) (i) Proving link between food rate & food guns in practice & gun sites, (ii) siting of gun batteries for A.A. defence London, (iii) Checking of intelligence reports on the number of bombers shot down (b) with Coastal Command, (c) best depth setting for depth charge (ii) right choice for pairing anti-submarine aircraft, (iii) Calculation of probabilities of success of various methods of patrolling a given area (c) with Dept of Naval OR in 1942 (i) anti-submarine campaign exclusively in the vectors of 1943, (ii) Controversies about diversion of heavy bombers from the bombing of Germany to U-Boat hunting and about the optimum size of Convoys : Blackett's difficulties in making people believe in O.R. – In fact (ii) alone won the "Battle of the Atlantic" since U-Boats left North Atlantic because it was not profitable – Historically the first O.R. problem was concerned as to how to set the time fuze of a bomb to be dropped from an aircraft on to a submarine & it was solved by Blackett, who showed that the fuze should be set to explode on impact, and not as the Royal Navy & Air Force believed it should be set to explode after a delay period. —

Origin of O.R. in U.S., Conception of U.K. & U.S. in early days of 1942 – Tizard's arrival in U.S., Cecil and A. Tizard's Report, Arrival of Conant to U.K., Bush's office in U.S. formed by Darwin, Hailing of reports & enclaves of reports – Bush's office as Director OSRD, Conant as Chairman NDRC – formation of Applied Math. Panel in 1943, appointment of K.T. Compton to inform OFS (Office of field service) of OSRD – Struggle of manpower in scientific personnel e.g. of Moseley killed in action – mobilisation of scientists quite well in O.R. – ASWORG (anti-submarine warfare OR group) scientists being mentioned in dispatches, Shea travelled over 300000 miles, British 'Hedgehog' & American 'Mousetrap' – Morse's Circus, by summer 1943 he had 46 men, 6 mathematicians, 14 actuaries, 18 physicists, 3 chemists, 2 biologists & an architect – formation of Central ORG under Morse – nine men taken from labs – work of OFS civilians accorded officers' privileges as to messing, travel & transportation but was a civilian, never uniform with badge "scientific Consultant"; few became more military than the military, example of 62 year old mathematician becoming a front runner, a metallurgist ranked as a major for 3 days in the emergency, a most successful & successful exploit this OFS

Compton Chief of OFS specified that if he be captured by enemy she is to be treated as Colin in reg. army, Gen. McArthur's welcome to Compton – More than 100 civilians in ASWORG in Navy; civilians loaned by Columbia Univ. to Navy greatly eliminated Nazi submarine threat – once prejudice against OFS men was broken, the demands for it became a crescendo – ASWORG → ORG – OFS had 464 at end of war, engineers, mathers, marine biologists & ornithologists, their veristability (p. 40' of "Notes on OR) – Requests made by services that vacancies to OFS be filled "the day before yesterday" – meetings of OFS Navy group & British Admiralty for discussion & rehearsing of papers, Blackett's remarks about quality of these papers being of FRS standard – Round table conference of OFS, AMD, OAD, ATAD men presided by H. P. Robertson deciding on a 6 to 8 weeks' course in OR – ORG dealt with 1600 reports a month, with 15000 items & maintained a machine section, & was called "combat team without guns" – Incident of Dr A A Bates (metallurgist) an OFS man ruling a conquered town for a few days – Selbert going to India & choosing Bangalore as the best place to repaint planes & make them invisible due to specular reflection – Brain-trust of ORS men in Gen. Richardson's area, his tribute

(3)

O.R. techniques (1) Experimental design (2) waiting problems (or Queueing theory) (3) linear programming, (4) symbolic logic, (5) decision theory, (6) information theory ~~handling~~ ~~Queueing theory~~ (7) theory of games, (8) Monte Carlo technique, (9) probability and statistical techniques, (10) cybernetics, (11) non-linear programming (or problems of bounded optimisation) (8) handling of uncontrolled data (Monte Carlo?) (12) computational techniques eg. to invert a ~~matrix~~ 20 x 20 matrix on a digital computers requires 108 years by usual method & by change of method can be done in a few hours ~~done~~ (13) Inventory problems eg. use of storage water in a hydroelectric system, scheduling of water collected during high river flows; if the future river flow be known at least a year in advance in detail, the O.R. plan of maximised expected return can be worked by calculus of variation & soln by digital computer.

