The Condition of Spherical Symmetry

Γ_{ik}

as applied to Γ^{l}_{ik} gives (Rao Proc. Nat. Inst. Sc India $\underline{19}$, 722, 1953)

$^{l}_{ik}$ $\Gamma^{\sigma}_{ik}\xi^{l}_{,\sigma} - \Gamma^{l}_{i\sigma}\xi^{\sigma}_{,k} - \Gamma^{l}_{\sigma k}\xi^{\sigma}_{,i} - \Gamma^{l}_{ik,\sigma}\xi^{\sigma} - \xi^{l}_{,ik} = 0$

$\left(\begin{smallmatrix}1\\11\end{smallmatrix}\right)$ $\Gamma^{1}_{11,\sigma}\xi^{\sigma} = 0$; $\left(\begin{smallmatrix}2\\11\end{smallmatrix}\right)$ $\Gamma^{\sigma}_{11}\xi^{2}_{,\sigma} - \Gamma^{2}_{11,\sigma}\xi^{\sigma} = 0$ $\Big\|$ ∴ $\Gamma^{2}_{11} = 0,\ \Gamma^{3}_{11} = 0$

$x \longrightarrow y.$

$\left(\begin{smallmatrix}3\\11\end{smallmatrix}\right)$ $\Gamma^{\sigma}_{11}\xi^{3}_{,\sigma} - \Gamma^{3}_{11,\sigma}\xi^{\sigma} = 0$

$\left(\begin{smallmatrix}4\\11\end{smallmatrix}\right)$ $\Gamma^{4}_{11,\sigma}\xi^{\sigma} = 0$

$\left(\begin{smallmatrix}1\\12\end{smallmatrix}\right)$ $\cancel{\Gamma^{\sigma}_{12}} - \Gamma^{1}_{1\sigma}\xi^{\sigma}_{,2} - \Gamma^{1}_{12,\sigma}\xi^{\sigma} = 0$ ∴ $-\Gamma^{1}_{12}\xi^{2}_{,2} - \Gamma^{1}_{13}\xi^{3}_{,2} - \Gamma^{1}_{12,2}\xi^{2} - \Gamma^{1}_{12,3}\xi^{3} = 0$

$\left(\begin{smallmatrix}1\\13\end{smallmatrix}\right)$ $-\Gamma^{1}_{1\sigma}\xi^{\sigma}_{,3} - \Gamma^{1}_{13,\sigma}\xi^{\sigma} = 0$ ∴ $-\Gamma^{1}_{12}\xi^{3}_{,2} - \Gamma^{1}_{13}\xi^{3}_{,3} - \Gamma^{1}_{13,2}\xi^{2} - \Gamma^{3}_{13,3}\xi^{3} = 0$

Two equations of this type: $\Gamma^{4}_{42} = \Gamma^{1}_{12} = x \quad \Gamma^{1}_{13} = y = \Gamma^{4}_{43}$

$$\left.\begin{aligned} x\xi^{2}_{,2} + y\xi^{3}_{,2} + x_{,2}\xi^{2} + \cancel{x}_{3}\xi^{3} = 0 \\ x\xi^{3}_{,2} + y\xi^{3}_{,3} + y_{,2}\xi^{2} + y_{,3}\xi^{3} = 0 \end{aligned}\right\}\sigma$$

$\cancel{\Gamma^{1}_{21}} =$

$\cancel{\Gamma^{1}_{13}} = x$

$\left(\begin{smallmatrix}1\\21\end{smallmatrix}\right)$ $\cancel{\Gamma^{1}_{2\sigma}\xi^{\sigma}} + \Gamma^{1}_{\sigma 1}\xi^{\sigma}_{,2} + \Gamma^{1}_{21,\sigma}\xi^{\sigma} = 0$ $\quad \Gamma^{4}_{24} = \Gamma^{1}_{21} = x \quad \Gamma^{1}_{31} = y = \Gamma^{4}_{34}$

$\left(\begin{smallmatrix}1\\31\end{smallmatrix}\right)$

$\left(\begin{smallmatrix}2\\24\end{smallmatrix}\right)$ $\Gamma^{\sigma}_{24}\xi^{2}_{,\sigma} - \cancel{\Gamma^{2}_{2\sigma}\xi^{\sigma}_{,4}} - \Gamma^{2}_{\sigma 4}\xi^{\sigma}_{,2} - \Gamma^{2}_{24,\sigma}\xi^{\sigma} - \xi^{2}_{,42} = 0$

$\cancel{\Gamma^{2}_{24}\xi^{2}_{,\sigma}} + \Gamma^{3}_{24}\xi^{2}_{,3} - \cancel{\Gamma^{2}_{24}\xi^{2}_{,\sigma}} - \Gamma^{2}_{34}\xi^{3}_{,2} - \Gamma^{2}_{24,2}\xi^{2} - \Gamma^{2}_{24,3}\xi^{3} = 0$

$\left(\begin{smallmatrix}3\\34\end{smallmatrix}\right)$ $\Gamma^{\sigma}_{34}\xi^{3}_{,\sigma} - \cancel{\Gamma^{3}_{3\sigma}\xi^{\sigma}_{,4}} - \Gamma^{3}_{\sigma 4}\xi^{\sigma}_{,3} - \Gamma^{3}_{34,\sigma}\xi^{\sigma} - \xi^{3}_{,34} = 0$

$\Gamma^{2}_{34}\xi^{3}_{,2} + \cancel{\Gamma^{3}_{34}\xi^{3}_{,3}} - \cancel{\Lambda_{32}} - \cancel{\Gamma^{3}_{34}\xi^{3}_{,3}} - \Gamma^{3}_{24}\xi^{2}_{,3} - \Gamma^{3}_{34,\sigma}\xi^{\sigma} = 0$

$g_{24}\xi^{2}_{,3} + g_{34}\xi^{3}_{,3} + g_{34,\sigma}\xi^{\sigma} = 0$

$\cancel{g}$. g_{34} : $\cancel{g_{3\sigma}\xi^{\sigma}_{,4}} + g_{\sigma 4}\xi^{\sigma}_{,3} + g_{34,\sigma}\xi^{\sigma} = 0$ $\quad$ ~~$g_{24}\xi^{3}_{,2} + g_{34}\xi^{3}_{,3} + g_{34,\sigma}\xi^{\sigma} = 0$~~

g_{24} : $\cancel{g_{2\sigma}\xi^{\sigma}_{,4}} + g_{\sigma 4}\xi^{\sigma}_{,2} + g_{24,\sigma}\xi^{\sigma} = 0$ $\quad g_{24}\xi^{2}_{,2} + g_{34}\xi^{3}_{,2} + g_{24,\sigma}\xi^{\sigma} = 0$

$\underline{A_{i,\sigma}\xi^{\sigma} = 0} + A_{\sigma}\xi^{\sigma}_{,i} = 0$ $\quad A_{2,\sigma}\xi^{\sigma} + A_{\sigma}\xi^{\sigma}_{,2} = 0$

$A_{3,\sigma}\xi^{\sigma} + A_{\sigma}\xi^{\sigma}_{,3} = 0$

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$$g_{ik} = \begin{pmatrix} 0 & 0 & +p\sin\theta & a \\ 0 & -\beta & +f\sin\theta & 0 \\ -p\sin\theta & -f\sin\theta & -\beta\sin^2\theta & +q\sin\theta \\ a & 0 & +q\sin\theta & 0 \end{pmatrix} \quad \therefore g = p\sin\theta\left[\beta(aq\sin\theta)\right] - a\left[\beta(-pq + a\beta)\sin^2\theta + f\sin\theta\left[af\sin\theta\right]\right]$$

$$\therefore g = -a\left[a\beta^2 - 2\beta pq + af^2\right] \quad \cancel{g^{11}=} \; g^{12} = \frac{+f\sin\theta\, aq\sin\theta}{(\;)}$$

$$g^{1\underset{\vee}{2}} = 0 \qquad g^{14} = 0 \qquad g^{13} = \frac{\beta aq\sin\theta}{-a(a\beta^2 - 2\beta pq + af^2)\sin^2\theta}$$

etc. etc.

$$\cancel{g_{11}\Gamma^1_{11}} + g_{\underline{14}}\Gamma^4_{11} = +g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{11}} + g_{\underset{\vee}{3}1}\Gamma^3_{\underset{\vee}{11}} = 0 \qquad \therefore \Gamma^4_{11} = 0.$$

$$g_{22}\Gamma^2_{11} = g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{21}} + g_{\underset{\vee}{3}1}\Gamma^3_{\underset{\vee}{12}} = 2g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{21}} \qquad \cancel{g_{44}\Gamma^4_{11} + g_{41}}$$

$$g_{33}\Gamma^3_{\underset{\vee}{21}} = -\tfrac{1}{2}g_{1\underset{\vee}{3},2} - \tfrac{1}{2}g_{3\underset{\vee}{2},1} - g_{2\underset{\vee}{3}}\Gamma^3_{\underline{31}} - g_{\underset{\vee}{3}1}\Gamma^3_{\underline{23}}.$$

$$g_{33}\Gamma^3_{\underline{31}} = \tfrac{1}{2}g_{33,1} + g_{\underset{\vee}{3}1}\Gamma^1_{\underset{\vee}{31}} + \cancel{g_{31}\Gamma^3_{33}}$$

$$g_{33}\Gamma^3_{\underline{32}} = \tfrac{1}{2}g_{33,2} + g_{\underset{\vee}{3}1}\Gamma^1_{\underset{\vee}{32}} + \cancel{g_{32}\Gamma^3_{33}}$$

& 1, 2, 3, 4 all will attain perfect symmetry if we put $g_{11} \neq 0$, $g_{44} \neq 0$, $g_{\underline{14}} = 0$

∴ We take

$$g_{ik} = \begin{pmatrix} -\alpha & 0 & p\sin\theta & 0 \\ 0 & -\beta & f\sin\theta & 0 \\ -p\sin\theta & -f\sin\theta & -\beta\sin^2\theta & q\sin\theta \\ 0 & 0 & q\sin\theta & \gamma \end{pmatrix} \quad g = -\alpha\left[-\beta(-\beta\gamma + q^2) - f(-f\gamma)\right] + p\left[\beta(-p\gamma)\right]\sin^2\theta$$

$$= \left(-\alpha\beta^2\gamma + \alpha q^2\beta - \alpha f^2\gamma - \gamma\beta p^2\right)\sin^2\theta = -\left[\alpha\gamma(\beta^2 + f^2) + \beta(\alpha q^2 - \gamma p^2)\right]\sin^2\theta$$

$$g_{11}\Gamma^1_{11} = g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{11}} + g_{\underset{\vee}{3}1}\Gamma^3_{\underset{\vee}{11}} \cancel{=0} \; \tfrac{1}{2}g_{11,1} \qquad \therefore \Gamma^1_{11} = \frac{\alpha'}{2\alpha}.$$

$$g_{22}\Gamma^2_{11} = g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{21}} + g_{\underset{\vee}{3}1}\Gamma^3_{\underset{\vee}{12}} = 2g_{1\underset{\vee}{3}}\Gamma^3_{\underset{\vee}{21}}. \qquad \cancel{g_{33}\Gamma^3_{21} = -\tfrac{1}{2}g_{13,2} - g_{23}\Gamma^3_{31}}$$

$$g_{33}\Gamma^3_{\underset{\vee}{21}} = -\tfrac{1}{2}g_{1\underset{\vee}{3},2} - \tfrac{1}{2}g_{3\underset{\vee}{2},1} -$$

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We have seen that either $g_{\underset{\vee}{14}} = 0$ or $g_{\underset{\vee}{23}} = 0$. We put $g_{\underset{\vee}{23}} = 0$ and found that a soln. does not exist. ~~Has~~ We put $g_{\underset{\vee}{14}} = 0$ $g_{\underset{\vee}{23}} = 0$ and then also found that the solution did not exist. So we now take the last case $g_{\underset{\vee}{14}} = 0$.

$$g_{\underset{\vee}{23}} = f\sin\theta \neq 0$$

We know that we can make ~~$g_{13} = 0$~~ $g_{12} = 0$ $g_{24} = 0$. Also (r, t) can be chosen to make either $g_{13} = 0$ or $g_{34} = 0$. With it we can remove $g_{\underline{14}}$ if $g_{11}(g_{34})^2 + g_{44}(g_{\underset{\vee}{13}})^2 + 2g_{\underline{14}}\, g_{\underset{\vee}{13}}\, g_{\underset{\vee}{34}} \neq 0$. If however the last expression is zero then with the vanishing of g_{13} or g_{34} g_{44} or g_{11} automatically vanishes and the ~~$g_{\underline{\mu\nu}}$ gets considerably simplified~~ and the other of these two can be chosen to vanish.

We shall $\therefore$ first take $g_{\underline{14}} = 0$.

$$g_{\mu\nu} = \begin{pmatrix} -\alpha & 0 & p\sin\theta & 0 \\ 0 & -\beta & +f\sin\theta & 0 \\ \cancel{p\sin\theta} & \cancel{f\sin\theta} & -p\sin^2\theta & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$

$$\therefore g = \gamma\left[-\alpha(\beta^2 + f^2)\sin^2\theta + p\sin\theta(-\beta p)\sin\theta\right]$$

$$= -\gamma\left[\alpha(\beta^2 + f^2) + \beta p^2\right]\sin^2\theta$$

$$g^{\underset{\vee}{12}} = \frac{fp\gamma}{\quad} = 0. \qquad g^{13} = \frac{-\beta p\gamma\sin\theta}{\cancel{\ }-\gamma[\quad]\sin^2\theta}$$

$$g^{14} = 0$$

444 $\therefore \boxed{\Gamma^{4}_{44} = \frac{\dot\gamma}{2\gamma}}$ (1) ~~441 $g_{11}\Gamma^{1}_{44} = -\frac{1}{2}g$~~

$g_{44}\Gamma^{4}_{44} = \frac{1}{2}g_{44,4}$

441. $g_{11}\Gamma^{1}_{44} = -\frac{1}{2}g_{44,1}$ $\therefore \boxed{\Gamma^{1}_{44} = \frac{\gamma'}{2\alpha}}$ (2)

$g_{44}\Gamma^{4}_{\underline{14}} = \frac{1}{2}g_{44,1} + g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{44}}$ $\therefore \boxed{\Gamma^{4}_{\underline{14}} = \frac{\gamma'}{2\gamma}}$ (3) $g_{44}\Gamma^{4}_{\underset{\vee}{14}} = -g_{1\underset{\vee}{3}}\Gamma^{3}_{44}$.

$g_{33}\Gamma^{3}_{44} = 0$ $\therefore \boxed{\Gamma^{4}_{\underset{\vee}{14}} = 0,\ \Gamma^{3}_{44} = 0}$ (5) $g_{22}\Gamma^{2}_{44} = 0$ $\boxed{\Gamma^{2}_{44} = 0}$ (6)

$g_{44}\Gamma^{4}_{\underline{24}} = g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{44}} = 0$ $\boxed{\Gamma^{4}_{\underline{24}} = 0}$ (7) $g_{44}\Gamma^{4}_{\underset{\vee}{24}} = -g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{44}} = 0$ $\boxed{\Gamma^{4}_{\underset{\vee}{24}} = 0}$ (8)

$g_{44}\Gamma^{4}_{\underline{34}} = g_{\underset{\vee}{31}}\Gamma^{1}_{\underset{\vee}{44}} + g_{3\underset{\vee}{2}}\Gamma^{2}_{\underset{\vee}{44}} = \overset{0}{\cancel{-f\sin\theta\frac{\gamma'}{2\alpha}}}$ $\therefore \boxed{\Gamma^{4}_{\underline{34}} = 0\cancel{\frac{\text{ϐ}}{\alpha}\frac{\gamma'}{2\gamma}\sin\theta}}$ (9)

$g_{44}\Gamma^{4}_{\underset{\vee}{34}} = -g_{\underset{\vee}{31}}\Gamma^{1}_{\underline{44}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{44}}$ $\therefore \boxed{\Gamma^{4}_{\underset{\vee}{34}} = \frac{\text{ϐ}}{\alpha}\frac{\gamma'}{2\gamma}\sin\theta}$ (10)

$g_{11}\Gamma^{1}_{\underline{14}} = \frac{1}{2}g_{11,4} + g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{14}}$;(11) $g_{33}\Gamma^{3}_{\underset{\vee}{14}} = -\frac{1}{2}g_{\underset{\vee}{31},4} - g_{1\underset{\vee}{3}}\Gamma^{3}_{\underline{34}}$ (12)

$g_{33}\Gamma^{3}_{\underline{34}} = \frac{1}{2}g_{33,4} + g_{\underset{\vee}{31}}\Gamma^{1}_{34} + g_{3\underset{\vee}{2}}\Gamma^{2}_{\underset{\vee}{34}}$;(13) $g_{11}\Gamma^{1}_{\underset{\vee}{34}} = -\frac{1}{2}g_{1\underset{\vee}{3},4} - g_{\underset{\vee}{31}}\Gamma^{1}_{\underline{14}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{14}}$ (14)

$g_{22}\Gamma^{2}_{\underset{\vee}{34}} = -\frac{1}{2}g_{2\underset{\vee}{3},4} - g_{\underset{\vee}{31}}\Gamma^{1}_{\underline{24}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{24}}$ (15); $g_{22}\Gamma^{2}_{\underline{14}} = g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{24}}$. (16)

$g_{11}\Gamma^{1}_{\underline{24}} = g_{2\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{14}}$ (17). $g_{22}\Gamma^{2}_{\underline{24}} = \frac{1}{2}g_{22,4} + g_{2\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{24}}$, (18)

$g_{33}\Gamma^{3}_{\underset{\vee}{24}} = -\frac{1}{2}g_{\underset{\vee}{32},4} - g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{34}}$. (19)

$-\alpha\Gamma^{1}_{\underline{14}} + \frac{\dot\alpha}{2} = \text{ϐ}\Gamma^{3}_{\underset{\vee}{14}}$ An ~~index~~ symbol with an upper 3 ~~will have~~
~~to be taken as if it has been~~ stands for the original
[illegible] lower 3 is cosec θ x [illegible].
symbol into sinθ. * [illegible]

$\therefore \Gamma^{1}_{\underline{14}} = \frac{\dot\alpha}{2\alpha} - \frac{\text{ϐ}}{\alpha}\Gamma^{3}_{\underset{\vee}{14}}$. $-\beta\Gamma^{3}_{\underset{\vee}{14}} = \frac{\dot{\text{ϐ}}}{2} - \text{ϐ}\Gamma^{3}_{\underline{34}}$ $\therefore \Gamma^{3}_{\underline{34}} = \frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \frac{\beta}{\text{ϐ}}\Gamma^{3}_{\underset{\vee}{14}}$.

$\therefore -\beta\Gamma^{3}_{\underset{\vee}{24}} = \frac{\dot f}{2} - f\Gamma^{3}_{\underline{34}}$ $\therefore -\beta\Gamma^{3}_{\underset{\vee}{24}} = \frac{\dot f}{2} - f\left[\frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \frac{\beta}{\text{ϐ}}\Gamma^{3}_{\underset{\vee}{14}}\right]$

$\therefore \Gamma^{3}_{\underset{\vee}{24}} = -\frac{\dot f}{2\beta} + \frac{f}{\beta}\frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \frac{f}{\text{ϐ}}\Gamma^{3}_{\underset{\vee}{14}}$. $-\beta\Gamma^{2}_{\underline{24}} = -\frac{\dot\beta}{2} + f\Gamma^{3}_{\underset{\vee}{24}}$

$\therefore \Gamma^{2}_{\underline{24}} = \frac{\dot\beta}{2\beta} - \frac{f}{\beta}\left[-\frac{\dot f}{2\beta} + \frac{f}{\beta}\frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \frac{f}{\text{ϐ}}\Gamma^{3}_{\underset{\vee}{14}}\right]$.

