Saturday, 11th January 1958

# IAS NOTES

**Indian Science Congress**

Lovely Madras was the venue of the 45th Session of the Indian Science Congress. The home of Raman and Ramanujam played the host to about two thousand delegates from all over the world who had gathered there to exchange ideas. It is a matter of pride for us that Professor Madhava Rao should have been elected as the President of the Mathematics Section of the Congress. An interesting feature in this Section this year was the symposium on Ballistics in which quite a number of Defence Scientists took part. The Institute was represented at the Symposium by Dr. Patnaik who read a paper on "Meteorological factors."

## SUBSCRIPTIONS

This is just to remind that subscriptions for the CME Weekly News for 1958 are overdue. Subscribers are requested to send their subscriptions for 1958 at an errly date.

## CINEMA NEWS

**HARKIRAT SINGH THEATRE**

Sunday, 12th January 58.

## Summaries of Addresses of Presidents of Sections, Forty-fifth Session, Indian Science Congress

### MODERN ALGEBRA & THEORY OF ELEMENTARY PARTICLES

THE development of modern algebraic concepts such as matrices, differential operators, groups, integral operator, tensors etc., and their increasing application in the solution of problems of theoretical physics formed the subject of the address delivered by Prof. B. S. Madhava Rao to the Section of Mathematics. The progress in theoretical physics as a result of these concepts, Prof. Rao said, has in turn led to the development of new algebraic concepts. The address reviews briefly the recent developments in modern algebra relating to invariance under charge conjugation, generalized gauge transformations, space and time inversions and conservation laws of isotopic spin and parity as applied to the " strange " particles. These developments are based essentially on the representations of the full Lorentz group and their topological significance. Prof. Rao has also referred to the mathematical implications of possible relativistic theories of discrete space-time and momentum-space of function spaces in quantum theory and of the logical foundations of causality.

The increasing use of abstract mathematics has been a striking feature of modern theoretical physics, specially in the field of the theory of elementary particles. This can be traced to a great extent to the change brought about by quantum mechanics in the meaning to be attached to a physical quantity, viz. as an operator yielding not one but a large set of numbers which can, however, be consistently used with the aid of a statistical theory. This widening of the concepts of theoretical physics has stimulated questions which have spurred on the progress of mathematics itself. A clear example of this is the development of the theory of abstract spaces and the knowledge that the analytical problems of quantum mechanics can be thought of in terms of linear transformations in an abstract Hilbert space.

The relationship between algebra and physics has been, until recently, rather a loose one. Although group theory has played a useful role in theoretical physics, it is perhaps true that progress in group theory and modern algebra has not always been chiefly motivated by physics. Researches on the theory of group representations, on abstract algebras and on continuous groups were carried on independently as a result of the fusion of several fundamental ideas in mathematics itself. But recent advances in theory of elementary particles have brought about an intimate connection between algebra and physics which is likely to help the growth of both the subjects.

In the early stages of the development of elementary particles when the Dirac theory of the electron, positron proton and neutron, Pauli's neutrino and Yukawa's meson held the field, it was found possible to set up a general theory which could be used consistently so as to include the above mentioned types of particles and their interactions. This theory consisted essentially of two stages, the c-number theory satisfying the postulates of special relativity, and the q-number theory making a transition from a one-particle to a many particle picture on the basis of quantum mechanics. The former theory was based essentially on the setting up of a Lagrangian function L which is to be invariant under transformation of the proper Lorentz group, i.e. having the determinant $+1$, and not reversing the direction of time. The quantities transforming according to irreducible representations of the above group are the so-called spinors; of the three fundamental properties, the mass, charge and spin of elementary particles, the last which is perhaps not the simplest one from the experimental point of view, has come up, remarkably, as the first in the mathematical formalism. Spin could be easily defined in terms of spinors, and the c-number theory itself yielded some very general results, viz. that for particles of half-integral spin, the total energy is not necessarily positive, and for particles of integral spin, the charge density is not necessarily positive. A deeper analysis showed that the uniqueness and definite positive character of charge and energy density is characteristic for the lower spins 0, $\frac{1}{2}$ and 1. Next, making the transition to the q-number theory, the essential step is the expression of the non-commutivity of the field quantities $\bar{U}(x)$ at different points of space-time $x$ and $x'$ in the form of the $\pm$ bracket expressions, the so-called commutation rules.