(3) ~~(4)~~ used in determining optimum utilisation of machines, most efficient scheduling procedures best system of salary scales & capital budgeting

(9) Stochastic processes (14) optimum distribution of effort, its naval aspects

(2) operational property of waiting lines that the longer they get, the longer they tend to get. This is a self-aggravating problem a non-linear O.R. problem — another example is slowing down in free flowing traffic bringing in a traffic jam — a third example of converse type is that of soldiers' bath tubs, here the change 2+2 → 3+1 should reduce waiting by 50%, but actually it became zero (15) measure of effectiveness & countermeasure (16) Automatic Data Processing System (ADPS) — relation to (8) ~~to (9)~~, (17) overt propaganda, covert propaganda, Strategic propaganda, Tactical propaganda & counterpropaganda — see (15) also —

(9) ~~(10)~~ (9) Uncertainty of probability — consideration of atomic weapons upon tactical planning & fire support coordination is essential in this atomic age

(10) Cybernetics not to be easily dismissed — leads to simplicity, breadth & depth of thinking — suggests possibility of automatic management of concerns

(4) Symbolic logic — Enunciation from Lewis Carroll to be modified

(8) Random walks of drunkard

~~Examples of O.R. being used~~

Fields in which O.R. is used

(1) O.R. & accounting & inspection & budgeting

(2) Capacity of roads (Road Research Laboratories) – Highway traffic – Accidents, time lost

(3) Hydroelectric schemes

(4) Factory production & sales – Petroleum companies in particular – optimisation of crude oil cracking problem

(5) Municipal administration – City housing & municipal facilities

(6) Railways – ~~tickets~~ – Transportation problems – Mechanising ticket printing issue – Development plans, key position of O.R.

(7) Heavy industries – Spare machinery

(8) Criminology & prediction – Borstal research project – use of Wald's work on Statistical decision fns used to find why more people come back to crime after release from jail

(9) Educational topics – 10 year forecasts of requirements on a rational basis for school populations

(2) Risk taken in crossing a road, study indicating where controlled crossings are necessary

(2) Analogy between flow of cars on a highway & flow of molecules in a gas as interpreted thro' kinetic theory; good qualitative agreement

(6) Transportation problem consisting of in finding a pattern of shipments between m origins with given surpluses & n destinations with given deficits such that total cost of transportation is minimised

(4) Arrangement of sales effort – Production planning

(10) O.R. in telephone operation

(11) " house heating (for eg. in event of next war should coal, oil or electric heating be encouraged in US?)

(12) Psychiatric research

(13) Work against disease & deterioration

(6) US Rys have O.R. division

(4) Production scheduling

(2) Shockwaves on the highway – theory of traffic flow

(3) Criteria for selection of water power projects

(14) Travelling salesman problem

(15) Machine repair as waiting line problem

(16) Hospital planning

(5) ~~4~~

Military OR

(1) In Air force, Navy & Army – Radar, anti-sub, anti-aircraft

(2) Stacking of aeroplanes (Monte Carlo)

(3) Time fuze of a bomb to be set

(4) Four classes of work in military OR

(a) to find operational effectiveness of weapons & equipment

(b) analysis of results of operation in war, or in peace of exercises to determine effectiveness of tactics, influence of weapons on tactics & conversely

(c) prediction of the results of future operations

(d) analysis of efficiency of organisations or of methods

(5) Methods of coping with atomic attack

(6) Psychological warfare

(7) Infantry problems – It is usually taught that you must never fire a long burst from a machine gun because the gun ~~vibrates~~ vibrated so much that it shook off your axis, it was only the first 2 or 3 bullets out of a burst that hit the target. A young OR. scientist who did not believe anything he was told took some coloured crayons & coloured a bunch of bullets & he was able to show that, consistently, the only rounds that hit the target were the last rounds & not the first