$= \frac{\dot\beta}{2\beta} + \frac{f\dot f}{2\beta^{2}} - \frac{f^{2}}{\beta^{2}}\frac{\dot{\text{ϐ}}}{2\text{ϐ}} - \frac{f^{2}}{\text{ϐ}\beta}\Gamma^{3}_{\underset{\vee}{14}}$.

$\Gamma^{1}_{\underline{14}} + \Gamma^{2}_{\underline{24}} + \Gamma^{3}_{\underline{34}} + \Gamma^{4}_{44} = \frac{\dot\gamma}{2\gamma} + \frac{\dot A}{2A}$. $A = \alpha(\beta^{2} + f^{2}) + \beta\text{ϐ}^{2}$

$\frac{\dot\alpha}{2\alpha} + \frac{\dot\beta}{2\beta} + \frac{f\dot f}{2\beta^{2}} - \frac{f^{2}}{\beta^{2}}\frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \frac{\dot{\text{ϐ}}}{2\text{ϐ}} + \Gamma^{3}_{\underset{\vee}{14}}\cancel{\left\{-\frac{\text{ϐ}}{\alpha}\right.} + \frac{\dot\gamma}{2\gamma}$

$+\Gamma^{3}_{\underset{\vee}{14}}\left\{-\frac{\text{ϐ}}{\alpha} + \frac{\beta}{\text{ϐ}} - \frac{f^{2}}{\text{ϐ}\beta}\right\} = \frac{\dot\gamma}{2\gamma} + \frac{\dot A}{2A}$

$\therefore \Gamma^{3}_{\underset{\vee}{14}} =$ – – These nine equations are then easily solved.

$g_{11}\Gamma^{1}_{14} = -g_{13}\Gamma^{3}_{\underline{14}}$; $g_{33}\Gamma^{3}_{\underline{14}} = g_{13}\Gamma^{3}_{34}$.

$g_{33}\Gamma^{3}_{34} = -g_{31}\Gamma^{1}_{\underline{34}} - g_{32}\Gamma^{2}_{\underline{34}}$; $g_{11}\Gamma^{1}_{\underline{34}} = g_{31}\Gamma^{1}_{14} + g_{32}\Gamma^{2}_{14}$

$g_{22}\Gamma^{2}_{\underline{34}} = g_{31}\Gamma^{1}_{24} + g_{32}\Gamma^{2}_{24}$; $g_{22}\Gamma^{2}_{14} = -g_{13}\Gamma^{3}_{\underline{24}}$

$g_{11}\Gamma^{1}_{24} = -g_{23}\Gamma^{3}_{\underline{14}}$; $g_{22}\Gamma^{2}_{24} = -g_{23}\Gamma^{3}_{\underline{24}}$

$g_{33}\Gamma^{3}_{\underline{24}} = g_{23}\Gamma^{3}_{34}$.

$$\therefore \boxed{\begin{gathered}\Gamma^{1}_{14} = 0,\ \Gamma^{2}_{24} = 0,\ \Gamma^{3}_{34} = 0,\ \Gamma^{3}_{\underline{14}} = 0,\ \Gamma^{1}_{\underline{34}} = 0,\ \Gamma^{2}_{\underline{34}} = 0.\\ \Gamma^{2}_{14} = 0,\ \Gamma^{1}_{24} = 0,\ \Gamma^{3}_{\underline{24}} = 0\end{gathered}} \quad (28)$$

All symbols involvg index 4 have are known

$g_{33}\Gamma^{3}_{11} = g_{13}\Gamma^{3}_{31} + g_{31}\Gamma^{3}_{13}$; $g_{33}\Gamma^{3}_{31} = -g_{31}\Gamma^{1}_{\underline{31}} - g_{32}\Gamma^{2}_{\underline{31}} - g_{31}\Gamma^{3}_{33}$.

$g_{11}\Gamma^{1}_{\underline{31}} = \cancel{g_{31}\Gamma^{1}_{11}} + \cancel{g_{32}\Gamma^{2}_{11}} + g_{31}\Gamma^{3}_{31}$; ~~g_{22}~~ $g_{22}\Gamma^{2}_{\underline{31}} = g_{31}\Gamma^{1}_{21} + g_{32}\Gamma^{2}_{21} + \overset{g_{31}\Gamma^{3}_{23}}{\cancel{g_{31}\Gamma^{3}_{33}}}$

$g_{33}\Gamma^{3}_{33} = \cancel{g_{31}\Gamma^{1}_{33}} + \cancel{g_{32}\Gamma^{2}_{33}} + \cancel{g_{13}\Gamma^{1}_{33}} + \cancel{g_{23}\Gamma^{2}_{33}} = 0$

$g_{11}\Gamma^{1}_{21} = -g_{23}\Gamma^{3}_{\underline{11}}$ ~~$g_{22}\Gamma^{2}_{21} = -$~~ $-g_{31}\Gamma^{3}_{\underline{21}}$

$g_{22}\Gamma^{2}_{21} = -g_{23}\Gamma^{3}_{\underline{21}} - g_{31}\Gamma^{3}_{22}$; $g_{33}\Gamma^{3}_{\underline{32}} = -g_{31}\Gamma^{1}_{\underline{32}} - g_{32}\Gamma^{2}_{\underline{32}} - g_{32}\Gamma^{3}_{\underline{33}}$

$g_{33}\Gamma^{3}_{\underline{21}} = +g_{23}\Gamma^{3}_{31} + g_{31}\Gamma^{3}_{23}$; $g_{33}\Gamma^{3}_{22} = g_{23}\Gamma^{3}_{32} + g_{32}\Gamma^{3}_{23}$.

$$\therefore \boxed{\begin{gathered}\Gamma^{3}_{11} = 0,\ \Gamma^{1}_{\underline{31}} = 0,\ \Gamma^{2}_{\underline{31}} = 0,\ \Gamma^{3}_{33} = 0,\ \Gamma^{3}_{\underline{21}} = 0,\ \Gamma^{3}_{22} = 0\\ \Gamma^{3}_{13} = 0,\ \Gamma^{1}_{21} = 0\ \ \Gamma^{2}_{12} = 0,\ \Gamma^{3}_{23} = 0\end{gathered}} \quad (38)$$

Since $\Gamma^{3}_{\underline{33}} = 0$ and $\Gamma^{1}_{\underline{31}} = \Gamma^{3}_{33} = \Gamma^{4}_{\underline{34}} = 0$ $\therefore \boxed{\Gamma^{2}_{\underline{32}} = 0}$ (39).

$g_{11}\Gamma^{1}_{31} = \frac{1}{2}g_{31,1} - \frac{1}{2}g_{13,1} - g_{32}\Gamma^{2}_{\underline{11}} - g_{31}\Gamma^{1}_{11} - g_{31}\Gamma^{3}_{\underline{31}}$.

$g_{22}\Gamma^{2}_{11} = g_{13}\Gamma^{3}_{21} + g_{31}\Gamma^{3}_{12}$; $g_{11}\Gamma^{1}_{11} = \frac{1}{2}g_{11,1} + \cancel{g_{13}\Gamma^{3}_{11}}$

$g_{33}\Gamma^{3}_{\underline{31}} = \frac{1}{2}g_{33,1} + g_{31}\Gamma^{1}_{31} + g_{32}\Gamma^{2}_{31} + \cancel{g_{31}\Gamma^{3}_{33}}$

$g_{33}\Gamma^{3}_{21} = -\frac{1}{2}g_{13,2} - \frac{1}{2}g_{32,1} - g_{23}\Gamma^{3}_{\underline{31}} - g_{31}\Gamma^{3}_{\underline{23}}$

$g_{22}\Gamma^{2}_{31} = \frac{1}{2}g_{31,2} - \frac{1}{2}g_{23,1} - g_{31}\Gamma^{1}_{\underline{21}} - g_{32}\Gamma^{2}_{\underline{21}} - g_{31}\Gamma^{3}_{\underline{32}}$

$g_{33}\Gamma^{3}_{\underline{32}} = \frac{1}{2}g_{33,2} + g_{31}\Gamma^{1}_{32} + g_{32}\Gamma^{2}_{32} + \cancel{g_{32}\Gamma^{3}_{33}}$

$g_{11}\Gamma^{1}_{\underline{21}} = \cancel{g_{23}\Gamma^{3}_{11}}$ ~~=~~ $+\ g_{31}\Gamma^{3}_{21}$

$$g_{11}\Gamma^{1}_{3\underset{\vee}{2}} = \tfrac{1}{2} g_{3\underset{\vee}{2},1} - \tfrac{1}{2} g_{1\underset{\vee}{3},2} - g_{3\underset{\vee}{1}}\Gamma^{1}_{\underline{\underline{12}}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{1\underline{2}} - g_{3\underset{\vee}{2}}\Gamma^{3}_{\underline{31}}.$$

$$g_{22}\Gamma^{2}_{2\underline{1}} = \tfrac{1}{2} g_{22,1} + g_{2\underset{\vee}{3}}\Gamma^{3}_{2\underset{\vee}{1}} + \cancel{g_{3\underset{\vee}{1}}\Gamma^{3}_{2\underset{\vee}{2}}}$$

$$g_{22}\Gamma^{2}_{3\underset{\vee}{2}} = +\tfrac{1}{2} g_{3\underset{\vee}{2},2} - \tfrac{1}{2} g_{2\underset{\vee}{3},2} - g_{3\underset{\vee}{1}}\Gamma^{1}_{22} - g_{3\underset{\vee}{2}}\Gamma^{2}_{22} - g_{3\underset{\vee}{2}}\Gamma^{3}_{\underline{32}}$$

$$g_{11}\Gamma^{1}_{22} = -\tfrac{1}{2} g_{22,1} + g_{2\underset{\vee}{3}}\Gamma^{3}_{1\underset{\vee}{2}} + g_{32}\Gamma^{3}_{2\underset{\vee}{1}}.$$

$$g_{22}\Gamma^{2}_{22} = g_{2\underset{\vee}{3}}\cancel{\Gamma^{3}_{2\underset{\vee}{2}}} + g_{3\underset{\vee}{2}}\cancel{\Gamma^{3}_{2\underset{\vee}{2}}} = 0$$

$$\boxed{\Gamma^{1}_{11} = \frac{\alpha'}{2\alpha}, \quad \Gamma^{2}_{22} = 0} \quad (41)$$

The last relations can be replaced by

$$\cancel{\Gamma^{x}_{1\underline{2}}} \quad \Gamma^{1}_{11} + \Gamma^{2}_{1\underline{2}} + \Gamma^{3}_{1\underline{3}} + \Gamma^{4}_{1\underline{4}} = \frac{\gamma'}{2\gamma} + \frac{A'}{2A}.$$

$$\Gamma^{1}_{2\underline{1}} + \cancel{\Gamma^{2}_{22}} + \Gamma^{3}_{2\underline{3}} + \cancel{\Gamma^{4}_{2\underline{4}}} = \cot\theta.$$

We shall find out $\Gamma^{3}_{2\underset{\vee}{1}}$.

$$-\alpha\Gamma^{1}_{2\underline{1}} = -\phi\Gamma^{3}_{2\underset{\vee}{1}} \quad \therefore \quad \Gamma^{1}_{2\underline{1}} = +\frac{\phi}{\alpha}\Gamma^{3}_{2\underset{\vee}{1}}. \quad \Gamma^{3}_{2\underline{3}} = \cot\theta \mp \frac{\phi}{\alpha}\Gamma^{3}_{2\underset{\vee}{1}}.$$

$$-\beta\Gamma^{2}_{1\underline{2}} = -\frac{\beta'}{2} + f\Gamma^{3}_{2\underset{\vee}{1}} \quad \therefore \quad \Gamma^{2}_{1\underline{2}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}\Gamma^{3}_{2\underset{\vee}{1}}.$$

Lastly $-\beta\Gamma^{3}_{2\underset{\vee}{1}} = -\frac{\phi}{2}\cot\theta + \frac{f'}{2} - f\Gamma^{3}_{3\underline{1}} + \phi\Gamma^{3}_{2\underline{3}}$

$$\therefore f\Gamma^{3}_{3\underline{1}} = \beta\Gamma^{3}_{2\underset{\vee}{1}} - \frac{\phi}{2}\cot\theta + \frac{f'}{2} + \phi\left(\cot\theta \mp \frac{\phi}{\alpha}\Gamma^{3}_{2\underset{\vee}{1}}\right)$$

$$= \frac{\phi}{2}\cot\theta + \frac{f'}{2} + \left(\beta \mp \frac{\phi^2}{\alpha}\right)\Gamma^{3}_{2\underset{\vee}{1}}.$$

$$\therefore \Gamma^{3}_{3\underline{1}} = \frac{\phi}{2f}\cot\theta + \frac{f'}{2f} + \frac{1}{f}\left(\beta \mp \frac{\phi^2}{\alpha}\right)\Gamma^{3}_{2\underset{\vee}{1}}.$$

Now use $\Gamma^{1}_{11} + \Gamma^{2}_{1\underline{2}} + \Gamma^{3}_{1\underline{3}} + \Gamma^{4}_{1\underline{4}} = \frac{\gamma'}{2\gamma} + \frac{A'}{2A}$

$$\therefore \frac{\alpha'}{2\alpha} + \frac{\beta'}{2\beta} + \frac{\phi}{2f}\cot\theta + \frac{f'}{2f} + \frac{\gamma'}{2\gamma} + \Gamma^{3}_{2\underset{\vee}{1}}\left(-\frac{f}{\beta} + \frac{1}{f}\left(\beta \mp \frac{\phi^2}{\alpha}\right)\right)$$

$$= \frac{\gamma'}{2\gamma} + \frac{A'}{2A}.$$

$$\therefore \Gamma^{3}_{2\underset{\vee}{1}} = \left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f} - \frac{\phi}{2f}\cot\theta\right]\frac{\alpha\beta f}{\alpha(\beta^2 - f^2) \mp \beta\phi^2}$$

But $\textcircled{g}^{3\underset{\vee}{1}}_{,1} = 0 \quad \therefore \left[\beta\phi\sqrt{\frac{\gamma}{A}}\right]' = 0 \quad \therefore \frac{\beta'}{\beta} + \frac{\phi'}{\phi} + \frac{\gamma'}{2\gamma} - \frac{A'}{2A} = 0$

$$\therefore \frac{A'}{2A} = \frac{\beta'}{\beta} + \frac{\gamma'}{2\gamma} + \frac{\phi'}{\phi}$$

$$\Gamma^{3}_{\underset{v}{21}} = B ~~\phi~~ - \frac{~~\alpha\phi f~~\ \alpha\beta\phi}{\alpha(\beta^2-\phi^2) - ~~\alpha~~\beta\phi^2} \cdot \frac{\cot\theta}{2}.$$

$$= B - C\cot\theta. \qquad \boxed{\Gamma^{3}_{\underset{v}{21}} = B - C\cot\theta} \quad (42)$$

$$\boxed{\Gamma^{2}_{\underline{12}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}B + \frac{f}{\beta}C\cot\theta} \quad (43)$$

$$\boxed{\Gamma^{3}_{\underline{13}} = \frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} + \frac{f}{\beta}B - \frac{f}{\beta}C\cot\theta} \quad (44)$$

$$-\alpha\,\Gamma^{1}_{22} = \frac{\beta'}{2} - 2f(B - C\cot\theta)$$

$$\therefore \boxed{\Gamma^{1}_{22} = -\frac{\beta'}{2\alpha} + \frac{2f}{\alpha}(B - C\cot\theta)} \quad (45)$$

$$-\alpha\,\Gamma^{1}_{\underline{21}} = -\phi\,\Gamma^{3}_{\underset{v}{21}} \quad \therefore \boxed{\Gamma^{1}_{\underline{21}} = \frac{\phi}{\alpha}(B - C\cot\theta)} \quad (46)$$

$$\Gamma^{3}_{\underline{23}} = \cot\theta - \frac{\phi}{\alpha}B + \frac{\phi}{\alpha}C\cot\theta \quad \therefore \boxed{\Gamma^{3}_{\underline{23}} = -\frac{\phi}{\alpha}B + \left(1 + \frac{\phi}{\alpha}C\right)\cot\theta} \quad (47)$$

$$-\beta\,\Gamma^{2}_{11} = 2\phi\,\Gamma^{3}_{\underset{v}{21}} \quad \therefore \boxed{\Gamma^{2}_{11} = -\frac{2\phi}{\beta}(B - C\cot\theta)} \quad (48)$$

$$-\beta\,\Gamma^{2}_{\underset{v}{31}} = -\frac{\phi}{2}\cot\theta - \frac{f'}{2} + \phi\cot\theta + f\left(\frac{\beta'}{2\beta} - \frac{f}{\beta}B + \frac{f}{\beta}C\cot\theta\right)$$

$$\therefore \boxed{\Gamma^{2}_{\underset{v}{31}} = \frac{f}{\beta}\left(\frac{f'}{2f} - \frac{\beta'}{2\beta} ~~\gamma~~ + \frac{f}{\beta}B\right) ~~\phi~~ - \cot\theta\left(\frac{f^2}{\beta^2}C + \frac{\phi}{2\beta}\right)} \quad (49)$$

~~$-\alpha\,\Gamma^{1}_{\underset{v}{31}} = -\frac{\phi'}{2} - \frac{\phi'}{2} + f\,\frac{2\phi}{\beta}(B - C\cot\theta) + \phi[$~~

$$-\beta\,\Gamma^{2}_{\underset{v}{32}} = -\frac{f}{2}\cot\theta - \frac{f}{2}\cot\theta + \phi\left(-\frac{\beta'}{2\alpha} + \frac{2f}{\alpha}(B - C\cot\theta)\right)$$
$$+ f\left(\cot\theta - \frac{\phi}{\alpha}(B - C\cot\theta)\right)$$

$$= \phi\left(-\frac{\beta'}{2\alpha} + \frac{f}{\alpha}(B - C\cot\theta)\right)$$

$$\therefore \boxed{\Gamma^{2}_{\underset{v}{32}} = \frac{\phi}{\alpha}\left(\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right)} \quad (50)$$

~~$\Gamma^{1}_{\underset{v}{31}} = -\frac{\phi}{\alpha}\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]$ (51)~~

$$\Gamma^{1}_{\underset{v}{31}} = -\Gamma^{2}_{\underset{v}{32}} - \Gamma^{4}_{\underset{v}{34}} \quad \therefore \boxed{\Gamma^{1}_{\underset{v}{31}} = -\frac{\phi}{\alpha}\left[\frac{\gamma'}{2\gamma} + \frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]} \quad (51)$$

$$-\alpha\,\Gamma^{1}_{\underset{v}{32}} = -\frac{f'}{2} - \frac{\phi}{2}\cot\theta + \frac{\phi^2}{\alpha}(B - C\cot\theta) + f\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha}\right)$$

$$\therefore \boxed{\Gamma^{1}_{\underset{\vee}{3}2} = \frac{f}{\alpha}\left[\frac{f'}{2f} - \frac{A'}{2A} + \frac{\alpha'}{2\alpha}\right] - \frac{\phi^2}{\alpha^2}(B - C\cot\theta) + \frac{\phi}{2\alpha}\cot\theta} \qquad (52)$$

