Topology

Lecture delivered at the Maharaja's College, Mysore

on Thursday 12th Sept 1940, at 6. P. M.

(1) Introduction:

Mr President & gentlemen

~~I want to speak to you this evening about one of the m~~

I want to speak to you this evening in a popular way about one of the most important topics of modern pure mathematics. I shall first say something about it without telling you what it ~~is~~ actually deals with. The subject might be said to have originated in 1736 with Euler, but ~~the~~ the official name of topology was given to it in 1847 by Listing. Like any mighty river its origin ~~was in~~ is very ~~humble surroundings like pebbles~~, inconspicuous the earliest problems it dealt with being such frivolous topics like tracing of labyrinths, traversing of bridges, colouring maps and similar recreational topics. It was only at the turn of the present century that Poincaré grasped its potentialities and placed the subject in its proper perspective with respect to other branches of mathematics. Since then topology has flourished as no other branch has done, and ~~is threatening~~ to research in the subject is proceeding at a tremendous pace. If you take any math. journal of some standing the odds are ten to one against not finding an article or contribution to topology. It was in Sept 1937 that an Internation Conference on topology was held at Moscow attend~~ing~~ by the leading topologists of the world at which the present position of the subject and the possible lines of its future development were discussed and taken stock of. This is an indication that the subject has attained maturity and is still in growth and flux influencing the other branches of mathematics. In fact a prominent topologist once declared that a cardinal principle of modern mathematical research may be stated as a maxim "one must always topologise". Another enthusiast has made the assertion that roughly speaking all mathematics could be divided into algebra and topology, the symbols so to say of the discontinuous and continuous entities. Topology has been a great unifying force in mathematics connecting as it does several branches like group theory, analysis and geometry. Another most interesting feature of topological research which is becoming evident in the last five years and to which I want to call your special attention, is the fact that of all the branches of mathematics it ~~is~~ appears to be the one best suited for applications to the social sciences. I have hardly seen any applications of it to physics except perhaps a single article by Rosenthal on the kinetic theory of gases. But, there is already a branch of psychology called "topological psychology" and Bertrand Russel in his book on "The analysis of matter" has applied it to philosophy. Mathematical logic, probability theory ~~and~~, statistics and mathematical economics have all problems very amenable to topological methods.

What is topology! most difficult thing from layman's point of view or eminent men in other fields – topography & toposheets – (top – ology) – Defn as properties of sets & invariant under continuous one-one transformation (p. 600 of Dashiell) is meaningless to a non-math – idea of continuous, homeomorphism in abstract spaces makes it still obscure (show deformation p. 600 on rubber sheet) – rubber-sheet geometry – Δ^{le}, $\odot^{le}$, ellipse, polygon are equivalent topologically – hunt for invariant – crumpled map ~~straightening~~ to pull ~~your leg~~ – ~~recreational methods~~ – simple curve – arc – Peano curves – dimension – closedness – connectedness – pulling your leg – recreational

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(2) Graphs – Bridge problem – Mahomet's signature – labyrinths.

(3) Water-gas-electricity problem – ~~[illegible]~~ Jordan's theorem –

~~Kuratowski's theorem~~ – Do on torus (Show torus) – Kuratowski's theorem

(4) Regular polyhedra – Betti number – ~~Sphere & torus~~ plane regions also – Sphere – Torus – connectivity – Hamilton's problem (vertex traversing)

(5) Four colour problem – Figures – ~~cubic~~ five colours – cubic map – 32 regions ~~x~~ torus map – regular polyhedra on sphere – Equivalence of plane & sphere

(6) Mobius strip – Orientability & two sidedness – Sphere and torus – 180° twist is one sided – 360° (2 turns) is 2 sided but edge not knotted (same as 180° cut up) – This further cut forms 2 interwoven strips – 540° unilateral (~~not~~ like mobius strip) but knotted edge. Is there a bilateral surface with knotted edge – Fig – topological invariants of higher manifolds – answer by math. logic

(7) Knots – ~~topological invariant~~ none in 2 dimensions (2nd dim) – cutting the Gordian knot – Pannwitz invariant – Show table – Dehn's lemma – Connectivity or genus – Relns (least possible) between these invariants – knot to be or knot not to be –

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(8) Point set topology – topological space – space – Klein's Erlanger programme – continuum – ∞ dim spaces – Hilbert spaces – metric spaces – separable – compact – complete – imbedding – simple plane curve as knot in E_3 – Menger – Nöbeling compact metric sp in E_{2n+1} –

(9) Applications – Psychology problem – p. 600.

(1) Konigsberger Bridge problem ✓

(2). Labyrinth ✓

(3) Mahomet's signature ✓

(4) maid's challenge ✓

(Mypore)

(5) ~~Hat~~ Dodecahedron ✓ – Jumping model (?) – Regular solids (?) 4 sphere ✓

(6) Rubber sheet. ✓

(7) 4-color problem sheet ✓

(8) Torus ✓

(9) Mobius bands ✓

(10) Knots ✓

(11) Reidemeister's book "Knotentheorie". ✓

(12) ~~Rosenthe~~ Bertrand Russell's book "Analysis of matter"

(13) Topology of psychology.

(14) ~~A torn~~ A map (Mypore)

(15) Rubber torus (")