(8) Time & motion study in Armed Services – (a) Ex. of gundrills in Artillery showing than anti-tank guns could be more effectively ~~managed~~ manned by three men instead of six. When the report was written & passed on to the gunners, they said "We never use more than 3 men in action, but we want enough men to do the cooking, bring up the ammunition & go scrounging around for supplies from the Officers' Mess etc, and here you are showing us how we can get rid of these men. Burn the Report" & it was done. (b) Of the gundrill of the 25-pounder. Movies taken, everything was good except that there was one man who spent a long time just standing still doing nothing. He was supposed to be holding the horses! Horses had disappeared 20 years earlier, but this task was not eliminated when the gun drill was re-written.

(9) 25-pounder gun problem – What is its purpose? No agreement among military people. Majority view is that it is anti-personnel – Next stage to see if it is effective & it was not at all – deficiencies due to effects of irregularities in ground height – Example of enthusiastic mathematician who found that if he used the poles that surveyors used, he could stand on his head in the middle of an array of poles & read off the heights. One day found by Police doing this in the middle of Barnes Common in London & marched off to fail because he was nothing except to say that it was highly secret – great difficulty in [illegible]

(10) The tank-gun weapons systems – German tanks in Libya & OR work on this

(11) Locating launching sites of V-2 rockets by sound ranging i.e. listening to the shock waves

(12) Line up of soldiers washing their mess kits after eating at a field mess station – Instead of two tubs for washing & two for rinsing, an OR-worker suggested three for washing & one for rinsing, because former took 3 times as much time. Change was made & a line of waiting soldiers not only diminished & on most days there was no line

(13) Putting AA guns on merchant vessels – problem of finding the problem – terrifying chance of their being sunk

(14) theoretical work on the various strategic distn of forces

(15) Submarine avoiding searching aircraft – tactics used in air engagement between bomber & opposing fighters

(16) Optimum search for enemy vessel, or submarine or aircraft

(17) Depth setting of anti-submarine depth charges to improve U-Boat sinkings

(18) arrival & docking of ships in harbour

(19) Submarine vs Aircraft as anti-ship weapons

(20) anti-ship vs anti-city bombing

(21) Evaluation of equipment performance – OR workers to be at the front

(22) Anti-torpedo nets – OR results showed that programme of fitting with nets had not paid itself

(23) Magnetic Airborne detector of submarines – useless for random search, equivalent to a blind man trying to draw a pencil line through a single dot on a large floor – but effective as a blockade across restricted area eg Italian campaign against Nazi U-Boats thro' Gibraltar

(24) Aircraft anti-submarine depth bombs – O.R. analyses made the doctrine to set depth of explosion for aerial depth bombs at 25 ft & not to drop bombs if submarine had already submerged for more than half a minute

(25) Radar bombrights – radar bombing equipment to be used acc'g to OR only at nights & only in non-restricted areas

(26). Lots of examples in Morse & Kimball's Methods of OR (1950) – a veritable mine of information

(27) Air cover to convoys against U-Boats – outstanding success of OR in this

(28) Shooting from tanks – O.R. recommendation accepted

(29) Laying mines – "

(30) Bird echoes – Reducing these spurious echoes & differentiating them from echoes due to aircraft – by E.J. Williams a biologist

(31) Logistics – use of OR in it – "maître du Loge" from French – not logic

(32) 'wiping' of ships ie ships natural mag. field be neutralised by magnetising ships permanently in opp. dir^n – freedom from torpedoes.

(33) Air-ground cooperation (lack of it killed Gen. McNair by one of our own bombs)

(34) Counter-counter measures – antigamming Committee of OSRD contract with Harvard Univ – involving most duels of wit in history of war & did a mighty job for early victory; deception in covering landings in Normandy

(35) Use of bathing thermograph in mines & torpedoes

(36) Proximity fuze – next to atom bomb most remarkable scientific achievement of war.