~~So we can now write down R_{11}.~~

Let us find Γ^4_{11}, Γ^4_{22}, Γ^4_{33}.

$$g_{44}\Gamma^4_{11} = -\tfrac{1}{2}g_{11,4} + g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{1}} + g_{3\underset{\vee}{1}}\Gamma^3_{1\underset{\vee}{4}}. \qquad (53)$$

$$g_{44}\Gamma^4_{22} = -\tfrac{1}{2}g_{22,4} + g_{2\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{2}} + g_{3\underset{\vee}{3}}\Gamma^3_{24} \qquad (54)$$

$$g_{44}\Gamma^4_{33} = -\tfrac{1}{2}g_{33,4} + 2g_{3\underset{\vee}{1}}\Gamma^1_{4\underset{\vee}{3}} + 2g_{3\underset{\vee}{2}}\Gamma^2_{43} \qquad (55)$$

$$g_{44}\Gamma^4_{\underline{12}} = g_{13}\Gamma^3_{4\underset{\vee}{2}} + g_{3\underset{\vee}{2}}\Gamma^3_{1\underset{\vee}{4}} \qquad (56)$$

$$g_{44}\Gamma^4_{1\underset{\vee}{2}} = -g_{1\underset{\vee}{3}}\Gamma^3_{\underline{42}} - g_{3\underset{\vee}{2}}\Gamma^3_{\underline{14}} = 0 \qquad (57)$$

$$g_{44}\Gamma^4_{\underline{13}} = g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{3}} + g_{1\underset{\vee}{3}}\Gamma^1_{1\underset{\vee}{4}} + g_{2\underset{\vee}{3}}\Gamma^2_{1\underset{\vee}{4}} = 0 \qquad (58)$$

$$g_{44}\Gamma^4_{1\underset{\vee}{3}} = +\tfrac{1}{2}g_{1\underset{\vee}{3},4} - g_{1\underset{\vee}{3}}\Gamma^3_{\underline{43}} - g_{2\underset{\vee}{3}}\Gamma^2_{\underline{14}} - g_{1\underset{\vee}{3}}\Gamma^1_{1\underline{4}} \qquad (59)$$

$R_{11} = +\Gamma^s_{11,s} - \Gamma^t_{1s}\Gamma^s_{t1} - \Gamma^s_{1\underline{s},1} + \Gamma^s_{11}\Gamma^t_{s\underline{t}}$. Taking θ terms

$$R_{11} = \Gamma^2_{11,2} - \Gamma^2_{11}\Gamma^1_{21} - \Gamma^1_{12}\Gamma^2_{11} - \Gamma^2_{12}\Gamma^2_{21} - \Gamma^3_{\underset{\vee}{12}}\Gamma^2_{\underset{\vee}{31}} - \cancel{\Gamma^1_{13}\Gamma^3_{11}}$$

$$- \Gamma^2_{\underset{\vee}{13}}\Gamma^3_{\underset{\vee}{21}} - \Gamma^3_{13}\Gamma^3_{31} + \Gamma^2_{11}\cot\theta.$$

$$= \pm\frac{2\phi}{\beta}C\,\mathrm{cosec}^2\theta + 2\cdot\frac{2\phi}{\beta}(B - C\cot\theta)^2\frac{\phi}{\alpha} - \left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]^2$$

$$+2(B - C\cot\theta)\left[\frac{f}{\beta}\left(\frac{f'}{2f} - \frac{\beta'}{2\beta}\right) + \frac{f^2}{\beta^2}(B - C\cot\theta) - \frac{\phi}{2\beta}\cot\theta\right]$$

$$-\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} + \frac{f}{\beta}(B - C\cot\theta)\right]^2 \pm \frac{2\phi}{\beta}(B - C\cot\theta)\cot\theta.$$

$$= \cot^2\theta\left[\cancel{\frac{2\phi C}{\beta}} + \frac{4\phi^2}{\alpha\beta}C^2 - \cancel{\frac{f^2}{\beta^2}C^2} + 2\cancel{C^2\frac{f^2}{\beta^2}} + 2C\frac{\phi}{2\beta} - \cancel{\frac{f^2}{\beta^2}C^2}\right.$$

$$\left.\cancel{+2f} + \cancel{\frac{2\phi C}{\beta}}\right] + \cot\theta\left[\mp\frac{8\phi^2}{\alpha\beta}BC + \frac{f^2\,\cancel{2BC}}{\beta^2}\right.$$

$$+ \cancel{2\frac{\beta'}{2\beta}\frac{f}{\beta}(-C)} - \cancel{2BC\frac{f^2}{\beta^2}} - 2B\frac{\phi}{2\beta} - 2C\frac{f}{\beta}\left(\frac{f'}{2f} - \cancel{\frac{\beta'}{2\beta}}\right)$$

$$\left. - \cancel{2BC\frac{f^2}{\beta^2}} + \cancel{2BC}\frac{f^2}{\beta^2} \pm 2C\left(\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta}\right)\frac{f}{\beta} \mp \frac{2\phi}{\beta}B\right]$$

$$= \cot^2\theta\,\frac{2C\phi}{\alpha\beta}(\alpha - 4\phi C) + \cot\theta(\quad)$$

$$= \frac{\phi^2\cot^2\theta}{2[\alpha(\beta^2 - f^2) + \beta\phi^2]}\left[\alpha \mp \frac{2\phi^2\alpha\beta}{\alpha(\beta^2 - f^2) \mp \alpha\beta\phi^2}\right] + \cot\theta(\quad)$$

$$= \frac{\cot^2\theta\ \phi^2\alpha}{2\left[\alpha(\beta^2-f^2)+\beta\phi^2\right]^2}\quad\left[\alpha(\beta^2-f^2)+\beta\phi^2\right]+\cot\theta c\,)$$

The coeff of $\cot^2\theta$ is $\underline{\underline{\text{not}}}$ g. So ~~this is not~~ there is a change after all. Let us work out these symbols again

$$g_{22}\,\Gamma^{2}_{11} = 2g_{1\underset{\vee}{3}}\,\Gamma^{3}_{2\underset{\vee}{1}}. \qquad ; \quad g_{11}\Gamma^{1}_{11} = \tfrac12 g_{11,1}.$$

$$g_{33}\,\Gamma^{3}_{\underline{31}} = \tfrac12 g_{33,1} + g_{3\underset{\vee}{1}}\,\Gamma^{1}_{3\underset{\vee}{1}} + g_{3\underset{\vee}{2}}\,\Gamma^{2}_{3\underset{\vee}{1}} + \cancel{g_{\underline{3}\underset{\vee}{1}}\,\Gamma^{3}_{3\underset{\vee}{3}}}.$$

$$g_{33}\,\Gamma^{3}_{\underline{32}} = \tfrac12 g_{33,2} + g_{3\underset{\vee}{1}}\,\Gamma^{1}_{3\underset{\vee}{2}} + g_{3\underset{\vee}{2}}\,\Gamma^{2}_{3\underset{\vee}{2}} + \cancel{g_{3\underset{\vee}{2}}\,\Gamma^{3}_{3\underset{\vee}{3}}}$$

$$g_{11}\,\Gamma^{1}_{1\underline{2}} = \cancel{g_{1\underset{\vee}{3}}\,\Gamma^{3}_{1\underset{\vee}{1}}} + g\, g_{1\underset{\vee}{3}}\,\Gamma^{3}_{1\underset{\vee}{2}} + \cancel{g_{3\underset{\vee}{2}}\,\Gamma^{3}_{1\underset{\vee}{1}}}$$

$$g_{22}\,\Gamma^{2}_{\underline{21}} = \tfrac12 g_{22,1} + g_{2\underset{\vee}{3}}\,\Gamma^{3}_{2\underset{\vee}{1}} + \cancel{g_{3\underset{\vee}{1}}\,\Gamma^{3}_{2\underset{\vee}{2}}}$$

$$g_{11}\,\Gamma^{1}_{22} = -\tfrac12 g_{22,1} + 2\,g_{2\underset{\vee}{3}}\,\Gamma^{3}_{1\underset{\vee}{2}}$$

$$g_{22}\,\Gamma^{2}_{22} = 0$$

$$g_{11}\,\Gamma^{1}_{1\underset{\vee}{3}} = \tfrac12 g_{1\underset{\vee}{3},1} - \tfrac12 g_{3\underset{\vee}{1},1} - g_{1\underset{\vee}{3}}\,\Gamma^{3}_{\underline{13}} - g_{1\underset{\vee}{3}}\,\Gamma^{1}_{11} - g_{2\underset{\vee}{3}}\,\Gamma^{2}_{11}$$

$$g_{33}\,\Gamma^{3}_{2\underset{\vee}{1}} = -\tfrac12 g_{1\underset{\vee}{3},2} - \tfrac12 g_{3\underset{\vee}{2},1} - g_{2\underset{\vee}{3}}\,\Gamma^{3}_{\underline{31}} - g_{3\underset{\vee}{1}}\,\Gamma^{3}_{\underline{23}}$$

$$g_{22}\,\Gamma^{2}_{3\underset{\vee}{1}} = \tfrac12 g_{3\underset{\vee}{1},2} - \tfrac12 g_{2\underset{\vee}{3},1} - g_{3\underset{\vee}{1}}\,\Gamma^{1}_{\underline{21}} - g_{3\underset{\vee}{2}}\,\Gamma^{2}_{\underline{21}} - g_{3\underset{\vee}{1}}\,\Gamma^{3}_{\underline{32}}$$

$$g_{11}\,\Gamma^{1}_{3\underset{\vee}{2}} = \tfrac12 g_{3\underset{\vee}{2},1} - \tfrac12 g_{1\underset{\vee}{3},2} - g_{3\underset{\vee}{1}}\,\Gamma^{1}_{\underline{12}} - g_{3\underset{\vee}{2}}\,\Gamma^{2}_{1\underline{2}} - g_{\underset{\vee}{3}2}\,\Gamma^{3}_{\underline{31}}$$

$$g_{22}\,\Gamma^{2}_{3\underset{\vee}{2}} = \tfrac12 g_{3\underset{\vee}{2},2} - \tfrac12 g_{2\underset{\vee}{3},2} - g_{3\underset{\vee}{1}}\,\Gamma^{1}_{\underline{22}} - \cancel{g_{3\underset{\vee}{2}}\,\Gamma^{2}_{22}} - g_{\underset{\vee}{3}2}\,\Gamma^{3}_{\underline{32}}$$

$$\Gamma^{1}_{11} + \Gamma^{2}_{1\underline{2}} + \Gamma^{3}_{\underline{13}} + \Gamma^{4}_{\underline{14}} = \frac{A'}{2A} + \frac{\gamma'}{2\gamma}. \qquad \Gamma^{1}_{11} = \frac{\alpha'}{2\alpha}. \quad \Gamma^{4}_{\underline{14}} = \frac{\gamma'}{2\gamma}$$

$$-\beta\,\Gamma^{2}_{1\underline{2}} = -\frac{\beta'}{2} + f\,\Gamma^{3}_{2\underset{\vee}{1}} \qquad \therefore\quad \Gamma^{2}_{1\underline{2}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}\,\Gamma^{3}_{2\underset{\vee}{1}}.$$

Next $\Gamma^{1}_{2\underline{1}} + \Gamma^{2}_{22} + \Gamma^{3}_{2\underline{3}} + \Gamma^{4}_{2\underline{4}} = \cot\theta$

$$\cancel{\cos\Gamma^{1}_{2\underline{1}}} \quad -\alpha\,\Gamma^{1}_{2\underline{1}} = \phi\,\cancel{\sin\theta}\,\Gamma^{3}_{1\underset{\vee}{3}} \quad \therefore\ -\alpha\,\Gamma^{1}_{2\underline{1}} = -\phi\,\Gamma^{3}_{2\underset{\vee}{1}}$$

$$\therefore\ \Gamma^{1}_{2\underline{1}} = \frac{\phi}{\alpha}\,\Gamma^{3}_{2\underset{\vee}{1}} \qquad \therefore\ \Gamma^{3}_{2\underline{3}} = \cot\theta - \frac{\phi}{\alpha}\,\Gamma^{3}_{2\underset{\vee}{1}}.$$

Now $-\beta\,\Gamma^{3}_{2\underset{\vee}{1}} = -\dfrac{\phi}{2}\cot\theta + \dfrac{f'}{2} - f\,\Gamma^{3}_{\underline{31}} + \phi\left[\cot\theta - \dfrac{\phi}{\alpha}\,\Gamma^{3}_{2\underset{\vee}{1}}\right]$

$$\therefore \quad f\,\Gamma^{3}_{3\underline{1}} = \frac{\phi}{2}\cot\theta + \frac{f'}{2} + \Gamma^{3}_{2\underset{\vee}{1}}\left(\beta - \frac{\phi^2}{\alpha}\right).$$

$$\therefore \quad \Gamma^{3}_{3\underline{1}} = \frac{\phi}{2f}\cot\theta + \frac{f'}{2f} + \frac{1}{f}\left(\beta - \frac{\phi^2}{\alpha}\right)\Gamma^{3}_{2\underset{\vee}{1}}.$$

$$\therefore \quad \Gamma^{1}_{11} + \Gamma^{2}_{1\underline{2}} + \Gamma^{3}_{1\underline{3}} + \Gamma^{4}_{4\underline{4}} = \frac{A'}{2A} + \frac{\gamma'}{2\gamma} \quad \text{gives}$$

$$\frac{\alpha'}{2\alpha} + \frac{\beta'}{2\beta} - \frac{f}{\beta}\Gamma^{3}_{2\underset{\vee}{1}} + \frac{\phi}{2f}\cot\theta + \frac{f'}{2f} + \frac{1}{f}\left(\beta - \frac{\phi^2}{\alpha}\right)\Gamma^{3}_{2\underset{\vee}{1}} + \frac{\gamma'}{2\gamma} = \frac{A'}{2A} + \frac{\gamma'}{2\gamma}$$

$$\therefore \quad \Gamma^{3}_{2\underset{\vee}{1}}\left[\frac{1}{f}\left(\beta - \frac{\phi^2}{\alpha}\right) - \frac{f}{\beta}\right] = \frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f} - \frac{\phi}{2f}\cot\theta.$$

$$\therefore \quad \Gamma^{3}_{2\underset{\vee}{1}}\,\frac{\alpha\beta^2 - \beta\phi^2 - \alpha f^2}{\alpha\beta f} = \frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f} - \frac{\phi}{2f}\cot\theta$$

$$\therefore \quad \Gamma^{3}_{2\underset{\vee}{1}} = \frac{\alpha\beta f}{\alpha(\beta^2 - f^2) - \beta\phi^2}\cdot\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f}\right] - \frac{\phi\alpha\beta}{\alpha(\beta^2 - f^2) - \beta\phi^2}\cdot\frac{\cot\theta}{2}$$

$$= B - C\cot\theta$$

$$B = \frac{\alpha\beta f}{\alpha(\beta^2 - f^2) - \beta\phi^2}\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f}\right]$$

$$C = \frac{\phi\alpha\beta}{\alpha(\beta^2 - f^2) - \beta\phi^2}\cdot\frac{1}{2}$$

$$\therefore \quad \boxed{\Gamma^{3}_{2\underset{\vee}{1}} = (B - C\cot\theta)\,\operatorname{cosec}\theta} \; (40) \qquad \boxed{\Gamma^{1}_{11} = \frac{\alpha'}{2\alpha}, \quad \Gamma^{2}_{22} = 0} \; (42)$$

$$\boxed{\Gamma^{2}_{11} = -\frac{2\phi}{\beta}(B - C\cot\theta)} \; (43)$$

$-\beta\,\Gamma^{2}_{11} = 2\phi\,\Gamma^{3}_{2\underset{\vee}{1}} \quad \therefore$ ~~$\Gamma^{2}_{11} = -\frac{\phi}{2\beta}$~~

$$\boxed{\Gamma^{3}_{2\underline{3}} = \cot\theta - \frac{\phi}{\alpha}(B - C\cot\theta)} \; (45)$$

$$\boxed{\Gamma^{1}_{2\underline{1}} = \frac{\phi}{\alpha}(B - C\cot\theta)} \; (44)$$

~~$-\alpha\,\Gamma^{1}_{22} = \frac{\beta'}{2} \mp 2f\,(B$~~

$$\boxed{\Gamma^{2}_{1\underline{2}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)} \; (46)$$

$$\boxed{\Gamma^{3}_{1\underline{3}} = \frac{A'}{2A} - \frac{\beta'}{2\beta} - \frac{\alpha'}{2\alpha} + \frac{f}{\beta}(B - C\cot\theta)} \; (47)$$

$$\therefore \quad \boxed{\Gamma^{1}_{22} = -\frac{\beta'}{2\alpha} + \frac{2f}{\alpha}(B - C\cot\theta)} \; (48)$$

$$-\alpha\,\Gamma^{1}_{22} = \frac{\beta'}{2} \overset{-}{\cancel{\phi}}\, 2f\,(B - C\cot\theta)$$

$$-\beta\,\Gamma^{2}_{3\underset{\vee}{2}} = -\frac{f}{2}\cot\theta - \frac{f}{2}\cot\theta + \phi\left[-\frac{\beta'}{2\alpha} + \frac{2f}{\alpha}(B - C\cot\theta)\right] + f\left[\cot\theta - \frac{\phi}{\alpha}(B - C\cot\right.$$

$$\therefore \quad -\beta\,\Gamma^{2}_{3\underset{\vee}{2}} = \frac{\phi}{\alpha}\left[-\frac{\beta'}{2} + 2f(B - C\cot\theta) - f(B - C\cot\theta)\right]$$

$$\therefore \quad \boxed{\Gamma^{2}_{3\underset{\vee}{2}} = \frac{\phi}{\alpha}\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]} \; (49)$$

~~$\sin\theta$~~

~~$\Gamma^{1}_{1\underset{\vee}{3}} = \frac{\phi}{\alpha}\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]$~~ ~~(50)~~

~~$\sin\theta$~~

$$\boxed{\Gamma^{1}_{3\underset{\vee}{1}} = -\frac{\phi}{\alpha}\left[\frac{\gamma'}{2\gamma} + \frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]} \; (50)$$

$$-\beta\,\Gamma^{2}_{\underset{\vee}{31}} = -\frac{p}{2}\cot\theta - \frac{f'}{2} + \beta\cot\theta + f\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]$$

$$\therefore -\beta\,\Gamma^{2}_{\underset{\vee}{31}} = \frac{p}{2}\cot\theta - \frac{f'}{2} + f\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)\right]$$

$$\therefore \boxed{\Gamma^{2}_{\underset{\vee}{31}} = \frac{f}{\beta}\left[-\frac{\beta'}{2\beta} + \frac{f'}{2f} + \cancel{(B - C\cot\theta)} - \frac{p}{2f}\cot\theta\right] + \frac{f}{\beta}(B - C\cot\theta)} \quad (51)$$

$$-\alpha\,\Gamma^{1}_{\underset{\vee}{32}} = -\frac{f'}{2} - \frac{p}{2}\cot\theta + \frac{p^{2}}{\alpha}\left[B - C\cot\theta\right] + f\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha}\right]$$

$$\therefore \boxed{\Gamma^{1}_{\underset{\vee}{32}} = \frac{f}{\alpha}\left[-\frac{A'}{2A} + \frac{\alpha'}{2\alpha} + \frac{f'}{2f}\right] + \frac{p}{2\alpha}\cot\theta - \frac{p^{2}}{\alpha^{2}}(B - C\cot\theta)} \quad (52)$$