$$[\bar{U}(x), \bar{U}^*(x')]^{\pm} = D(x, x') \quad \ldots (1)$$

the $+$ or $-$ being taken according as the particles satisfy the Fermi and Bose statistics. The transformation properties of the $\bar{U}$'s under the proper Lorentz transformation require that the $D$'s also should transform in a certain way. The requirements that $D$ should be a function only of the invariant distance between $x$ and $x'$, and that $D=0$ is each of $x$ and $x'$ lies outside the light cone of the other are stringent enough to determine the form of $D$ uniquely. These considerations led to the well-known connection between spin and statistics, which is perhaps one of the most important applications of the theory of representations of the Lorentz Group.

A further development was to put the wave equations of the particles in the Dirac form

$$\frac{\partial}{\partial x^{\kappa}} \beta_{\kappa} \Psi + \chi \Psi = 0 \quad \ldots\ldots (2)$$

representing particles of a single spin, and this form in which Dirac originally derived the wave equation of the electron and which was the basis of relativistic quantum mechanics, can be said to represent the particle aspect of the theory of elementary particles. The $\beta_k'^s$ in equation (2) are matrices satisfying different types of commutation relations depending on the spin, and generate in virtue of these relations, a system of hypercomplex numbers. The theorems relating to representations of finite groups can be extended, to group rings and hence also to hypercomplex numbers satisfying certain conditions. Thus an intimate connection could be set up between particles of specified spin and general types of algebras, e.g. the Dirac algebra, Meson algebra and so on. This also showed the possibility of several representations, with different orders, of the same algebra being equivalent to different types of elementary particles having the same spin.

The limiting case of particles of rest mass zero could be brought into the general theory by the further requirement that the field quantities representing them be invarient under the gauge group. Similarly the other limiting case of particles of zero charge could also be treated satisfactorily by postulating invariance under transformations of charge/conjugation or the so-called charge invariance.

Recent experimental work in high energy physics has resulted in the discovery of a large number of elementary particles, called the 'strange particles', like heavy mesons and hyperons of masses greater than that of the proton; the interactions between themselves, and between them and the known elementary particles, and also the decay of the strange particles are now of great interest. This has led to a re-examination of the questions of invariance under several types of Lorentz transformations like those confined to space-time displacements and rotations only, and those which are not. Questions relating to invariance under charge conjugation, generalized gauge transformations and space and time inversions, and conservation laws of isotopic spin and parity as applied to the strange particles have all raised problems of great algebraic significance. A new quantum number called the "strangeness quantum number "(S)" has been introduced to explain interactions among the new particles, and the relation between invariance of strangeness and invariance of parity (i.e. distinction between right and left) has been investigated. A particular problem of current interest is the question about the conservation laws in two types of interactions, viz. the strong ones including the electromagnetic interactions, and the weak ones like the decay interactions of mesons and hyperons and various Fermi interactions, specially the question of invariance of parity in these two types. Thus while parity is conserved in strong interactions, the same cannot be said of weak interactions, and the question of invariance of parity under these has recently been examined by Lee and Yang with reference to the heavy mesons $\theta^+$ and $\tau^+$ which have very close masses and they have come to the conclusion that they have the same spin but opposite parities, i.e. they are parity doublets. Thus one could speak of parity conjugation in relation to these two paricles. It appears that we have here a new conservation of law of physics, viz. that all particles of odd strangeness have parity doublets. Also, there does not exist at present experimental proof of the invariance of weak interactions under charge conjugation. A confirmation of the above conservation law is afforded by the hyperons, $\Lambda$ and $\Sigma$, which have the same spin $\frac{1}{2}$ and the same parity, the value of S assigned being even.