(37) Searching for a submarine with sound detection gear as portables prepared by ORG Curtains of OFS – analytical wizards in the ORG – work in intercepting submarines crossing the Mediterranean

(38) Work of Steinhardt (trained biochemist) to stop German blockade runners crossing Atlantic bringing strategic materials to Japan, surprise to Navy

(39) Uehling's counter measures for Nazi acoustic torpedo found in one month – Quintfounded Nazis

(40) Problem of military escort protecting a convoy of food lorries against guerilla attack.

(31) Automation & psychology of logistics – Transportation aspects – Storage in motion – Electronic traffic management – Stoppages & machine failures – Military Traffic Management Agency – French General de Logis is Quarto Mastro General meaning art of military movement & maintenance

(31) Logic in logistics – Operation of tactician & logistician in peace & in war

(31) Military Govt logistics includes a ccg to Dictionary of U.S. Army terms "p. 2 later notes" – Example of Napoleon losing Waterloo

(31) Logistics in future atomic war – keep it small, keep it moving

(41) Evaluation of weapons, tactics & organisational concepts

(42) – (45) – Military articles from J. OPSA of 1959

Other articles from JORSA, 1957

Status & scope of O.R. — Doctorates in O.R. in M.I.T. & Case I.T. — ORSA recognised by National Research Council — 2000 members of ORSA — 20 Universities giving seminars — Shorter courses at many institutions — Societies in most countries — number of journals specially devoted to O.R. — 100 companies do their own O.R. in U.S. — Ultimately U.S. may have 10000 O.R. workers & no reqd > recruits available, already noticeable — Pay twice as much as for Engineers — Report of Committee on O.R. (i) diff. eqns, probability & math statistics, sampling theory, machine computation & symbolic logic as prerequisites (ii) course on O.R. to emphasise methodology rather than special techniques (iii) social scientists to be acquainted with physical sciences & vice versa (iv) team research (v) to understand relations bet. maths & science (vi) desirable, analysis, servo-mechanisms, numerical analysis, calculus of variations, group theory & topology — Classes in O.R. in several Universities — Formation of Committee on O.R. of the U.S. National Research Council — O.R. Club in London — more details in O.R.Q. for June, 1954 — Contr. O.R. Societies in most countries — ~~Bankers~~ U.S. Rys. have an O.R. division — Also O.R.Q., Sept. 55 — Library services — 2 courses of ten lectures each during 58-59 at Brunel College of Technology — Examination Courses available in U.K. — Syllabuses of these course — a comprehensive syllabus — Methods of teaching O.R.

(1) All Officers from South Carolians in I.A.S. play Tennis

(2) None but ~~Cols~~ the Cols ~~Majors~~ dines at the High table.

(3) None of the Tennis players rows.

(4) My friends in I.A.S. are all Carolians from South

(5) All Cols. are rowing men.

Conclusion - None of my friends dines at a high table

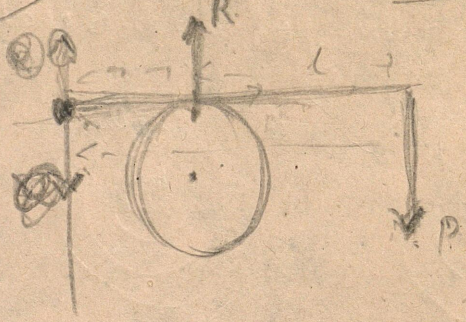


~~S.0070~~

~~S.0071~~

S.0106.
S.0107
S.0108

$P.l = R.a$



~~$Q.a = P.l$~~

$Q.(l-a) =$

$Q.a = P(l-a)$

$R = P + Q = P + \frac{P(l-a)}{a}$

$= \frac{Pl}{a}$

$M_C = \mu R . r .$

$= \mu . \frac{Pl}{a} . r .$

Notes for 2 lectures Pt. III to 8th T.S.O.'s (1)

on 28/10/59 & 4/11/59

(I) My review of Monte Carlo Methods

(II) Note from Military Review

(1) Jan. 1959, p. 17 – Automatic data processing system (ADPS) – The flexibility of ADPS is being studied in an ever-widening number of fields in the Army, to include centralized data processing for financial management, personnel, supply, maintenance fns; finance operations; transportation operations including traffic management & vessel scheduling; medical applns in the field of diagnoses, casualty reports & stats. Applns to problems of the Army in the field include handling of intelligence information, preparing intelligence summaries, troops units, & staff organization; computing supply & transportation requirements; handling data usable in air defence control or nuclear weapon fire planning – ADPS is a potential source of personnel savings or of a more efficient performance