~~$g_{33}\Gamma^{3}_{22} = \frac{1}{2} g_{23} \quad g_{23}\Gamma^{3}_{\underset{\vee}{32}} + g_{\underset{\vee}{32}}\Gamma^{3}_{\underset{\vee}{23}} = 0$~~ $\boxed{\Gamma^{1}_{\underline{32}} = 0}$ (53)

~~$g_{33}\Gamma^{3}_{\underset{\vee}{23}} = 0 \quad g_{11}\Gamma^{1}_{\underline{32}} =$~~

$$g_{11}\Gamma^{1}_{33} = -\frac{1}{2} g_{33,1} + 2 g_{\underset{\vee}{31}}\Gamma^{1}_{\underset{\vee}{13}}$$

$R_{11} = \Gamma^{s}_{11,s} - \Gamma^{t}_{1s}\Gamma^{s}_{t1} - \Gamma^{s}_{1\underline{s},1} + \Gamma^{s}_{11}\Gamma^{t}_{\underline{st}}$. The θ terms are

$$\Gamma^{2}_{11,2} - \Gamma^{2}_{11}\cdot\Gamma^{1}_{\underline{21}} - \Gamma^{1}_{1\underline{2}}\Gamma^{2}_{11} - \Gamma^{2}_{1\underline{2}}\Gamma^{2}_{\underline{21}} - \Gamma^{3}_{\underset{\vee}{12}}\Gamma^{2}_{\underset{\vee}{31}}$$

$$- \cancel{\Gamma^{1}_{\underset{\vee}{13}}\Gamma^{3}_{\underline{11}}} - \Gamma^{2}_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{21}} - \Gamma^{3}_{1\underline{3}}\Gamma^{3}_{\underline{31}} + \Gamma^{2}_{11}\Gamma^{t}_{2\underline{t}}$$

$$= -\frac{2p}{\beta}C\,\mathrm{cosec}^{2}\theta - 2\cdot(-2)\frac{p^{2}}{\alpha\beta}(B - C\cot\theta)^{2} - \left[\frac{\beta'}{2\beta} - \frac{f}{\beta}B + \frac{f}{\beta}C\cot\theta\right]^{2}$$

$$- 2(-)\frac{f}{\beta}(B - C\cot\theta)\left[-\frac{\beta'}{2\beta} + \frac{f'}{2f} + \frac{f}{\beta}B - \left(\frac{p}{2f} + \frac{fC}{\beta}\right)\cot\theta\right]$$

~~$+ \frac{p^{2}}{\alpha^{2}}\left[\frac{\gamma'}{2\gamma} + \frac{\beta'}{2\beta} - \frac{f}{\beta}B + \frac{f}{\beta}C\cot\theta\right]^{2}$~~

$$- \left[\frac{A'}{2A} - \frac{\beta'}{2\beta} - \frac{\alpha'}{2\alpha} + \frac{f}{\beta}B - \frac{f}{\beta}C\cot\theta\right]^{2} - \frac{2p}{\beta}(B - C\cot\theta)\cot\theta$$

~~$= \cot^{2}\theta$ of~~ [The coeff. of $\cot^{2}\theta$ in the above

is

$$-\frac{2p}{\beta}C + 4\frac{p^{2}C^{2}}{\alpha\beta} - \overset{\checkmark}{\frac{f^{2}C^{2}}{\beta^{2}}} + \frac{2fC}{\beta}\overset{\checkmark}{\left(\frac{p}{2f} + \frac{fC}{\beta}\right)}$$

$$+ \cancel{\frac{p^{2}}{\alpha^{2}}\frac{f^{2}C^{2}}{\beta^{2}}} - \frac{f^{2\checkmark}}{\beta^{2}}C^{2} + \frac{2pC}{\beta}$$

$$= \frac{4p^2c^2}{\alpha\beta} + \frac{pc}{\beta} + \cancel{\frac{\frac{p^2}{\alpha^2}f^2c^2}{\beta^2}}$$

$$= \frac{pc}{\alpha\beta}\left[\alpha + \quad 4pc\right]$$

$$= \frac{pc}{\alpha\beta}\left[\alpha + \frac{\overset{2}{\cancel{4}}p\, p\alpha\beta}{\alpha(\beta^2-f^2)-\beta p^2}\cdot\frac{1}{\cancel{2}}\right]$$

$$= \frac{\alpha pc}{\alpha\beta}\left[\frac{\alpha(\beta^2-f^2)-\beta p^2 + 2\beta p^2}{\alpha(\beta^2-f^2)-\beta p^2}\right]$$

$$= \frac{\alpha p\cdot p\cancel{\alpha\beta}}{[\alpha(\beta^2-f^2)-\beta p^2]^2}\cdot\frac{\frac{1}{2}}{\cancel{\alpha\beta}}\left[\alpha(\beta^2-f^2)+\beta p^2\right]$$

For this coeff to vanish, we must have

$p \neq 0$, $\alpha \neq 0$ $\therefore$ $\alpha(\beta^2-f^2)+\beta p^2 = 0$. $\therefore \alpha\beta^2+\beta p^2 = \alpha f^2$

Then ~~A =~~ $g = -\gamma[\alpha(\beta^2+f^2)+\beta p^2]\sin^2\theta$

$$= -\gamma[\alpha f^2+\alpha f^2]\sin^2\theta$$

$$g = -2\gamma\alpha f^2\sin^2\theta.$$

$$A = 2\alpha f^2 \qquad \frac{A'}{2A} = \frac{\alpha'}{2\alpha}+\frac{f'}{f}.$$

Also

$$B = \frac{\alpha\beta f}{\alpha(\beta^2-f^2)-\beta p^2}\cdot\left[\frac{A'}{2A}-\frac{\alpha'}{2\alpha}-\frac{\beta'}{2\beta}-\frac{f'}{2f}\right]$$

$$= \frac{\alpha\beta f}{-2\beta p^2}\left[\frac{f'}{2f}-\frac{\beta'}{2\beta}\right] = -\frac{\alpha f}{2p^2}\left(\frac{f'}{2f}-\frac{\beta'}{2\beta}\right)$$

$$C = -\frac{p\alpha\beta}{2\beta p^2}\cdot\frac{1}{2} = \cancel{0}-\frac{\alpha}{4p}.$$

The coeff of $\cot\theta$ in the above is

$$-\frac{8p^2BC}{\alpha\beta} - \frac{2f}{\beta}C\left(\frac{\beta'}{2\beta}-\cancel{\frac{f}{\beta}B}\right) \overset{-}{\cancel{+}} \frac{2f}{\beta}B\left(\cancel{0}\ \frac{p}{2f}+\cancel{\frac{fc}{\beta}}\right)$$

$$-\frac{2f}{\beta}C\left(\cancel{+\frac{\beta'}{2\beta}}+\frac{f'}{2f}+\cancel{\frac{f}{\beta}B}\right)+\frac{2f}{\beta}C\left(\frac{A'}{2A}-\frac{\beta'}{2\beta}-\frac{\alpha'}{2\alpha}+\cancel{\frac{f}{\beta}B}\right)$$

$$-2pB/\beta.$$

$$= -\frac{8p^2BC}{\alpha\beta} - \frac{Bp}{\beta} + \cancel{\frac{2fC}{\beta}\left[\frac{A'}{2A} - \frac{\beta'}{2\beta} - \frac{\alpha'}{2\alpha} + \frac{\beta'}{2\beta}\right]}$$

$$+ \frac{2fC}{\beta}\left[\frac{A'}{2A} - \frac{\beta'}{2\beta} - \frac{\alpha'}{2\alpha} - \frac{f'}{2f}\right] - \frac{2pB}{\beta}$$

But $\alpha(\beta^2 - f^2) + \beta p^2 = 0 \quad \therefore \alpha\beta^2 + \beta p^2 = \alpha p^2$

$\therefore A = \alpha(\beta^2 + f^2) + \beta p^2 = 2\alpha f^2$

$$\therefore \frac{A'}{2A} = \frac{\alpha'}{2\alpha} + \frac{f'}{f}$$

$$\therefore B = \frac{\alpha\beta f}{\alpha(\beta^2 - f^2) - \beta p^2}\left[\frac{A'}{2A} - \frac{\alpha'}{2\alpha} - \frac{\beta'}{2\beta} - \frac{f'}{2f}\right]$$

$$= \frac{\alpha\beta f}{-2\beta p^2}\left[\frac{f'}{2f} - \frac{\beta'}{2\beta}\right]$$

$$= \frac{\alpha f}{\cancel{2}p^2} \quad -\frac{\alpha f}{2p^2}\left(\frac{f'}{2f} - \frac{\beta'}{2\beta}\right)$$

$\therefore$ coeff ~~cot~~ of $\cot\theta$ is

$$-\frac{8p^2BC}{\alpha\beta} - \frac{3Bp}{\beta} + \frac{2fC}{\beta}\left(-\frac{2p^2}{\alpha f}\right)B$$

$$= -\frac{12p^2BC}{\alpha\beta} - \frac{3Bp}{\beta}$$

$= -\frac{3Bp}{\beta}\left[\frac{4pC}{\alpha} + 1\right]$ ~~= 0 wh~~ which

vanishes obviously being the same as the coeff of $\cot^2\theta$.

Hence with

$$\alpha(\beta^2 - f^2) + \beta p^2 = 0$$

θ terms in R_{11} vanish. There are no θ terms in R_{44}. Let us now take R_{22}

$R_{22} = \Gamma_{22,s}^{s} - \Gamma_{2s}^{t}\Gamma_{t2}^{s} \pm \Gamma_{2\underline{s},2}^{s} + \Gamma_{22}^{s}\Gamma_{s\underline{t}}^{t}$. θ terms are

$$\Gamma_{22,1}^{1} - \Gamma_{2\underline{1}}^{1}\Gamma_{1\underline{2}}^{1} - \Gamma_{2\underline{1}}^{2}\Gamma_{22}^{1} - \Gamma_{2\underset{\checkmark}{1}}^{3}\Gamma_{3\underset{\checkmark}{2}}^{1} - \Gamma_{22}^{1}\Gamma_{12}^{2}$$

$$- \Gamma_{2\underset{\checkmark}{3}}^{1}\Gamma_{1\underset{\checkmark}{3}}^{3} - \cancel{\Gamma_{2\underset{\checkmark}{3}}^{2}\Gamma_{2\underset{\checkmark}{2}}^{3}} - \Gamma_{2\underline{3}}^{3}\Gamma_{3\underline{2}}^{3} + \operatorname{cosec}^2\theta + \Gamma_{22}^{1}\Gamma_{1\underline{t}}^{t}.$$

$$= \left(-\frac{2fC}{\alpha}\right)'\cot\theta - \frac{\phi^2}{\alpha^2}(B - C\cot\theta)^2 \cancel{-2}$$

$$-2\left[-\frac{\beta'}{2\alpha} + \frac{2f}{\alpha}B - \frac{2f}{\alpha}C\cot\theta\right]\left[\frac{\beta'}{2\beta} - \frac{f}{\beta}B + \frac{f}{\beta}C\cot\theta\right]$$

$$-2(B - C\cot\theta)\left[\frac{f}{\alpha}\left(-\frac{A'}{2A} + \frac{\alpha'}{2\alpha} + \frac{f'}{2f}\right) - \frac{\phi^2}{\alpha^2}B + \left(\frac{\phi}{2\alpha} + \frac{\phi^2}{\alpha^2}C\right)\underset{\cot\theta}{\cot\theta}\right]$$

$$-\left[-\frac{\phi}{\alpha}B + \left(1 + \frac{\phi}{\alpha}C\right)\cot\theta\right]^2 + \cot^2\theta \cancel{+} - \frac{2f}{\alpha}C\left(\frac{\gamma'}{2\gamma} + \frac{A'}{2A}\right)$$

coeff. of $\cot^2\theta$ is

$$-\frac{\phi^2}{\alpha^2}C^2 + \frac{4f^2}{\alpha\beta}C^2 + 2C\left(\frac{\phi}{2\alpha} + \frac{\phi^2}{\alpha^2}C\right) - \left(1 + \frac{\phi}{\alpha}C\right)^2$$

$$+1 \qquad = \cancel{-\frac{\phi^2}{\alpha^2}C^2} + \frac{4f^2C^2}{\alpha\beta} + \frac{\phi C}{\alpha} + \cancel{\frac{2\phi^2C^2}{\alpha^2}} \cancel{-1} - \cancel{\frac{\phi^2C^2}{\alpha^2}}$$

$$-\frac{2\phi C}{\alpha} \cancel{+1} = \frac{4f^2C^2}{\alpha\beta} - \frac{\phi C}{\alpha}$$

$$= \frac{4f^2 \cdot \phi^2\alpha^2\beta^2}{16\,\cancel{k}\phi^2\phi^4 \cdot \alpha\beta} - \frac{\phi \cdot \phi\alpha\beta}{\alpha \cdot (-2\beta\phi^2) \cdot \cancel{k}} \cdot \frac{1}{2}$$

$$= \frac{1}{4}\left[\frac{\alpha f^2}{\beta} + \phi^2\right]$$

$\therefore$ for this coeff to vanish $\alpha f^2 = -\beta\phi^2$

Let us take it again $C = -\dfrac{\alpha}{4\phi}$

$$\therefore \frac{4f^2C^2}{\alpha\beta} - \frac{\phi C}{\alpha} = \frac{4f^2\alpha^2}{16\phi^2\alpha\beta} + \frac{\phi \cdot \alpha}{4\phi\alpha} = \frac{f^2\alpha}{4\phi^2\beta} + \frac{1}{4}$$

$$\left(\frac{f^2\alpha}{\phi^2\beta} + 1\right)\frac{1}{4} \qquad \text{Hence } f^2\alpha = -\phi^2\beta.$$

$\therefore$ ultimately $\alpha\beta^2 + 2\phi^2\beta = 0$ $\qquad \alpha\beta + 2\phi^2 = 0.$

and g becomes $-\gamma[\alpha\beta^2]\sin^2\theta$ the usual form and f becomes proportional to β

$2f^2 = \beta^2$. $\therefore \frac{f'}{f} = \frac{\beta'}{\beta}$ $\therefore B = \frac{f'}{2f} - \frac{\beta'}{2\beta} = 0$.

The coeff of $\cot\theta$ will be

$$-\left(\frac{2fC}{\alpha}\right)' + \frac{\beta'}{\alpha}\cdot\frac{f}{\beta}C + \frac{\beta'}{\beta}\frac{2f}{\alpha}C$$

$$+ 2C\left[\frac{f}{\alpha}\left(-\frac{A'}{2A} + \frac{\alpha'}{2\alpha} + \frac{f'}{2f}\right)\right] - \frac{2fC}{\alpha}\left(\frac{\gamma'}{2\gamma} + \frac{A'}{2A}\right) =$$

$$\therefore \left(\frac{2fC}{\alpha}\right)\left[-\frac{f'}{f} - \frac{C'}{C} + \frac{\alpha'}{\alpha} + \frac{1}{2}\frac{\beta'}{\beta} + \frac{\beta'}{\beta}\right.$$

$$\left. - \frac{A'}{2A} + \frac{\alpha'}{2\alpha} + \frac{f'}{2f} - \frac{\gamma'}{2\gamma} - \frac{A'}{2A}\right]$$

$$= \frac{2fC}{\alpha}\left[-\frac{f'}{f} - \frac{\alpha'}{\alpha} + \frac{\phi'}{\phi} + \frac{\alpha'}{\alpha} + \frac{3\beta'}{2\beta} - \frac{\alpha'}{\alpha} - 2\frac{f'}{f}\right.$$

$$\left. + \frac{\alpha'}{2\alpha} + \frac{f'}{2f} - \frac{\gamma'}{2\gamma}\right]$$

$$= \frac{2fC}{\alpha}\left[\frac{\phi'}{\phi} - \frac{5f'}{2f} + \frac{3\beta'}{2\beta} - \frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma}\right]$$

$$= \frac{2fC}{\alpha}\left[\frac{\phi'}{\phi} - \frac{\alpha'}{2\alpha} - \frac{\gamma'}{2\gamma} - \frac{f'}{f}\right]$$

Now $$g^{13} = \frac{-\beta\phi\gamma\sin\theta}{-\alpha\beta^2\gamma\sin^2\theta} = \frac{\phi}{\alpha\beta\sin\theta}$$

$$\therefore \textcircled{g}^{13} = \frac{\phi}{\alpha\beta\sin\theta}\cdot\sqrt{\alpha\gamma}\cdot\beta\sin\theta$$

$$= \phi\sqrt{\frac{\gamma}{\alpha}}$$

$\therefore \textcircled{g}^{31}_{,1} = 0$ gives $\frac{\phi'}{\phi} + \frac{\gamma'}{2\gamma} - \frac{\alpha'}{2\alpha} = 0$

But $f^2 = 2\beta^2$ $\therefore \frac{f'}{f} = \frac{\beta'}{\beta}$

∴ coeff of cot θ becomes

$$\frac{2fC}{\alpha}\left[\frac{\phi'}{\phi}-\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}-\frac{\beta'}{\beta}\right]$$

But $\alpha\beta=-2\phi^2$ $\therefore \frac{\alpha'}{\alpha}+\frac{\beta'}{\beta}=\frac{2\phi'}{\phi}$ $\therefore \frac{\alpha'}{\alpha}-\frac{2\phi'}{\phi}=-\frac{\beta'}{\beta}$

$$=\frac{2fC}{\alpha}\left[\frac{\phi'}{\phi}-\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}+\frac{\alpha'}{\alpha}-\frac{2\phi'}{\phi}\right]$$

$$=\frac{2fC}{\alpha}\left[-\frac{\phi'}{\phi}+\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}\right]$$

$= 0$ by $\textcircled{g}^{13}{}_{,1} = 0$

But this means that ~~our~~ two relations

~~viz $\alpha f^2 = -\beta\phi^2$ and $\alpha\beta + 2\phi^2 = 0$~~

~~will lead $\textcircled{g}^{13}{}_{,1} = 0$ to $\gamma' = 0$.~~

~~$\therefore \textcircled{g}^{31}{}_{,1} = 0 \longrightarrow \frac{\phi'}{\phi}+\frac{\gamma'}{2\gamma}-\frac{\alpha'}{2\alpha}=0$~~

~~Now $\alpha f^2 = -\beta\phi^2$ $\therefore 2\alpha f^2 = -2\beta\phi^2$ $\therefore 2\alpha f^2 = \beta\alpha\beta$~~

~~$\therefore 2f^2=\beta^2$ $2f^2=\beta^2$ $\therefore \alpha^2 f^4 = \beta^2$~~

$\alpha\beta = -2\phi^2$ $\therefore \frac{\alpha'}{\alpha}+\frac{\beta'}{\beta}=\frac{2\phi'}{\phi}$

$\therefore \frac{\alpha'}{\alpha}+\frac{f'}{f}=\frac{2\phi'}{\phi}$ $\therefore \frac{\alpha'}{2\alpha}+\frac{f'}{2f}=\frac{\phi'}{\phi}$

$\therefore \frac{\alpha'}{2\alpha}+\frac{f'}{2f}=\frac{\alpha'}{2\alpha}-\frac{\gamma'}{2\gamma}$ $\therefore \frac{f'}{2f}+\frac{\gamma'}{2\gamma}=0$

$\therefore (f\gamma)'=0$ $\therefore f\gamma = 1$ by properly choosing t.