Data-processing associated with handling of information by machine – DPS is a combination of two or more machines which can communicate with one another to accomplish a variety of things – Decision & judgment – Ability to think – Terminology (internal memory, storage circuit, arithmetic unit, control unit) – Input & output devices – communications network – Elements of an ADPS system – Binary number – Applns to feasibility determination – Systems analyses – Programming

(2) Jan, 58, p. 32 – Automation & the psychology of logistics by Capt. Carl M. Guelzo

Transportation aspects – Storage in motion – Electronic traffic management – Effects & machine failures – Military Traffic Management Agency

(3) Feb, 58, p. 82 – Logistics are logistic by Lt Col. A. Green of Australia

Connection bet. military noun "logistics" & math. adjective "logistic": military term was imported from the American adaptation of the French for quarters (Loges) [The French General de Loges is literally Quartermaster General) to mean the art of military movement and maintenance

Three things necessary to win in war: a sword in the right hand, a gift in the left hand, and soft words in the mouth.

—

(4) May, 1958 — Logic in Logistics by Lt. Col. R. C. Biggs, p. 38 — ~~Cop~~ Cooperation of tactician & logistician working together in practice with provide an effectively managed military service in peace & a highly capable combat force in war

(5) August, 1958 — Operations Research — from article by Major J. Barber in "Revue de Defense Nationale" (France), Oct. 57

Ex. of Napoleon & Clausewitz — greatest general could not win a battle if his forces were < ½ of adversary — Complexity — definitions — Errors in approach, limits cannot be found — Refers to 'maximitis' etc.

(6) Nov. 1952 — Max. security with min. forces

(7) Sept. 53 — Mil. Govt. logistics, p. 56. — Logistics includes acceq. as to ~~U.S.A.~~

Dictionary of U.S. Army terms "production, procurement, storage, transportation, distn, maintenance & evacuation of personnel, supplies & equipment, with induction, classification, assignment, welfare & separation of personnel & with facilities reqd for support of the military establishment including constn & operation thereof. It comprises both planning & implementation"

(8) Oct. 53, p. 82 — For the lack of a nail, a shoe was lost
" shoe, horse "
" horse, rider "
" rider, an order "
" order, a battle "
} referring to Napoleons defeat at Waterloo

(9) May, 1953, The principles of war by Vice Admiral R. L. Conolly U.S. Navy, p. 22

Napoleons 115 reduced by Clausewitz to 7 — Nelson's ten principles: objective, simplicity, cooperation, offensive, manouvre, mass, economy of force, surprise and security — New list given by author :- Objective (master principle), Simplicity, Control, The Offensive, Exploitation, Mobility, Concentration, Economy of force, surprise, security, Readiness (logistic & otherwise) — over & above these imaginative thought, logical analysis, common sense (good judgment), broad professional knowledge & moral qualities of leadership

(10) April, 53. – Unsolved problems of ordnance, p. 41 – John Slessor wrote

"If we are ever we are to be governed by intelligence, not by things & words, science must have something to say about what we do, and not merely about how we may do it most easily & economically"

p. 84 – Sea power & air power. p. 87 – Tanks & anti-tank defence

(11) Feb, 53, p. 32 – organisation for joint operations.

(12) Nov. 53, p. 58 – Psychological warfare – Overt propaganda, Covert propaganda, Strategic propaganda, Tactical propaganda & Counter propaganda – measure & counter measure

(13) June 57, p. 56 – Keep it small – Keep it moving! by Major T. McDonald – Logistics in future atomic warfare

(14) Nov, 1954, p. 5 – Planning the counterattack – Counterattack is an offensive operation executed from a defensive situation – Planning counterattacks – Operational plan.

" p. 52 – Changing aspect of war – Success will come to the side that can most intelligently grasp the significance of the changing nature of war & plan searchingly for attainment of the maximum benefits in the diplomatic, Strategic & tactical arts

(15) Dec, 54, p. 39 – A business look at the Army – Efficient management requires organisational structure & control processes similar to those in business

(16) Nov, 57, p. 10. – Development of doctrine at USA - CGSC

(17) May, 57, p. 94 – Science in War.

(18) April, 57, p. 14 – Optimum integrated strategy

(19) August 55, p. 13 – Probability & atomic support – understanding of probability Considerations of atomic weapons upon tactical planning & fire support Coordination is essential in this atomic age

(20) July, 57, p. 35, Realistic decision making at division level

(21) July, 54, p. 31, Evaluation of weapons, tactics & organisational concepts

The Army needs a combat development establishment which is capable of

Applying scientific procedure to the evaluation of equipment, weapons and organisational & tactical concepts for the Army in the field – Lt Col. Thomas J. B. Manley.

Game theory, linear programming, decision theory, logistics, cost analysis, industrial economics, electronic systems, operations engineering, military operations development planning, weapons systems effectiveness analysis

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Logistics – theories involving mathematical analysis of military supply, maintenance, transportation, equipment obsolescence, spare part and inventory control considerations

Methodology.

Electronic countermeasures, Air defence systems, Air traffic control, Advanced nuclear weapons effects, Digital data processing, Project programming analysis

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weapon systems & space research systems – in planning, Conceptual design and evaluation – Chief of operations analysis

Large-scale automated systems, man-machine relationships in such systems in a number of fields including air operations – use of advanced digital computer techniques to optimise these relationships – System analysis, Forecasts, Cost analyses, Operational gaming, Design analysis, Performance evaluation

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Advanced planning & systems synthesis, missile & space programs

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electronic computing service to help O.R. teams

~~Op~~ Computing – data processing analysis.

Industrial dynamics – Company planning, organisation, operation & control, – Advanced coding techniques, Simulation studies, Data processing & handling techniques, creative statistics

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Communications, information storage & retrieval systems, satellite orbits & automatic programming

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Analysis of tactical situations, employment of future weapon systems, numerical analyses

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detailing consequences of fall out from a nuclear attack

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game theory, linear programming, sequencing & dynamic programming.

Planning of crafts – Strategy & market structure – Competition, Oligopoly & theory of games

Missile testing, tracking & data processing

—

Data handling, communication networks, Combat surveillance, information theory, queueing theory

—

Target analysis, missile feasibility & evaluation

—

Military systems analysis – nuclear & space craft systems – Commercial air transport studies, Industrial O.R

—

Quality control, sampling processes, experimental design & analysis, non-parametric & order statistics; Monte Carlo processes & Markoff processes, reliability evaluation, statistical decision theory

—

Process Engineering, Mathematical, Operational & diagnostic programming

—

Inventory control & production smoothing – dynamic programming

—

Symbolic logic – information handling

—

8. Alexpon Daudig:

$x_2 - 2x_1 \geq 2$

$x_1 - 2x_2 \geq 2$

$x_1 + x_2 \geq 5$

Minimise $x_2 - x_1 = C$

$x_1 \geq 0, x_2 \geq 0$

$x_3, x_4, x_5 \geq 0$

$-2x_1 + x_2 + x_3 = 2$

$x_1 - 2x_2 + x_4 = 2$

$x_1 + x_2 + x_5 = 5$

$C = x_2 - x_1$

$x_1 = 0, x_2 = 0, x_3 = 2, x_4 = 2, x_5 = 5$

(i) $x_3 = 2 + 2x_1 - x_2$

(ii) $x_4 = 2 - x_1 + 2x_2$

(iii) $x_5 = 5 - x_1 - x_2$

Express C in terms of x_1 & x_2.

Increasing x_1 makes decrease C & thus increasing x_1 must not make $x_3, x_4, x_5 < 0$.

We can increase x_1 till $x_1 = 2$ as seen from (ii) (with $x_2 = 0$) & this makes $x_4 = 0$.