Thus we have three relations.

(1) ~~$2\phi^2$~~ $2f^2=\beta^2$, $\alpha\beta=-2\phi^2$, $f\gamma=\pm1$. (± depends on the sign of f).

we have only to determine ϕ and f.

$$g_{11}\Gamma^1_{33} = -\tfrac{1}{2}g_{33,1} + 2g_{31}\Gamma^1_{\underset{\checkmark}{13}} + 2g_{\underset{\checkmark}{32}}\Gamma^2_{\underset{\checkmark}{13}}$$

$$\therefore -\alpha\Gamma^1_{33} = \frac{\beta'}{2} \mp 2\frac{pc}{\alpha}\cot\theta - 2f\left[\frac{p}{2\beta}\cot\theta + \frac{f^2}{\cancel{4}\beta^2}c\cot\theta\right]$$

We simplify our 3-index symbols now

We use $\gamma f = 1$, $2f^2 = \beta^2$ and $\alpha\beta = -2p^2$

or $\frac{\gamma'}{\gamma} = -\frac{f'}{f}$; $\frac{\beta'}{\beta} = \frac{f'}{f}$ ~~$\frac{\alpha'}{2\alpha}$~~, $\frac{\alpha'}{\alpha} + \frac{\beta'}{\beta} = \frac{2p'}{p}$.

$\therefore \frac{\alpha'}{\alpha} = \frac{2p'}{p} - \frac{f'}{f}$; $\frac{\beta'}{\beta} = \frac{f'}{f}$, $\frac{\gamma'}{\gamma} = -\frac{f'}{f}$.

$\therefore B = 0$, $C = -\frac{\alpha}{p}\frac{1}{4}$, $\frac{A'}{2A} = \frac{\alpha'}{2\alpha} + \frac{f'}{f} = \frac{\cancel{2}p'}{p} + \frac{f'}{2f}$

$\therefore \Gamma^3_{\underset{\checkmark}{21}} = \frac{\alpha}{p}\cdot\frac{\cot\theta}{4}$ $\qquad \Gamma^2_{11} = -\frac{2p}{\beta}\frac{\alpha}{4p}\cot\theta = -\frac{\alpha}{\beta}\frac{\cot\theta}{2}$

$\Gamma^1_{\underline{12}} = -\frac{p}{\alpha}\cdot\frac{\alpha}{p4}\cdot\cot\theta \therefore = \frac{1}{4}\cot\theta$, $\qquad \Gamma^2_{\underline{12}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}\frac{\alpha}{4p}\cot\theta$

$$\Gamma^3_{\underline{13}} = \frac{\cancel{2}p'}{p} + \frac{f'}{2f} ~~-\frac{p'}{p}+\frac{f'}{2f}~~ - \frac{f'}{2f} - \frac{p'}{p} + \frac{f'}{2f} + \frac{\cancel{2}f}{\beta}\frac{\alpha}{p4}\cot\theta$$

$= \frac{f'}{2f} + \frac{f\alpha}{\beta p}\cdot\frac{1}{4}\cot\theta$. $\qquad \Gamma^1_{22} = -\frac{\beta}{\alpha}\left(\frac{f'}{2f}\right) + \frac{2f}{\alpha}\cdot\frac{\alpha}{4p}\cot\theta$

$$\Gamma^2_{\underset{\checkmark}{32}} = \frac{p}{\alpha}\left[\cancel{\frac{\beta'}{2\beta}}\,\frac{f'}{2f} - \frac{f}{\beta}\frac{\alpha}{4p}\cot\theta\right]$$

$$\Gamma^1_{\underset{\checkmark}{31}} = -\frac{p}{\alpha}\left[-\frac{f'}{2f} + \frac{f'}{2f} - \frac{f}{\beta}\frac{\alpha}{4p}\cot\theta\right]$$

$$\Gamma^2_{\underset{\checkmark}{31}} = \frac{f}{\beta}\left[\frac{f}{\beta}\cdot\frac{\alpha}{4p}\cot\theta - \frac{p}{2f}\cot\theta\right]$$

$$= \frac{f}{\beta}\left[\frac{f^2\alpha - 2\beta p^2}{4f\beta p}\right]\cot\theta$$

$$= \frac{f}{\beta}\left[\frac{\alpha\beta^2 - 4\beta p^2}{8f\beta p}\right]\cot\theta = \frac{f}{\beta}\cdot\frac{\beta[\alpha\beta - 4p^2]}{8f\beta p}\cot\theta$$

$$= -\frac{6p^2}{8\beta p}\cot\theta = -\frac{3}{4}\frac{p}{\beta}\cot\theta$$

$$\Gamma^1_{\underset{\checkmark}{32}} = \frac{f}{\alpha}\left[-\frac{f'}{f} + \frac{f'}{2f}\right] + \left(\frac{p}{2a} - \frac{p^2}{\alpha^2}\frac{\alpha}{4p}\right)\cot\theta\Big]$$

$$= -\frac{f}{\alpha}\frac{f'}{2f} + \frac{p}{4\alpha}\cot\theta$$

$$g_{11}\Gamma^1_{33} = -\tfrac12 g_{33,1} + 2g_{31}\Gamma^1_{13} + 2g_{32}\Gamma^2_{13}$$

$$\therefore -\alpha\Gamma^1_{33} = \frac{\beta'}{2} + 2\rho\frac{f}{4\beta}\cot\theta + 2(-f)\frac34\frac{\rho}{\beta}\cot\theta$$

$$= \frac{\beta'}{2} - \frac{\rho f}{\beta}\cot\theta$$

$$\therefore \Gamma^1_{33} = -\frac{\beta}{\alpha}\left(\frac{\beta'}{2\beta}\right) + \frac{\rho f}{\alpha\beta}\cot\theta$$

$$g_{22}\Gamma^2_{33} = -\tfrac12 g_{33,2} + 2g_{31}\Gamma^1_{23} + 2g_{32}\Gamma^2_{23}$$

$$\therefore -\beta\Gamma^2_{33} = +\beta\cot\theta - 2\rho\left[\frac{f}{\alpha}\left(\frac{f'}{2f}\right) - \frac{\rho}{4\alpha}\cot\theta\right] \cancel{-2f[}$$

$$-2f\left[-\frac{\beta}{\alpha}\left(\frac{f'}{2f}\right) + \frac{f}{4\beta}\cot\theta\right]$$

$$= +\beta\cot\theta + \left(\frac{\rho^2}{2\alpha} - \frac{f^2}{2\beta}\right)\cot\theta$$

$$= +\beta\cot\theta + \left(\frac{-\alpha\beta}{4\alpha} - \frac{\beta^2}{4\beta}\right)\cot\theta$$

$$= +\beta\cot\theta - \frac{\beta}{2}\cot\theta = \cancel{\frac{3\beta}{2}\cot\theta}\ \frac{\beta}{2}\cot\theta$$

$$\therefore \Gamma^2_{33} = \cancel{\frac{\beta}{2}\cot\theta} - \frac12\cot\theta.$$

We can now work out θ-terms in all $R_{\mu\nu}$. $R_{11} = +\Gamma^s_{11,s} - \Gamma^t_{1s}\Gamma^s_{t1} - \Gamma^s_{1\underline{s},1} + \Gamma^s_{11}\Gamma^t_{s\underline{t}}$

$$= \Gamma^2_{11,2} - \Gamma^2_{11}\Gamma^1_{21} - \Gamma^1_{12}\Gamma^2_{11} - \Gamma^2_{12}\Gamma^2_{21} - \Gamma^3_{12}\Gamma^2_{31} - \cancel{\Gamma^1_{13}\Gamma^3_{11}}$$

$$- \Gamma^2_{13}\Gamma^3_{21} - \Gamma^3_{13}\Gamma^3_{31} + \Gamma^2_{11}\Gamma^t_{2\underline{t}}$$

$$= \frac{\alpha}{\beta}\frac{\operatorname{cosec}^2\theta}{2} - 2\cdot\left(-\frac{\alpha}{\beta}\right)\frac{\cot\theta}{2}\,\frac14\cot\theta - \left(\frac{\beta'}{2\beta} - \frac{\alpha f}{\beta\rho}\frac{\cot\theta}{4}\right)^2$$

$$-2\left(-\frac{\alpha}{\rho}\right)\frac{\cot\theta}{4}\left(-\frac34\right)\frac{\rho}{\beta}\cot\theta - \left(\frac{f'}{2f} + \frac{\alpha f}{\beta\rho}\frac{\cot\theta}{4}\right)^2$$

$$-\frac{\alpha}{\beta}\frac{\cot\theta}{2}\cdot\cot\theta$$

$$= \frac{\alpha}{\cancel{2}\beta} + \cot^2\theta\left[\frac{\alpha}{2\beta} + \frac{\alpha}{4\beta} - \frac{\alpha^2 f^2}{\beta^2\rho^2}\cdot\frac1{16} - \frac38\frac{\alpha}{\beta} - \frac{\alpha^2 f^2}{\beta^2\rho^2}\cdot\frac1{16} - \frac{\alpha}{2\beta}\right]$$

$$= \cot^2\theta\left[\frac{\alpha}{4\beta}\left(1 - \frac32 - \frac12(-1)\right)\right] = 0.$$

$$R_{22} = \Gamma^{s}_{22,s} - \Gamma^{t}_{2s}\Gamma^{s}_{t2} - \Gamma^{s}_{2\underline{s},2} + \Gamma^{s}_{22}\Gamma^{t}_{s\underline{t}}$$

$$= \Gamma^{1}_{22,1} - \Gamma^{1}_{21}\Gamma^{1}_{12} - \Gamma^{2}_{21}\Gamma^{1}_{22} - \Gamma^{3}_{21}\Gamma^{1}_{32} - \Gamma^{1}_{22}\Gamma^{2}_{12}$$

$$- \Gamma^{1}_{23}\Gamma^{3}_{12} - \cancel{\Gamma^{2}_{23}\Gamma^{3}_{22}} - \Gamma^{3}_{23}\Gamma^{3}_{32} - \Gamma^{3}_{2\underline{3},2} + \Gamma^{1}_{22}\Gamma^{t}_{1\underline{t}}$$

$$= \left(\frac{f}{2\phi}\right)'\cot\theta - \frac{1}{16}\cot^2\theta - 2\left[-\frac{\beta}{\alpha}\frac{f'}{2f} + \frac{f}{2\phi}\cot\theta\right]\left[\frac{\beta'}{2\beta} - \frac{\alpha f}{\beta\phi}\frac{\cot\theta}{4}\right]$$

$$-2\frac{\dot{\alpha}}{\cancel{\phi}}\frac{\cot\theta}{4}\left(-\frac{f}{\alpha}\frac{f'}{2f} + \frac{\phi}{4\alpha}\cot\theta\right) - \frac{9}{16}\cot^2\theta + \operatorname{cosec}^2\theta$$

$$+ \frac{f}{2\phi}\cot\theta\left(\frac{\alpha'}{2\alpha} + \frac{\gamma'}{2\gamma} + \frac{f'}{f}\right)$$

$$= \cot^2\theta\left[-\frac{1}{16} + 2\frac{\alpha f^2}{\beta\phi^2}\cdot\frac{1}{8} - \frac{2}{16} - \frac{9}{16} + 1\right]$$

$$+ \frac{f}{2\phi}\cot\theta\left[\frac{f'}{f} - \frac{\phi'}{\phi} - \frac{1}{2}\frac{\beta}{\alpha}\frac{f'}{2f}\cancel{\frac{\alpha f}{\beta\phi}} - 2\frac{f'}{2f}\right.$$

$$\left. + \frac{f'}{2f} + \frac{\cancel{2}\phi'}{\phi} - \frac{f'}{2f} - \frac{f'}{2f} + \frac{f'}{f}\right]$$

$$= 0 + 0$$

$$R_{33} = \Gamma^{s}_{33,s} - \Gamma^{t}_{3s}\Gamma^{s}_{t3} - \Gamma^{s}_{3\underline{s},3} + \Gamma^{s}_{33}\Gamma^{t}_{s\underline{t}}$$

$$= \Gamma^{1}_{33,1} + \Gamma^{2}_{33,2} - \Gamma^{1}_{31}\Gamma^{1}_{13} - \Gamma^{2}_{31}\Gamma^{1}_{23} - \Gamma^{3}_{31}\Gamma^{1}_{33}$$

$$- \Gamma^{1}_{32}\Gamma^{2}_{13} - \Gamma^{2}_{32}\Gamma^{2}_{23} - \Gamma^{3}_{32}\Gamma^{2}_{33} - \Gamma^{1}_{33}\Gamma^{3}_{13} - \Gamma^{2}_{33}\Gamma^{3}_{23}$$

$$+ \Gamma^{2}_{33}\Gamma^{t}_{2\underline{t}} \qquad \frac{\cot^2\theta - 1}{\cancel{1\ \cot\theta}} \qquad + \Gamma^{1}_{33}\Gamma^{t}_{1\underline{t}}$$

$$= \left(\frac{\phi f}{\alpha\beta}\right)'\cot\theta\sin^2\theta + \left(-\frac{1}{2}\right)\left(\cancel{\cot^2\theta - 1}\right)\sin^2\theta$$

$$+ \frac{f^2}{16\beta^2}\cot^2\theta\sin^2\theta - 2\left(-\frac{3}{4}\frac{\phi}{\beta}\right)\cot\theta\sin^2\theta\left(\frac{f}{\alpha}\frac{f'}{2f} - \frac{\phi}{4\alpha}\cot\theta\right)$$

$$-2\left(\frac{f'}{2f} + \frac{\alpha f}{\beta\phi}\frac{\cot\theta}{4}\right)\left[-\frac{\beta}{\alpha}\left(\frac{\beta'}{2\beta}\right) + \frac{\phi f}{\alpha\beta}\cot\theta\right]\sin^2\theta$$

$$+\left(\frac{\phi}{\alpha}\frac{f'}{2f} - \frac{f}{4\beta}\cot\theta\right)^2\sin^2\theta$$

$$-2\,\frac{3}{4}\cot\theta\cdot\left(-\frac{1}{2}\right)\cot\theta\sin^2\theta - \frac{1}{2}\cot\theta\sin^2\theta$$

$$= \cot^2\theta\sin\theta\left(-\frac{1}{2} + \frac{f^2}{16\beta^2} - \frac{3}{8}\frac{p^2}{\alpha\beta} - \frac{1}{2}\frac{f^2}{\beta^2} + \frac{f^2}{16\beta^2} + \frac{3}{4} - \frac{1}{2}\right) + \cdot$$

coeff of $\cot^2\theta\sin\theta$ is $-\frac{1}{2} + \frac{1}{32} + \frac{3}{16}\boxed{-\frac{1}{4}} + \frac{1}{32} + \boxed{\frac{3}{4} - \frac{1}{2}}$

$$= \frac{\cancel{-16+1+6-8+1+24-16}}{32} = -\frac{9}{32}$$

$$= -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$$ which does not vanish

coeff of $\cot\theta\sin^2\theta$ is $\frac{f}{2p}\left[-\frac{f'}{f} + \frac{p'}{p}\right] + \frac{3}{4}\frac{p}{\beta}\frac{f}{a}\frac{f'}{f} - 2\frac{f'}{f}\frac{pf}{\alpha\beta}$

$$+ 2\frac{\beta}{\alpha}\frac{f'}{2f}\frac{\alpha f}{\beta p}\cdot\frac{1}{4} - 2\frac{p}{a}\frac{f'}{2f}\cdot\frac{f}{4\beta}\Big]\frac{a}{\beta} - \frac{f}{2p}\left(\frac{\gamma'}{2\gamma} + \frac{A'}{2A}\right)$$

$$= \frac{f}{2p}\left[-\frac{f'}{f} + \frac{p'}{p} - \frac{3}{4}\frac{f'}{f} + \frac{f'}{f} + \frac{1}{2}\frac{f'}{f} + \frac{1}{4}\frac{f'}{f}\right]\frac{a}{a} - \left(-\frac{f'}{2f} + \frac{p'}{p} + \frac{f'}{2f}\right)$$

$$= \frac{f}{2p}\left[\frac{p'}{p} - \frac{p'}{p}\right] = 0$$

So θ terms can not be removed

~~Let us now take~~ The only possibility to be discussed is the one in which you cannot remove a.

In that case g_{44} automatically vanishes and we can choose (r, t) such that g_{11} also vanishes.

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & p\sin\theta & a \\ 0 & -\beta & f\sin\theta & 0 \\ -p\sin\theta & -f\sin\theta & -\beta\sin^2\theta & 0 \\ a & 0 & 0 & 0 \end{pmatrix} \quad \therefore g = \cancel{-a[-p\sin\theta} = -a^2(\beta^2+f^2)\sin^2\theta.$$

$$g^{13}_{\nu} = 0, \quad g^{34}_{\nu} = -\frac{p\sin\theta\, a\beta}{-a^2(\beta^2+f^2)\sin\theta} =$$

$\cancel{\Gamma^{4}_{44}}$ 4

$\cancel{g_{44}\Gamma^{4}_{44}} + g_{41}\Gamma^{1}_{44} = 0 \quad \therefore \Gamma^{1}_{44} = 0 \qquad \Gamma^{2}_{44} = 0.$

$\cancel{g_{11}\Gamma^{1}_{44}} + g_{14}\Gamma^{4}_{44} = g_{14,4} \quad \therefore \Gamma^{4}_{44} = \frac{\dot{a}}{a}. \qquad \Gamma^{3}_{44} = 0.$