2nd Step: $x_4 = 0, x_2 = 0$ & in terms of these

$x_1 = 2 + 2x_2 - x_4$

$x_3 = 2 + 2(2 + 2x_2 - x_4) - x_2 = 6 + 3x_2 - 2x_4$

$x_5 = 5 - (2 + 2x_2 - x_4) - x_2 = 3 - 3x_2 + x_4$

$C = x_2 - (2 + 2x_2 - x_4) = -2 - x_2 + x_4$

Further decrease C by increasing x_2 & limiting increase of x_2 is $x_2 = 1$ & this makes both x_1 & x_3 positive i.e. $x_3 = 9, x_1 = 4$. & makes $x_5 = 0$ i.e. obvious $x_4 = 0, x_5 = 0$.

& expressing x_1, x_2, x_3 in terms of x_4 & x_5

$3x_2 = 3 + x_4 - x_5$

$x_1 = 2 + \frac{2}{3}(3 + x_4 - x_5) - x_4$

$x_3 = 6 + 3 + x_4 - x_5 - 2x_4$

$x_1 = 4 - \frac{x_4}{3} - \frac{2x_5}{3}$

$x_2 = 1 + \frac{x_4}{3} - \frac{x_5}{3}$

$x_3 = 9 - x_4 - x_5$

$C = -2 - \frac{1}{3}(3 + x_4 - x_5) + x_4 = -3 + \frac{2x_4}{3} + \frac{x_5}{3}$

Any further increase of x_4 or x_5 would not decrease C. Hence required soln is $x_4 = 0, x_5 = 0, C = -3$ & hence $x_1 = 4, x_2 = 1, x_3 = 9$.

3/11/59 (1)

(A) ORQ – Vol. 9. No. 1 – March 58

(1) Simulation techniques in OR – Monte Carlo

(2) A Monte Carlo problem in underground communications in British collieries – A method by which to ~~obtain~~ calculate the times to warn each man & clear the colliery by taking into account the various probabilities of obtaining an answer from some of the telephone points

(B) Ibid No. 2 – June 58 – Z. M. Tarkowski – Problems in decision taking – Symposium talk – Although this symposium has been primarily for the benefit of non-specialists & managers unacquainted with O.R, we have heard the following words used – Δ^r & exp. dist, sub-optimisation, Monte Carlo method, sampling error, hydraulic, gaseous & probability models, χ^2-dist & matrix algebra" It might be salutary to remember occasions ~~like this~~ when several doctors in our hearing were discussing our ailments in technical terms. To quote: "– thou hast men about thee that usually talk of a noun and a verb, and such abominable words as no Christian ~~ear~~ ear can endure to hear" (Shakespeare, King Henry VI, Part II, Act IV, Scene VII).

(2) In discussion – "it had already been necessary to explain to those not in the secret, that JORSA is not the Goddess of OR – her demise was already remarked upon")

(3) A paper on location of depots in a petroleum Company

(4) Two courses of ten lectures each during 1958-59 on O.R at the Brunel College of Technology

(C) Ibid No. 4, Dec. 58

(1) Teaching of O.R. by M. G. Kendall – Examines courses available in U.K – Syllabuses of those courses – a possible comprehensive syllabus – Methods of teaching.

(2) OR today & tomorrow by Ant Span – A plea for simplicity, and for depth & breadth of thinking – Methods used in industrial OR – Cybernetics is dismissed in some quarters as airy-fairy nonsense – feedback in cybernetics may suggest that automatic management ~~[illegible]~~ may be round the corner – Symbolic logic not to be dismissed; a magnificent example of the self-evident statement is the couplet by the Scottish poet, William McGonagall

"On yonder hill there stands a coo,
If she's awa she's no there the noo."

Eg. Lewis Carroll's "Exercises in Logic" (Lewis Carroll = Rev. C.L. Dodgson ?) Problem

(1) All the Oxford men in this College play cricket (2) None but the Scholars dine at the high table (3) None of my friends are rowing men (4) My friends in this College all come from Eton (5) All the Scholars are rowing men.