$g_{22}\Gamma^{2}_{44} = 0 \quad g_{33}\Gamma^{3}_{44} = 0$

$$g_{\mu\nu} = \begin{matrix} 0 & 0 & f\sin\theta & a \\ 0 & -\beta & f\sin\theta & 0 \\ -f\sin\theta & f\sin\theta & -f\sin\theta & 0 \\ a & 0 & 0 & v \end{matrix}$$

$$g_{44}\cancel{\Gamma^4_{\underline{14}}} + g_{4\underline{1}}\Gamma^1_{1\underline{4}} = -\tfrac12 g_{14,4} + \tfrac12 g_{44,4}$$

$$+ g_{\underset{\vee}{1}3}\Gamma^3_{44} = 0$$

$$\therefore \Gamma^1_{\underline{14}} = 0, \quad \Gamma^1_{1\underset{\vee}{4}} = 0$$

$$g_{44}\cancel{\Gamma^4_{1\underset{\vee}{4}}} + g_{4\underline{1}}\Gamma^1_{1\underset{\vee}{4}} = -g_{1\underset{\vee}{3}}\Gamma^3_{44} = 0 \qquad g_{44}\Gamma^4_{2\underset{\vee}{4}} + g_{4\underline{1}}\Gamma^1_{2\underset{\vee}{4}} = -g_{2\underset{\vee}{3}}\Gamma^3_{44} = 0$$

$$g_{44}\cancel{\Gamma^4_{\underline{24}}} + g_{4\underline{1}}\Gamma^1_{2\underline{4}} = +g_{2\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{4}} = 0 \qquad \Gamma^1_{\underline{24}} = 0, \quad \Gamma^1_{\underset{\vee}{2}4} = 0$$

$$g_{44}\cancel{\Gamma^4_{\underline{34}}} + g_{4\underline{1}}\Gamma^1_{\underline{34}} = g_{3\underset{\vee}{1}}\Gamma^1_{4\underset{\vee}{4}} + g_{\underset{\vee}{3}2}\Gamma^2_{44} = 0 \qquad \Gamma^1_{\underline{34}} = 0$$

$$g_{44}\cancel{\Gamma^4_{3\underset{\vee}{4}}} + g_{4\underline{1}}\Gamma^1_{3\underset{\vee}{4}} = -g_{3\underset{\vee}{1}}\Gamma^1_{\underline{44}} - g_{\underset{\vee}{3}2}\Gamma^2_{44} = 0 \qquad \Gamma^1_{3\underset{\vee}{4}} = 0$$

$$\cancel{g_{44}\Gamma^4_{11} + g_{\underline{41}}\Gamma^1_{11} = g_{\underline{14},1} + 2g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{1}}}$$

$$g_{33}\Gamma^3_{4\underset{\vee}{1}} = -\tfrac12 g_{1\underset{\vee}{3},4} - g_{3\underset{\vee}{1}}\Gamma^3_{\underline{43}}$$

$$g_{33}\Gamma^3_{\underline{43}} = \tfrac12 g_{33,4} + g_{1\underset{\vee}{3}}\Gamma^1_{4\underset{\vee}{3}} + g_{\underset{\vee}{2}3}\Gamma^2_{4\underset{\vee}{3}}$$

$$\cancel{g_{11}\Gamma^1_{4\underset{\vee}{3}}} + g_{44}\cancel{\Gamma^4_{4\underset{\vee}{3}}} + g_{4\underline{1}}\Gamma_{43} = -g_{1\underset{\vee}{3}}\Gamma^1_{\underline{44}} = 0$$

$$g_{22}\Gamma^2_{4\underset{\vee}{3}} = -\tfrac12 g_{3\underset{\vee}{2},4} - g_{1\underset{\vee}{3}}\Gamma^1_{\underline{42}} - g_{2\underset{\vee}{3}}\Gamma^2_{\underline{42}}$$

$$g_{44}\cancel{\Gamma^4_{11}} + g_{4\underline{1}}\Gamma^1_{11} = g_{1\underline{4},1} + 2g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{1}}$$

$$g_{33}\Gamma^3_{4\underset{\vee}{1}} = -\tfrac12 g_{1\underset{\vee}{3},4} - g_{3\underset{\vee}{1}}\Gamma^3_{\underline{43}}$$

$$g_{33}\Gamma^3_{\underline{43}} = \tfrac12 g_{33,4} + g_{1\underset{\vee}{3}}\cancel{\Gamma^1_{4\underset{\vee}{3}}} + g_{2\underset{\vee}{3}}\Gamma^2_{4\underset{\vee}{3}}$$

$$g_{44}\cancel{\Gamma^4_{4\underset{\vee}{3}}} + g_{4\underline{1}}\Gamma^1_{4\underset{\vee}{3}} = -g_{1\underset{\vee}{3}}\Gamma^1_{44} - g_{2\underset{\vee}{3}}\Gamma^2_{44} = 0$$

$$g_{22}\Gamma^2_{4\underset{\vee}{3}} = -\tfrac12 g_{3\underset{\vee}{2},4} - \cancel{g_{1\underset{\vee}{3}}\Gamma^1_{\underline{42}}} - g_{2\underset{\vee}{3}}\Gamma^2_{\underline{42}}$$

$$g_{22}\Gamma^2_{\underline{42}} = \tfrac12 g_{22,4} + g_{3\underset{\vee}{2}}\Gamma^3_{4\underset{\vee}{2}}$$

$$g_{33}\Gamma^3_{4\underset{\vee}{2}} = -\tfrac12 g_{2\underset{\vee}{3},4} - g_{3\underset{\vee}{2}}\Gamma^3_{\underline{43}}$$

These give 6 ratios

Γ^1_{11}, $\Gamma^3_{\underline{43}}$ $\Gamma^2_{\underline{42}}$

$\Gamma^3_{4\underset{\vee}{1}}$ $\Gamma^2_{4\underset{\vee}{3}}$, $\Gamma^3_{4\underset{\vee}{2}}$.

$$g_{11}\cancel{\Gamma^1_{\underline{14}}} + g_{1\underline{4}}\Gamma^4_{\underline{14}} = -\tfrac12 g_{1\underline{4},1} + \tfrac12 g_{4\underline{1},1} + g_{1\underset{\vee}{3}}\Gamma^3_{14} = 0 \qquad \cancel{\Gamma^4_{\underline{14}} = 0}$$

$$\cancel{g_{11}\Gamma^1_{1\underset{\vee}{4}}} + g_{1\underline{4}}\Gamma^4_{1\underset{\vee}{4}} = -g_{1\underset{\vee}{3}}\Gamma^3_{\underline{14}} \qquad g_{22}\Gamma^2_{\underline{34}} = +\cancel{g_{3\underset{\vee}{1}}\Gamma^1_{2\underset{\vee}{4}}} + g_{3\underset{\vee}{2}}\Gamma^2_{2\underset{\vee}{4}}$$

$$g_{33}\Gamma^3_{\underline{14}} = \cancel{\tfrac12 g_{31,4}} + g_{1\underset{\vee}{3}}\Gamma^3_{3\underset{\vee}{4}} \qquad g_{22}\Gamma^2_{2\underset{\vee}{4}} = -g_{2\underset{\vee}{3}}\Gamma^3_{\underline{24}}$$

$$g_{33}\Gamma^3_{3\underset{\vee}{4}} = -\cancel{g_{3\underset{\vee}{1}}\Gamma^1_{\underline{34}}} - g_{3\underset{\vee}{3}}\Gamma^2_{\underline{34}}$$

$g_{33}\Gamma^{3}_{\underline{24}} = g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{34}}$; $\cancel{\Gamma^{4}_{\underset{\vee}{14}}}$ $\Gamma^{3}_{\underline{14}} = \Gamma^{2}_{\underline{34}} = \Gamma^{3}_{\underline{24}} = 0$ $\quad {}^{4}_{12}$

$\Gamma^{4}_{\underset{\vee}{14}} = \Gamma^{4}_{\underset{\vee}{34}} = \Gamma^{2}_{\underset{\vee}{24}} = 0$

~~$\cancel{g_{44}\Gamma^{4}_{\underline{12}}} + g_{\underline{41}}\Gamma^{1}_{\underline{12}} = g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{42}}$ Determins $\Gamma^{1}_{\underline{12}}$~~

~~$\cancel{g_{44}\Gamma^{4}_{\underset{\vee}{12}}} + g_{\underline{41}}\Gamma^{1}_{\underset{\vee}{12}} = -g_{\underset{\vee}{13}}\Gamma^{3}_{\underline{42}} - g_{\underset{\vee}{32}}\Gamma^{2}_{14} = 0$ ∴ $\Gamma^{1}_{\underset{\vee}{12}} = 0$~~

~~$g_{22}\Gamma^{2}_{\underline{14}} = g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{24}}$ Determins $\Gamma^{2}_{\underline{14}}$~~

~~$g_{22}\Gamma^{2}_{\underset{\vee}{14}} = -g_{\underset{\vee}{13}}\Gamma^{3}_{\underline{24}} = 0$ $\quad \Gamma^{2}_{\underset{\vee}{14}} = 0$~~

~~$g_{11}\Gamma^{1}_{\underline{24}} + g_{\underline{14}}\Gamma^{4}_{\underline{24}} = g_{\underset{\vee}{23}}\Gamma^{3}$~~

$\cancel{g_{44}\Gamma^{4}_{\underline{12}}} + g_{\underline{41}}\Gamma^{1}_{\underline{12}} = g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{42}} + g_{\underset{\vee}{32}}\Gamma^{3}_{\underset{\vee}{14}}$ Determs $\Gamma^{1}_{\underline{12}}$

$g_{44}\Gamma^{4}_{\underset{\vee}{12}} + g_{\underline{41}}\Gamma^{1}_{\underset{\vee}{12}} = -g_{\underset{\vee}{13}}\Gamma^{3}_{\underline{42}} - g_{\underset{\vee}{32}}\Gamma^{3}_{\underline{14}} = 0$ $\quad \Gamma^{1}_{\underset{\vee}{12}} = 0$.

$g_{22}\Gamma^{2}_{\underline{14}} = g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{24}}$ Determins $\Gamma^{2}_{\underline{14}}$

$g_{22}\Gamma^{2}_{\underset{\vee}{14}} = -g_{\underset{\vee}{13}}\Gamma^{3}_{\underline{24}} = 0$ $\quad \Gamma^{2}_{\underset{\vee}{14}} = 0$

$\cancel{g_{11}\Gamma^{1}_{\underline{24}}} + g_{\underline{14}}\Gamma^{4}_{\underline{24}} = g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{14}}$ Determins $\Gamma^{4}_{\underline{24}}$

$g_{11}\Gamma^{1}_{\underset{\vee}{24}} + g_{\underline{14}}\Gamma^{4}_{\underset{\vee}{24}} = -g_{\underset{\vee}{23}}\Gamma^{3}_{\underline{14}}$ $\quad$ ∴ $\Gamma^{4}_{\underset{\vee}{24}} = 0$

$\cancel{g_{44}\Gamma^{4}_{\underline{13}}} + g_{\underline{41}}\Gamma^{1}_{\underline{13}} = g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\nabla}{43}} + g_{\underset{\vee}{13}}\Gamma^{1}_{\underset{\vee}{14}} + g_{\underset{\vee}{23}}\Gamma^{2}_{\underset{\vee}{14}} = 0$ $\quad \Gamma^{1}_{\underline{13}} = 0$

$\cancel{g_{44}\Gamma^{4}_{\underset{\vee}{13}}} + g_{\underline{41}}\Gamma^{1}_{\underset{\vee}{13}} = \frac{1}{2}g_{\underset{\vee}{13},4} - g_{\underset{\vee}{13}}\Gamma^{3}_{\underline{43}} - g_{\underset{\vee}{13}}\cancel{\Gamma^{1}_{\underline{14}}} - g_{\underset{\vee}{23}}\Gamma^{2}_{\underline{14}}$ $\quad \Gamma^{1}_{\underset{\vee}{13}}$ Determine

$g_{33}\Gamma^{3}_{\underline{14}} = -g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{34}} = 0$ $\quad \Gamma^{3}_{\underline{14}} = 0$

$\cancel{g_{11}\Gamma^{1}_{\underline{34}}} + g_{\underline{14}}\Gamma^{4}_{\underline{34}} = g_{\underset{\vee}{31}}\Gamma^{1}_{\underset{\vee}{14}} + g_{\underset{\vee}{32}}\Gamma^{2}_{\underset{\vee}{14}} = 0$ $\quad \Gamma^{4}_{\underline{34}} = 0$

$g_{11}\cancel{\Gamma^{1}_{\underset{\vee}{34}}} + g_{\underline{14}}\Gamma^{4}_{\underset{\vee}{34}} = -\frac{1}{2}g_{\underset{\vee}{13},4} - g_{\underset{\vee}{31}}\cancel{\Gamma^{1}_{\underline{14}}} - g_{\underset{\vee}{32}}\Gamma^{2}_{\underline{14}}$ $\quad \Gamma^{4}_{\underset{\vee}{34}}$ is determined

$\cancel{g_{11}\Gamma_{22}}\,\cancel{g_{44}\Gamma^{4}_{22}} + g_{\underline{41}}\Gamma^{1}_{22} = -\frac{1}{2}g_{22,4} + 2g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{42}}$ $\quad \Gamma^{1}_{22}$ is determined

$\cancel{g_{44}\Gamma^{4}_{\underline{23}}} + g_{\underline{41}}\Gamma^{1}_{\underline{23}} = g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{43}} + g_{\underset{\vee}{13}}\Gamma^{1}_{\underset{\vee}{24}} + g_{\underset{\vee}{23}}\Gamma^{2}_{\underset{\vee}{24}}$ $\quad$ ∴ $\Gamma^{1}_{\underline{23}} = 0$

$\cancel{g_{44}\Gamma^{4}_{\underset{\vee}{23}}} + g_{\underline{41}}\Gamma^{1}_{\underset{\vee}{23}} = +\frac{1}{2}g_{\underset{\vee}{23},4} - g_{\underset{\vee}{23}}\Gamma^{3}_{\underline{43}} - g_{\underset{\vee}{13}}\Gamma^{1}_{\underline{24}} - g_{\underset{\vee}{23}}\Gamma^{2}_{\underline{24}}$ $\quad \Gamma^{1}_{\underset{\vee}{23}}$ is determined

$\cancel{g_{44}}\Gamma^{4}_{33} + g_{\underline{41}}\Gamma^{1}_{33} = -\frac{1}{2}g_{33,4} + 2g_{\underset{\vee}{31}}\cancel{\Gamma^{1}_{\underset{\vee}{43}}} + 2g_{\underset{\vee}{32}}\Gamma^{2}_{43}$ $\quad \Gamma^{1}_{33}$ is determined

$\cancel{g_{11}\Gamma^{1}_{11}} + g_{\underline{14}}\Gamma^{4}_{11} = \frac{1}{2}g_{11,1} = 0 \quad \therefore \Gamma^{4}_{11} = 0$

$g_{33}\Gamma^{3}_{11} = 2g_{1\underset{\vee}{3}}\Gamma^{3}_{3\underset{\vee}{1}} \;;\; g_{33}\Gamma^{3}_{3\underset{\vee}{1}} = -\cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{\underline{31}}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{31}} - g_{3\underset{\vee}{1}}\Gamma^{3}_{33}$

$g_{22}\Gamma^{2}_{\underline{31}} = \cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{2\underset{\vee}{1}}} + g_{3\underset{\vee}{2}}\Gamma^{2}_{2\underset{\vee}{1}} + g_{3\underset{\vee}{1}}\Gamma^{3}_{3\underset{\vee}{2}} \;;\; g_{33}\Gamma^{3}_{33} = 0$

$g_{22}\Gamma^{2}_{2\underset{\vee}{1}} = -g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{21}} - g_{3\underset{\vee}{1}}\Gamma^{3}_{22} \;;\; g_{33}\Gamma^{3}_{3\underset{\vee}{2}} = -\cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{\underline{32}}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{32}} - g_{3\underset{\vee}{2}}\Gamma^{3}_{33}$

$g_{33}\Gamma^{3}_{\underline{21}} = \cancel{g_{2\underset{\vee}{3}}\Gamma^{3}_{3\underset{\vee}{1}}} + \cancel{g_{3\underset{\vee}{1}}\Gamma^{3}_{2\underset{\vee}{3}}} \;;\; g_{33}\Gamma^{3}_{22} = 2\cancel{g_{2\underset{\vee}{3}}\Gamma^{3}_{3\underset{\vee}{2}}}$

$g_{22}\Gamma^{2}_{\underline{23}} = \cancel{g_{2\underset{\vee}{3}}\Gamma^{3}_{2\underset{\vee}{3}}} + \cancel{g_{1\underset{\vee}{3}}\Gamma^{1}_{2\underset{\vee}{2}}} - \cancel{g_{2\underset{\vee}{3}}\Gamma^{2}_{2\underset{\vee}{2}}} = 0$

$\Gamma^{3}_{11} = \Gamma^{2}_{3\underline{1}} = \Gamma^{3}_{33} = \Gamma^{3}_{\underline{21}} = \Gamma^{3}_{22} = \Gamma^{2}_{2\underline{3}} = 0 \;;\; \Gamma^{3}_{3\underset{\vee}{1}} = \Gamma^{2}_{2\underset{\vee}{1}} = 0 \quad \Gamma^{3}_{3\underset{\vee}{2}} = 0$

~~$g_{11}\cancel{\Gamma^{1}_{1\underline{2}}} + g_{1\underline{4}}\Gamma^{4}_{1\underline{2}} = g_{1\underset{\vee}{3}}\Gamma^{3}_{1\underset{\vee}{1}} + \cancel{g_{3\underset{\vee}{2}}\Gamma^{3}_{1\underset{\vee}{1}}}$~~

~~$g_{33}\Gamma^{3}_{1\underset{\vee}{2}} = -\frac{1}{2}g_{3\underset{\vee}{3},1}$~~

$g_{22}\Gamma^{2}_{11} = 2g_{1\underset{\vee}{3}}\Gamma^{3}_{2\underset{\vee}{1}}\,.$

$g_{33}\Gamma^{3}_{2\underset{\vee}{1}} = -\frac{1}{2}g_{1\underset{\vee}{3},2} + \frac{1}{2}g_{3\underset{\vee}{2},1} - g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{31}} - g_{3\underset{\vee}{1}}\Gamma^{3}_{\underline{23}}$

$g_{33}\Gamma^{3}_{\underline{31}} = \frac{1}{2}g_{33,1} + g_{3\underset{\vee}{1}}\Gamma^{1}_{3\underset{\vee}{1}} + g_{3\underset{\vee}{2}}\Gamma^{2}_{3\underset{\vee}{1}} + \cancel{g_{3\underset{\vee}{1}}\Gamma^{3}_{3\underset{\vee}{3}}}$

$g_{33}\Gamma^{3}_{\underline{32}} = \frac{1}{2}g_{33,2} + g_{3\underset{\vee}{1}}\Gamma^{1}_{3\underset{\vee}{2}} + g_{3\underset{\vee}{2}}\Gamma^{2}_{3\underset{\vee}{2}} + \cancel{g_{3\underset{\vee}{2}}\Gamma^{3}_{3\underset{\vee}{3}}}$

$g_{22}\Gamma^{2}_{3\underset{\vee}{1}} = \frac{1}{2}g_{3\underset{\vee}{1},2} - \frac{1}{2}g_{2\underset{\vee}{3},1} - g_{3\underset{\vee}{1}}\Gamma^{1}_{\underline{21}} - g_{3\underset{\vee}{2}}\Gamma^{2}_{\underline{21}} - g_{3\underset{\vee}{1}}\Gamma^{3}_{\underline{32}}$

$g_{22}\Gamma^{2}_{\underline{21}} = \frac{1}{2}g_{22,1} + g_{2\underset{\vee}{3}}\Gamma^{3}_{2\underset{\vee}{1}} + \cancel{g_{3\underset{\vee}{1}}\Gamma^{3}_{2\underset{\vee}{2}}}$

The ratios are interconnected Let us evaluate all of them

$$g_{\mu\nu} = \begin{pmatrix} 0 & 0 & f\sin\theta & a \\ 0 & -\beta & f\sin\theta & 0 \\ f\sin\theta & -f\sin\theta & -\beta\sin\theta & 0 \\ a & 0 & 0 & 0 \end{pmatrix} \quad \therefore g = -a^2(\beta^2+f^2)\sin^2\theta = -a^2A^2\sin^2\theta$$

$$A^2 = \beta^2 + f^2 \quad \therefore \frac{\dot{A}}{A} = \frac{\beta\dot{\beta} + f\dot{f}}{\beta^2+f^2}$$

$$g^{3\underset{\vee}{4}} = \frac{-f\sin\theta\, a\beta}{-a^2(\beta^2+f^2)\sin^2\theta} = \frac{\beta f}{aA^2\sin\theta} \quad \therefore \textcircled{g}^{3\underset{\vee}{4}} = \frac{\beta f}{aA^2\sin\theta}\cdot aA\sin\theta$$

$$= \frac{\beta f}{A} \qquad \textcircled{g}^{3\underset{\vee}{4}}{}_{,4} = 0 \quad \therefore \frac{\dot{\beta}}{\beta} + \frac{\dot{f}}{f} - \frac{\dot{A}}{A} = 0\,.$$

~~444~~, ~~441~~, ~~442~~, ~~443~~, ~~114~~, ~~224~~, ~~334~~, ~~124~~, ~~134~~, ~~234~~,

$g_{44}\Gamma^4_{44} + g_{4\underline{1}}\Gamma^1_{44} = 0 \therefore \Gamma^1_{44} = 0$ (1). $\cancel{g_{11}\Gamma^1_{44}} + g_{4\underline{4}}\Gamma^4_{44} = g_{4\underline{4},4} \therefore \Gamma^4_{44} = \frac{\dot a}{a}$ (2)

$\cancel{g_{44}\Gamma^4_{1\underline{4}}} + g_{4\underline{1}}\Gamma^1_{1\underline{4}} = -\frac{1}{2}g_{1\underline{4},4} + \frac{1}{2}g_{4\underline{4},4} + g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{4}} = 0$ $\Gamma^1_{1\underline{4}} = 0$ (3).