Conclusion — None of my friends dine at a high table (Graphical Solution)



(3) Article by Hawkins — Michael Faraday said that there were steps in useful research; the first to begin it, the second to end it, and the third to publish it — D.G.R: the last is to "Communicate" it.

(D) JORSA Vol 7, No. 1, Jan - Feb, 1959.

(1) The dynamics of military combat by H. Bracken — In this paper the essentials of warfare are given theoretical treatment in a particularly simple way. A theory of warfare a combat dynamics, is derived that is in close agreement with what men has learned about war through his use of it. To the theoretical work of earlier investigators, the present theory adds a reconnaissance ~~effect~~ concept & a weapons concept both being stated in explicit mathematical form. The theoretical dynamics appears to be applicable to warfare in the nuclear age.

(E) Ibid., No. 2, March - April, 59

(1) Some differential games of tactical interest and the value of a supporting weapon system by H.K. Weiss — The synthesis of weapon-system combat is often involves consideration of tactical models in which combat extends over a period of time with choices of tactics available to both sides & subject to change with time. This leads naturally to the formulation of the problems of differential games. A tactic that lends itself to analysis is the determination of which of several targets a weapon should choose. This tactic is incorporated in a combat model that includes a primary & supporting weapon system on each side. Optimum tactics are determined, and the implications on force structure determined upon the weapon range, cost, and parametrically specified performance.

(2) A vulnerability model for weapon sites with interdependent elements by S. D. Firstman

(3) Dist. & [illegible] effects of fallout in large nuclear weapon campaigns by Hugh Everett & G. E. Pugh — Purpose is to provide a simple way of evaluating the consequences of radioactive fallout from a large nuclear weapon campaign without resorting to detailed map studies. —

(4) optimum properties of defence strategy of equal attack against all targets by J. E. Walsh — Math. models of offence - defence interaction often specify that each member of an offence force receives the same attack from the defence. This is generalised here —

(F) Ibid. No. 3, May – June 59

(1) on optimum target assignments by 3 authors — Concerned with two target assignment models & uses a computational necessity

(G) Ibid. Vol. 4, No. 1. Feb. 58

(1) Shockwaves on the Highway

OR (H) June 56 (1) Influence of non-military inspiration has become more noticeable recently as shown by Case History — Commercial — 2, Lin. Prog — 2, Military — 6, Prod. Inv — 2, Statist — 1, Traffic & Queueing — 3, Others — 3

(I), March, 57 (1) Correspondence between Prof J. F. McCloskey (Case Inst of Tech) & A. W. Swan (Constantine Ltd) about name & nature of OR

(J) June 57 — Comparison of O.R. in U.S & U.K by R. L. Ackoff

(K) Sept, 57 (1) Discussion on dynamic instability of transport systems — Lighthill on Hydrodyn. analogy

(2) O.R. in hospital planning

(3) International Conference in OR

(4) The Ballad of Bonnie Dundee — A big poem by A. Battersby very interesting — Last two stanzas specially

(L) Dec, 57 — (1) Correspondence on above poem by A. W. Swan

Sir, May I point out an error in the poem? for 'shortcake' read 'shortbread'

Yours more in sorrow than in anger

A. H. Savage

(2) Sir, "Qu'ils mangent de la brioche" (attrib. Marie Antoinette)

Yrs faithfully

A. Battersby.

JORSA, 1957 – Articles only noted

(1) Bombing problems – a statistical approach

(2) Industrial applns of military O.R. methods

(3) Applns of design of expts & modelling to complex weapons systems

(4) Possible solns to the shortage of scientists

(5) Some civil defence problems in the ~~nations~~ nations Capital following widespread thermonuclear attack

(6) Monte Carlo approach of the ground controller approach system

(7) The theory of search III – by Koopman – optimum distn of searching effort.

(8) Queueing with impatient customers & indifferent clerks

(9) " & ordered service

(10) A tactical air game

(11) A laboratory for combat O.R

(12) Inadequacy of cost per "kill" as a measure of effectiveness

(13) Decision theory & O.R

(14) Evaluating man-machine links in weapon systems analysis.

Bulletin of ORSA, Vol. 7, Suppl. 1, 1959. – Mere enumeration is striking

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