$\cancel{g_{44}\Gamma^4_{\underset{\vee}{1}4}} + g_{4\underline{1}}\Gamma^1_{1\underset{\vee}{4}} = -g_{1\underset{\vee}{3}}\Gamma^3_{44}$; $g_{33}\Gamma^3_{44} = 0$ — $\Gamma^3_{44} = 0$, $\Gamma^1_{1\underset{\vee}{4}} = 0$ (5)

$\cancel{g_{44}\Gamma^4_{4\underline{3}}} + g_{4\underline{1}}\Gamma^1_{4\underline{3}} = g_{1\underset{\vee}{3}}\Gamma^1_{\underset{\vee}{4}4} + g_{2\underset{\vee}{3}}\Gamma^2_{4\underset{\vee}{4}} = 0$; $\cancel{g_{44}\Gamma^4_{4\underset{\vee}{3}}} + g_{4\underline{1}}\Gamma^1_{4\underset{\vee}{3}} = -g_{1\underset{\vee}{3}}\Gamma^1_{44} - g_{2\underset{\vee}{3}}\Gamma^2_{44}$

$g_{22}\Gamma^2_{44} = 0.$ $\Gamma^2_{44} = 0$, $\Gamma^1_{4\underline{3}} = 0$, $\Gamma^1_{4\underset{\vee}{3}} = 0$ (8), $\Gamma^1_{2\underline{4}} = 0$, $\Gamma^1_{2\underset{\vee}{4}} = 0$ (10)

$\cancel{g_{44}\Gamma^4_{2\underline{4}}} + g_{4\underline{1}}\Gamma^1_{2\underline{4}} = g_{2\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{4}} = 0$; $\cancel{g_{44}\Gamma^4_{2\underset{\vee}{4}}} + g_{4\underline{1}}\Gamma^1_{2\underset{\vee}{4}} = -g_{2\underset{\vee}{3}}\Gamma^3_{44} = 0$

$\cancel{g_{44}\Gamma^4_{11}} + g_{4\underline{1}}\Gamma^1_{11} = \frac{1}{2}g_{1\underline{4},1} + \frac{1}{2}g_{4\underline{1},1} + 2g_{1\underset{\vee}{3}}\Gamma^3_{4\underset{\vee}{1}}$:

$g_{33}\Gamma^3_{4\underset{\vee}{1}} = -\frac{1}{2}g_{1\underset{\vee}{3},4} - g_{\underset{\vee}{3}1}\Gamma^3_{4\underline{3}}$; $g_{33}\Gamma^3_{4\underline{3}} = \frac{1}{2}g_{33,4} + \cancel{g_{1\underset{\vee}{3}}\Gamma^1_{4\underset{\vee}{3}}} + g_{2\underset{\vee}{3}}\Gamma^2_{4\underset{\vee}{3}}$;

$g_{22}\Gamma^2_{4\underset{\vee}{3}} = -\frac{1}{2}g_{3\underset{\vee}{2},4} - \cancel{g_{1\underset{\vee}{3}}\Gamma^1_{4\underline{2}}} - g_{2\underset{\vee}{3}}\Gamma^2_{4\underline{2}}$; $g_{22}\Gamma^2_{4\underline{2}} = \frac{1}{2}g_{22,4} + g_{3\underset{\vee}{2}}\Gamma^3_{4\underset{\vee}{2}}$;

$g_{33}\Gamma^3_{4\underset{\vee}{2}} = -\frac{1}{2}g_{2\underset{\vee}{3},4} - g_{3\underset{\vee}{2}}\Gamma^3_{4\underline{3}}$. $-\beta\Gamma^2_{4\underline{2}} = -\frac{\dot\beta}{2} - f\Gamma^3_{4\underset{\vee}{2}}$ $\therefore \Gamma^2_{4\underline{2}} = \frac{\dot\beta}{2\beta} + \frac{f}{\beta}\Gamma^3_{4\underset{\vee}{2}}$.

$-\beta\Gamma^3_{4\underset{\vee}{2}} = -\frac{\dot f}{2} + f\Gamma^3_{4\underline{3}}$ $\therefore f\Gamma^3_{4\underline{3}} = \frac{\dot f}{2} - \beta\Gamma^3_{4\underset{\vee}{2}}$ $\therefore \Gamma^3_{4\underline{3}} = \frac{\dot f}{2f} - \frac{\beta}{f}\Gamma^3_{4\underset{\vee}{2}}$.

$\Gamma^1_{4\underline{1}} + \Gamma^2_{4\underline{2}} + \Gamma^3_{4\underline{3}} + \Gamma^4_{44} = \cancel{a}\frac{\dot a}{a} + \frac{\dot A}{A}$ $\therefore$ $\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} + \Gamma^3_{4\underset{\vee}{2}}\left(\frac{f}{\beta} - \frac{\beta}{f}\right) + \cancel{\frac{\dot a}{a}} = \cancel{\frac{\dot a}{a}} + \frac{\dot A}{A}$.

$\therefore \Gamma^3_{4\underset{\vee}{2}}\left(\frac{f^2-\beta^2}{f\beta}\right) = \frac{\dot A}{A} - \frac{\dot\beta}{2\beta} - \frac{\dot f}{2f}$ $\therefore \Gamma^3_{4\underset{\vee}{2}} = \frac{f\beta}{\beta^2-f^2}\left[\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right]$ (11)

$\Gamma^2_{4\underline{2}} = \frac{\dot\beta}{2\beta} + \frac{f^2}{\beta^2-f^2}\left[\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right] = \frac{\beta^2}{\beta^2-f^2}\cdot\frac{\dot\beta}{2\beta} + \frac{f^2}{\beta^2-f^2}\left(\frac{\dot f}{2f} - \frac{\dot A}{\cancel{2}A}\right)$ ~~(12)~~

$\Gamma^3_{4\underline{3}} = \frac{\dot f}{2f} - \frac{\beta^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) = \frac{\beta^2}{\beta^2-f^2}\left(\frac{\dot A}{\cancel{2}A} - \frac{\dot\beta}{2\beta}\right) - \frac{f^2}{\beta^2 f^2}\frac{\dot f}{2f}$ ~~(13)~~

$\Gamma^2_{4\underline{2}} = \frac{\beta\dot\beta + f\dot f}{2(\beta^2-f^2)} - \frac{f^2}{\beta^2-f^2}\frac{\beta\dot\beta + f\dot f}{\beta^2+f^2} = \frac{\beta\dot\beta + f\dot f\ \cancel{2}}{2(\beta^2+f^2)(\beta^2-f^2)}\left[\beta^2 + f^2 - 2f^2\right]$

$= \frac{\beta\dot\beta + f\dot f}{2(\beta^2+f^2)} = \frac{\dot A}{2A}$ (12) $\Gamma^3_{4\underline{3}} = \frac{\beta^2}{\beta^2-f^2}\frac{\beta\dot\beta + f\dot f}{\beta^2+f^2} - \frac{\beta\dot\beta + f\dot f}{2(\beta^2-f^2)}$

$\Gamma^3_{4\underline{3}} = \frac{\beta\dot\beta + f\dot f}{2(\beta^2+f^2)(\beta^2-f^2)}\left[2\beta^2 - \beta^2 - f^2\right] = +\frac{\beta\dot\beta + f\dot f}{2(\beta^2+f^2)} = \frac{\dot A}{2A}$ (13)

$-\beta\Gamma^2_{4\underset{\vee}{3}} = \frac{\dot f}{2} - f\frac{\dot A}{2A}$ $\therefore \Gamma^2_{4\underset{\vee}{3}} = \frac{f}{\beta}\left(\frac{\dot A}{2A} - \frac{\dot f}{2f}\right)$ (14)

$-\beta\Gamma^3_{4\underline{3}} = -\frac{\dot\beta}{2} + \frac{f^2}{\beta}\left(\frac{\dot A}{2A} - \frac{\dot f}{2f}\right)$ $\therefore \Gamma^3_{4\underline{3}} = \frac{\dot\beta}{2\beta} + \frac{f^2}{\beta^2}\frac{\dot f}{2f} - \frac{f^2}{\beta^2}\frac{\dot A}{2A}$

$\therefore \Gamma^3_{4\underline{3}} = \frac{\beta\dot\beta + f\dot f}{2\beta^2} - \frac{f^2}{\beta^2}\frac{\dot A}{2A} = \frac{\beta\dot\beta + f\dot f}{2\beta^2} - \frac{f^2}{\beta^2}\frac{\beta\dot\beta + f\dot f}{2(\beta^2+f^2)} = \frac{\beta\dot\beta + f\dot f}{2\beta^2(\beta^2+f^2)}(\beta^2 + f^2 - f^2)$

$= \frac{\beta\dot\beta + f\dot f}{2(\beta^2+f^2)} = \frac{\dot A}{2A}$. So verified

$-\beta\Gamma^3_{4\underset{\vee}{1}} = -\frac{\dot\phi}{2} + \phi\frac{\dot A}{2A}$ $\therefore \Gamma^3_{4\underset{\vee}{1}} = \cancel{\frac{\phi}{\beta}}\frac{\phi}{\beta}\left(\frac{\dot\phi}{2\phi} - \frac{\dot A}{2A}\right) = \frac{\phi}{\beta}\left(-\frac{\dot\beta}{2\beta}\right)$ (15)

$a\Gamma^1_{11} = a' + 2\phi\left(-\frac{\phi}{\beta}\right)\frac{\dot\beta}{2\beta}$ $\therefore \Gamma^1_{11} = \frac{a'}{a} - \frac{2\phi^2}{a\beta}\frac{\dot\beta}{2\beta}$ (16)

$\cancel{g_{11}\Gamma^{1}_{\underline{14}}} + g_{1\underline{4}}\Gamma^{4}_{\underline{14}} = -\frac{1}{2}g_{1\underline{4},1} + \frac{1}{2}g_{4\underline{1},1} + g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{14}} \therefore \Gamma^{4}_{\underline{14}} = \frac{\rho^2}{a\beta}\frac{\dot\beta}{2\beta}$ (17)

$\cancel{g_{11}\Gamma^{1}_{\underset{\vee}{14}}} + g_{1\underline{4}}\Gamma^{4}_{14} = -g_{1\underset{\vee}{3}}\Gamma^{3}_{\underline{14}}$; $g_{33}\Gamma^{3}_{\underline{14}} = g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{34}}$;

$g_{33}\Gamma^{3}_{\underset{\vee}{34}} = -\cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{\underline{34}}} - g_{3\underset{\vee}{3}}\Gamma^{2}_{\underline{34}}$; $g_{22}\Gamma^{2}_{\underline{34}} = \cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{\underset{\vee}{24}}} + g_{3\underset{\vee}{3}}\Gamma^{2}_{\underset{\vee}{24}}$

$g_{22}\Gamma^{2}_{\underset{\vee}{24}} = -g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{24}}$; $g_{33}\Gamma^{3}_{\underline{24}} = g_{2\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{34}}$.

$\therefore \Gamma^{4}_{\underset{\vee}{14}} = 0,\quad \Gamma^{3}_{\underline{14}} = 0,\quad \Gamma^{3}_{\underset{\vee}{34}} = 0,\quad \Gamma^{2}_{\underline{34}} = 0,\quad \Gamma^{2}_{\underset{\vee}{24}} = 0,\quad \Gamma^{3}_{\underline{24}} = 0$ (23)

$\cancel{g_{44}\Gamma^{4}_{22}} + g_{4\underline{1}}\Gamma^{1}_{22} = -\frac{1}{2}g_{22,4} + 2g_{2\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{42}}$

$\therefore a\Gamma^{1}_{22} = +\frac{\dot\beta}{2} + \frac{2f^2\beta}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)$

$\therefore \Gamma^{1}_{22} = \frac{\beta}{a}\left[+\frac{\dot\beta}{2\beta} + \frac{2f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right]$ (24)

$\cancel{g_{44}\Gamma^{4}_{33}} + g_{4\underline{1}}\Gamma^{1}_{33} = \cancel{-\frac{1}{2}g_{33,1} + 2g_{3\underset{\vee}{1}}\Gamma^{1}_{\underset{\vee}{13}} + 2g_{3\underset{\vee}{3}}\Gamma^{2}_{13}}$

$= -\frac{1}{2}g_{33,4} + 2\cancel{g_{3\underset{\vee}{1}}\Gamma^{1}_{\underset{\vee}{43}}} + 2g_{3\underset{\vee}{3}}\Gamma^{2}_{\underset{\vee}{43}}$

$\therefore a\Gamma^{1}_{33} = \frac{\dot\beta}{2} - \frac{2f^2}{\beta}\left(\frac{\dot A}{2A} - \frac{\dot f}{2f}\right) \therefore \Gamma^{1}_{33} = \frac{\beta}{a}\left[\frac{\dot\beta}{2\beta} + \frac{2f^2}{\beta^2}\left(\frac{\dot f}{2f} - \frac{\dot A}{2A}\right)\right]\sin^2\theta$ (25)

$\cancel{g_{44}\Gamma^{4}_{\underline{12}} + g_{4\underline{1}}\Gamma^{1}_{\underline{12}} = g_{3\underset{\vee}{2}}\Gamma^{3}_{\underset{\vee}{14}} \therefore a\Gamma^{1}_{\underline{12}} = -\frac{f\rho}{\beta}\frac{\dot\beta}{2\beta} \therefore \Gamma^{1}_{\underline{12}} = -\frac{\rho f}{a\beta}\frac{\dot\beta}{2\beta}\ (26)}$

$\cancel{g_{44}\Gamma^{4}_{\underline{12}}} + g_{4\underline{1}}\Gamma^{1}_{\underline{12}} = g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{42}} + g_{3\underset{\vee}{2}}\Gamma^{3}_{\underset{\vee}{14}} = \frac{\rho f\beta}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) - \frac{f\rho}{\beta}\left(\frac{\dot\beta}{2\beta}\right)$

$\therefore \Gamma^{1}_{\underline{12}} = \frac{\rho f}{a\beta}\left[\frac{f^2}{\beta^2-f^2}\frac{\dot\beta}{2\beta} + \frac{\beta^2}{\beta^2-f^2}\left(\frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right]$ (26)

$\cancel{g_{44}\Gamma^{4}_{\underset{\vee}{12}}} + g_{4\underline{1}}\Gamma^{1}_{\underset{\vee}{12}} = -g_{1\underset{\vee}{3}}\Gamma^{3}_{\underline{42}} - g_{3\underset{\vee}{2}}\Gamma^{3}_{\underline{14}} = 0 \quad \Gamma^{1}_{\underset{\vee}{12}} = 0$ (27)

$g_{22}\Gamma^{2}_{\underline{14}} = g_{1\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{24}} \therefore -\beta\Gamma^{2}_{\underline{14}} = -\frac{\rho f\beta}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) \therefore \Gamma^{2}_{\underline{14}} = \frac{\rho f}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)$ (28

$g_{22}\Gamma^{2}_{\underset{\vee}{14}} = -g_{1\underset{\vee}{3}}\Gamma^{3}_{\underline{24}} = 0 \quad \Gamma^{2}_{\underset{\vee}{14}} = 0$ (29)

$\cancel{g_{11}\Gamma^{1}_{\underline{24}}} + g_{1\underline{4}}\Gamma^{4}_{\underline{24}} = g_{2\underset{\vee}{3}}\Gamma^{3}_{\underset{\vee}{14}} \therefore a\Gamma^{4}_{\underline{24}} = \frac{f\rho}{\beta}\frac{\dot\beta}{2\beta} \therefore \Gamma^{4}_{\underline{24}} = \frac{f\rho}{a\beta}\frac{\dot\beta}{2\beta}$ (30)

$\cancel{g_{11}\Gamma^{1}_{\underset{\vee}{24}}} + g_{1\underline{4}}\Gamma^{4}_{\underset{\vee}{24}} = -g_{2\underset{\vee}{3}}\Gamma^{3}_{\underline{14}} = 0 \quad , \Gamma^{4}_{\underset{\vee}{24}} = 0$ (31)

$\cancel{g_{44}\Gamma^{4}_{\underline{13}}} + g_{4\underline{1}}\Gamma^{1}_{\underline{13}} = g_{1\underset{\vee}{3}}\cancel{\Gamma^{3}_{\underset{\vee}{43}}} + g_{1\underset{\vee}{3}}\cancel{\Gamma^{1}_{\underset{\vee}{14}}} + \cancel{g_{2\underset{\vee}{3}}\Gamma^{2}_{\underset{\vee}{14}}} = 0 \therefore \Gamma^{1}_{\underline{13}} = 0$ (32)

$\cancel{g_{44}\Gamma^{4}_{\underset{\vee}{13}}} + g_{4\underline{1}}\Gamma^{1}_{\underset{\vee}{13}} = \frac{1}{2}g_{1\underset{\vee}{3},4} - g_{1\underset{\vee}{3}}\Gamma^{3}_{\underline{43}} - g_{1\underset{\vee}{3}}\Gamma^{1}_{\underline{14}} - g_{2\underset{\vee}{3}}\Gamma^{2}_{\underline{14}}$

$\therefore a\Gamma^{1}_{\underset{\vee}{13}} = \frac{\dot\rho}{2} - \rho\frac{\dot A}{2A} - \frac{f\rho f}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)$

$\Gamma^{1}_{\underset{\vee}{13}} = \frac{\rho}{a}\left[\frac{\dot\rho}{2\rho} - \frac{\dot A}{2A} - \frac{f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right]$

$= \frac{\rho}{a}\left[-\frac{\dot\beta}{2\beta} - \frac{f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right]$

$$= -\frac{\phi}{a}\left[\frac{\beta^2}{\beta^2-f^2}\frac{\dot\beta}{2\beta} + \frac{f^2}{\beta^2 f^2}\frac{\dot f}{2f} - \frac{f^2}{\beta^2-f^2}\frac{\dot A}{A}\right]$$

$$= -\frac{\phi}{a}\left[\frac{\beta\dot\beta+f\dot f}{2(\beta^2-f^2)} - \frac{f^2}{\beta^2-f^2}\cdot\frac{\beta\dot\beta+f\dot f}{\beta^2+f^2}\right] = -\frac{\phi}{a}\frac{\beta\dot\beta+f\dot f}{2(\beta^2+f^2)(\beta^2-f^2)}\cdot(\beta^2+f^2-y^2)$$

$$= -\frac{\phi}{a}\frac{\beta\dot\beta+f\dot f}{2(\beta^2+f^2)} = -\frac{\phi}{a}\frac{\dot A}{2A}. \qquad \therefore\ \Gamma^1_{13} = -\frac{\phi}{a}\frac{\dot A}{2A} \qquad (33)$$

$$\Gamma^4_{3\underline{4}} = 0 \qquad (34)$$

$$\cancel{g_{33}}\ \cancel{g_{11}\Gamma^1_{3\underline{4}}} + g_{1\underline{4}}\Gamma^4_{3\underline{4}} = g_{31}\Gamma^1_{14} + g_{32}\Gamma^2_{14} = 0$$

$$\cancel{g_{11}}\Gamma^1_{34} + g_{1\underline{4}}\Gamma^4_{34} = -\tfrac{1}{2}g_{13,4} - \cancel{g_{31}\Gamma^1_{1\underline{4}}} - g_{33}\Gamma^2_{1\underline{4}}$$

$$\therefore\ a\Gamma^4_{34} = -\frac{\dot\beta}{2} + \cancel{f}\frac{f\phi f}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)$$

$$\therefore\ \Gamma^4_{34} = \frac{\phi}{a}\left(-\frac{\dot\beta}{2\phi} + \frac{f^2}{\beta^2 f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right)$$

$$= \frac{\phi}{a}\left(-\frac{\dot\phi}{2\phi} + \frac{\dot A}{2A} + \frac{f^2}{\beta^2 f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right)\right) - \frac{\phi}{a}\frac{\dot A}{2A}$$

$$\Gamma^4_{34} = 0 \qquad (35)$$

$$= \frac{\phi}{a}\frac{\dot A}{2A} - \frac{\phi}{a}\frac{\dot A}{2A} = 0.$$

This means that $\frac{f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) = \frac{\dot\phi}{2\phi}$

Let us verify it L.H.S. ~~$\frac{f^2}{\beta^2 f^2}($~~

or $\frac{f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) - \frac{\dot\phi}{2\phi} = 0$

L.H.S. $\frac{f^2}{\beta^2-f^2}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{A}\right) - \left(\frac{\dot A}{2A} - \frac{\dot\beta}{2\beta}\right)$

$$= \frac{\dot\beta}{2\beta}\left(\frac{f^2}{\beta^2-f^2} + 1\right) + \frac{f\dot f}{2(\beta^2-f^2)} - \frac{\dot A}{2A}\left(\frac{2f^2}{\beta^2-f^2} + 1\right)$$

$$= \frac{\beta\dot\beta+f\dot f}{2(\beta^2-f^2)} - \frac{\dot A}{2A}\frac{\beta^2+f^2}{\beta^2-f^2}.$$

$$= \frac{\beta\dot\beta+f\dot f}{2(\beta^2-f^2)} - \frac{\beta\dot\beta+f\dot f}{2\cancel{(\beta^2+f^2)}}\frac{\cancel{\beta^2+f^2}}{\beta^2-f^2} = 0 = \text{R.H.S.}$$

$$a\Gamma^1_{2\underline{3}} = 0 \qquad (36)$$

$$\cancel{g_{44}\Gamma^4_{2\underline{3}}} + g_{4\underline{1}}\Gamma^1_{2\underline{3}} = g_{23}\Gamma^3_{43} + g_{13}\Gamma^1_{24} + g_{23}\Gamma^2_{24}$$

$\therefore$ ~~$a\Gamma^1_{2\underline{3}} =$~~

$$\cancel{g_{44}\Gamma^4_{23}} + g_{4\underline{1}}\Gamma^1_{23} = \tfrac{1}{2}g_{23,4} - g_{23}\Gamma^3_{43} - \cancel{g_{13}\Gamma^1_{\underline{24}}} - g_{23}\Gamma^2_{\underline{24}}$$

$$\therefore\ a\Gamma^1_{23} = \frac{\dot f}{2} - f\frac{\dot A}{A}\ \cancel{\phi} \qquad \therefore\ \Gamma^1_{23} = \frac{f}{a}\left(\frac{\dot f}{2f} - \frac{\dot A}{A}\right) \qquad (37)$$

$$g_{33}\Gamma^3_{33} = 0 \quad \therefore\ \Gamma^3_{33} = 0.^{(38)} \quad \therefore\ \Gamma^1_{3\underline{1}} + \Gamma^2_{3\underline{2}} = 0 \quad \therefore\ \Gamma^2_{\underline{32}} = 0. \quad (39)$$

$$\Gamma^1_{31} + \Gamma^2_{32} = 0 \quad \therefore\ \Gamma^2_{32} = -\Gamma^1_{31} = -\frac{\phi}{a}\frac{\dot A}{2A} \quad (40)$$

$$\Gamma^1_{21} = 0,\ \Gamma^4_{24} = 0 \quad \therefore\ \Gamma^3_{23} = 0 \quad (42)$$

$$g_{22}\Gamma^2_{22} = 0. \quad (41)$$

$$\Gamma^1_{2\underline{1}} + \Gamma^3_{2\underline{3}} + \Gamma^4_{2\underline{4}} = \cot\theta \quad \therefore\ \Gamma^3_{\underline{23}} = \cot\theta - \frac{\phi f\beta}{a(\beta^2-f^2)}\left(\frac{\dot\beta}{2\beta} + \frac{\dot f}{2f} - \frac{\dot A}{2A}\right) + \frac{f\phi}{a\beta}\frac{\dot\beta}{2\beta}$$

$$- \frac{\phi f}{a\beta}\frac{\dot\beta}{2\beta} = \cot\theta - \frac{\phi\beta}{af}\frac{\dot\phi}{2\phi}. \quad (43)$$

$g_{22}\Gamma_{11}^{2} = 2g_{\underset{\vee}{13}}\Gamma_{\underset{\vee}{21}}^{3}$. ; $g_{33}\Gamma^{3}_{\underset{\vee}{21}} = -\frac{1}{2}g_{\underset{\vee}{13},2} - \frac{1}{2}g_{\underset{\vee}{32},1} - g_{\underset{\vee}{31}}\Gamma^{3}_{\underline{23}} - g_{\underset{\vee}{23}}\Gamma^{3}_{\underline{31}}$

$\therefore$ ~~$-\beta\Gamma^{3}_{\underset{\vee}{21}}$~~ $g_{33}\Gamma^{3}_{\underline{31}} = \frac{1}{2}g_{33,1} + g_{\underset{\vee}{31}}\Gamma^{1}_{\underset{\vee}{31}} + g_{\underset{\vee}{32}}\Gamma^{2}_{\underset{\vee}{31}} + g_{\underset{\vee}{31}}\cancel{\Gamma^{3}_{\underset{\vee}{33}}}$.

$g_{22}\Gamma^{2}_{\underset{\vee}{31}} = \frac{1}{2}g_{\underset{\vee}{31},2} - \frac{1}{2}g_{\underset{\vee}{23},1} - g_{\underset{\vee}{31}}\Gamma^{1}_{\underline{21}} - g_{\underset{\vee}{32}}\Gamma^{2}_{\underline{21}} - g_{\underset{\vee}{31}}\Gamma^{3}_{\underline{32}}$

$g_{22}\Gamma^{2}_{\underline{21}} = \frac{1}{2}g_{22,1} + g_{\underset{\vee}{23}}\Gamma^{3}_{\underset{\vee}{21}} + \cancel{g_{\underset{\vee}{31}}\Gamma^{3}_{\underset{\vee}{22}}}$

$\therefore -\beta\Gamma^{2}_{\underline{21}} = -\frac{\beta'}{2} + f\Gamma^{3}_{\underset{\vee}{21}}$ $\qquad \therefore \Gamma^{2}_{\underline{21}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}\Gamma^{3}_{\underset{\vee}{21}}$.

$$-\beta\Gamma^{3}_{\underset{\vee}{21}} = -\frac{\phi}{2}\cot\theta + \frac{f'}{2} + \phi\cot\theta - \frac{\phi^{2}\beta}{af}\frac{\dot{\phi}}{2\phi} - f\Gamma^{3}_{\underline{31}}$$

$$\therefore f\Gamma^{3}_{\underline{31}} = \beta\Gamma^{3}_{\underset{\vee}{21}} + \frac{\phi}{2}\cot\theta + \frac{f'}{2} - \frac{\phi^{2}\beta}{af}\frac{\dot{\phi}}{2\phi}$$

$$\therefore \Gamma^{3}_{\underline{31}} = \frac{\beta}{f}\Gamma^{3}_{\underset{\vee}{21}} + \frac{\phi}{2f}\cot\theta + \frac{f'}{2f} - \frac{\phi^{2}}{f^{2}}\frac{\beta}{a}\cdot\frac{\dot{\phi}}{2\phi}.$$

$\Gamma^{1}_{11} + \Gamma^{2}_{\underline{12}} + \Gamma^{3}_{\underline{13}} + \Gamma^{4}_{\underline{14}} = \frac{a'}{a} + \frac{A'}{A}$ ~~a~~

$$\therefore \frac{a'}{a} - \frac{2\phi^{2}}{a\beta}\frac{\dot{\beta}}{2\beta} + \frac{\beta'}{2\beta} + \frac{f'}{2f} - \frac{\phi^{2}\beta}{f^{2}a}\frac{\dot{\phi}}{2\phi} + \frac{\phi}{2f}\cot\theta + \frac{\phi^{2}}{a\beta}\frac{\dot{\beta}}{2\beta}$$

$$+ \frac{\beta^{2}-f^{2}}{f\beta}\Gamma^{3}_{\underset{\vee}{21}} = \frac{a'}{a} + \frac{A'}{A}$$

$$\therefore \Gamma^{3}_{\underset{\vee}{21}} = \left[\frac{A'}{A} - \frac{\beta'}{2\beta} - \frac{f'}{2f} + \frac{\phi^{2}}{a\beta}\left(\frac{\dot{\beta}}{2\beta} + \frac{\beta^{2}}{f^{2}}\frac{\dot{\phi}}{2\phi}\right) - \frac{\phi}{2f}\cot\theta\right]\frac{f\beta}{\beta^{2}-f^{2}}$$

$$= B - \frac{\phi\beta}{\beta^{2}-f^{2}}\cdot\frac{1}{2}\cot\theta = B - C\cot\theta \qquad (44)$$

$\Gamma^{2}_{11} = \frac{2\phi}{-\beta}(B - C\cot\theta)$ (45)

$\Gamma^{2}_{\underline{21}} = \frac{\beta'}{2\beta} - \frac{f}{\beta}(B - C\cot\theta)$ (46)

$\Gamma^{3}_{\underline{13}} = \frac{A'}{A} + \frac{\phi^{2}}{a\beta}\frac{\dot{\beta}}{2\beta} - \frac{\beta'}{2\beta} + \frac{f}{\beta}(B - C\cot\theta)$ (47)

$-\beta\Gamma^{3}_{\underline{31}} = -\frac{\beta'}{2} - \frac{\phi^{2}}{a}\frac{\dot{A}}{2A} - f\Gamma^{2}_{\underset{\vee}{31}}$. $\quad \Gamma^{3}_{\underline{31}} = \frac{\beta'}{2\beta} + \frac{\phi^{2}}{a\beta}\frac{\dot{A}}{2A} + \frac{f}{\beta}\Gamma^{2}_{\underset{\vee}{31}}$

$$\therefore \frac{f}{\beta}\Gamma^{2}_{\underset{\vee}{31}} = \frac{A'}{A} - \frac{\beta'}{2\beta} + \frac{\phi^{2}}{a\beta}\frac{\dot{\beta}}{2\beta} + \frac{f}{\beta}(B - C\cot\theta) - \frac{\beta'}{2\beta} - \frac{\phi^{2}}{a\beta}\frac{\dot{A}}{2A}$$

$$= \frac{A'}{A} - \frac{\beta'}{\beta} + \frac{\phi^{2}}{a\beta}\left(\frac{\dot{\beta}}{2\beta} - \frac{\dot{A}}{2A}\right) + \frac{f}{\beta}(B - C\cot\theta)$$

$$= \frac{A'}{A} - \frac{\beta'}{\beta} - \frac{\phi^{2}}{a\beta}\frac{\dot{\phi}}{2\phi} + \frac{f}{\beta}(B - C\cot\theta)$$

$$\therefore \Gamma^{2}_{\underset{\vee}{31}} = \frac{\beta}{f}\left[\frac{A'}{A} - \frac{\beta'}{\beta} - \frac{\phi^{2}}{a\beta}\frac{\dot{\phi}}{2\phi}\right] + B - C\cot\theta \qquad (48)$$

$g_{33}\Gamma^{3}_{11} = 2g_{\underset{\vee}{13}}\Gamma^{3}_{\underset{\vee}{31}}$; $g_{33}\cancel{\Gamma^{3}_{\underset{\vee}{31}}} = -\cancel{g_{\underset{\vee}{31}}\Gamma^{1}_{\underline{31}}} - g_{\underset{\vee}{32}}\Gamma^{2}_{\underline{31}} - \cancel{g_{\underset{\vee}{31}}}\Gamma^{3}_{\underline{33}}$

$g_{22}\Gamma^{2}_{\underline{31}} = g_{\underset{\vee}{31}}\cancel{\Gamma^{1}_{\underset{\vee}{21}}} + g_{\underset{\vee}{32}}\Gamma^{2}_{\underset{\vee}{21}} + \cancel{g_{\underset{\vee}{31}}\Gamma^{3}_{\underset{\vee}{32}}}$; $g_{22}\Gamma^{2}_{\underset{\vee}{21}} = -g_{\underset{\vee}{23}}\Gamma^{3}_{\underline{21}} - g_{\underset{\vee}{31}}\Gamma^{3}_{22}$

$g_{33}\Gamma^3_{\underline{21}} = \underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^3_{31}} + \cancel{\underset{\checkmark}{g_{31}}\underset{\checkmark}{\Gamma^3_{23}}}$; $g_{33}\Gamma^3_{22} = 2\cancel{\underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^3_{32}}}$

$\therefore \Gamma^3_{11} = 0, \; \Gamma^3_{22} = 0 \; \underset{\checkmark}{\Gamma^3_{31}} = 0, \; \Gamma^2_{\underline{31}} = 0, \; \Gamma^3_{\underline{21}} = 0 \; \underset{\checkmark}{\Gamma^2_{21}} = 0$ (54)

$\therefore \Gamma^4_{11} = 0$ (55)

$\cancel{g_{11}\Gamma^1_{11}} + g_{\underline{14}}\Gamma^4_{11} = 0$

$\cancel{g_{11}\Gamma^1_{\underline{12}}} + g_{\underline{14}}\Gamma^4_{\underline{12}} = \underset{\checkmark}{g_{13}}\underset{\checkmark}{\Gamma^3_{12}} + \cancel{\underset{\checkmark}{g_{32}}\underset{\checkmark}{\Gamma^3_{11}}} \; \therefore a\Gamma^4_{\underline{12}} = -p(B - C\cot\theta)$

$\therefore \Gamma^4_{\underline{12}} = -\frac{p}{a}(B - C\cot\theta)$ (56)

$\underset{\checkmark}{\Gamma^4_{12}} = 0$ (57)

$\cancel{g_{11}\underset{\checkmark}{\Gamma^1_{12}}} + g_{\underline{14}}\underset{\checkmark}{\Gamma^4_{12}} = -\underset{\checkmark}{g_{13}}\Gamma^3_{\underline{12}} - \underset{\checkmark}{g_{32}}\Gamma^3_{\underline{11}} = 0$

~~g_{14}~~ $\cancel{g_{11}\Gamma^1_{\underline{13}}} + g_{\underline{14}}\Gamma^4_{\underline{13}} = \underset{\checkmark}{g_{13}}\underset{\checkmark}{\Gamma^3_{13}} + \underset{\checkmark}{g_{13}}\underset{\checkmark}{\Gamma^1_{11}} + \underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^2_{11}} = 0 \; . \; \Gamma^4_{\underline{\cancel{1}3}} = 0$ (58)

$\cancel{g_{11}\underset{\checkmark}{\Gamma^1_{13}}} + g_{\underline{14}}\underset{\checkmark}{\Gamma^4_{13}} = \frac{1}{2}\underset{\checkmark}{g_{13,1}} - \frac{1}{2}\underset{\checkmark}{g_{31,1}} - \underset{\checkmark}{g_{13}}\Gamma^3_{\underline{13}} - \underset{\checkmark}{g_{23}}\Gamma^2_{11} - \underset{\checkmark}{g_{13}}\Gamma^1_{11}$

$\therefore a\underset{\checkmark}{\Gamma^4_{13}} = p' - p(\Gamma^3_{\underline{13}} + \Gamma^1_{11}) - f\left(-\frac{2p}{\beta}(B - C\cot\theta)\right)$

$\therefore a\underset{\checkmark}{\Gamma^4_{13}} = p' - p\left(\frac{A'}{A} + \frac{a'}{a} - \frac{p^2}{a\beta}\frac{\dot\beta}{2\beta} - \frac{\beta'}{2\beta} + \frac{f}{\beta}(B - C\cot\theta)\right)$

$\qquad + \frac{2pf}{\beta}(B - C\cot\theta)$

$\therefore \underset{\checkmark}{\Gamma^4_{13}} = \frac{p}{a}\left[\frac{p'}{p} - \frac{A'}{A} - \frac{a'}{a} + \frac{p^2}{a\beta}\frac{\dot\beta}{2\beta} + \frac{\beta'}{2\beta}\right] + \frac{pf}{a\beta}(B - C\cot\theta).$ (59)

$\cancel{g_{11}\Gamma^1_{22}} + g_{\underline{14}}\Gamma^4_{22} = -\frac{1}{2}g_{22,1} + 2\underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^3_{12}} \; \therefore a\Gamma^4_{22} = -2f(B - C\cot\theta)$ (60)

$\Gamma^4_{\underline{23}} = 0$ (61)

$\cancel{g_{11}\Gamma^1_{2\underline{3}}} + g_{\underline{14}}\Gamma^4_{2\underline{3}} = \cancel{g_{23}\underset{\checkmark}{\Gamma^3_{13}}} + \cancel{g_{13}\underset{\checkmark}{\Gamma^1_{21}}} + \cancel{\underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^2_{21}}}$

~~$g_{11}\underset{\checkmark}{\Gamma^1_{23}} + g_{14}\Gamma^4_{23} =$~~

~~$g_{33}\Gamma^3_{\underline{23}} = \frac{1}{2}g_{33,2} + \cancel{g_{23}\Gamma^3_{33}} + \underset{\checkmark}{g_{13}}\underset{\checkmark}{\Gamma^1_{23}} + \underset{\checkmark}{g_{23}}\underset{\checkmark}{\Gamma^2_{23}}$~~

~~$g_{11}\underset{\checkmark}{\Gamma^1_{23}} + g_{14}$~~ We shall find this afterwards. Let us take

$R_{11} = \Gamma^s_{11,s} - \Gamma^t_{1s}\Gamma^s_{t1} - \Gamma^s_{1\underline{s},1} + \Gamma^s_{11}\Gamma^t_{\underline{st}}$ Others are

$\cdot\, \Gamma^2_{11,2} - \Gamma^2_{11}\Gamma^1_{21} - \Gamma^1_{12}\Gamma^2_{11} - \Gamma^2_{12}\Gamma^2_{21} - \Gamma^3_{12}\Gamma^2_{31} - \Gamma^4_{12}\Gamma^2_{41} - \cancel{\Gamma^1_{13}\Gamma^3_{11}}$

$- \Gamma^2_{13}\Gamma^3_{21} - \Gamma^3_{13}\Gamma^3_{31} - \Gamma^4_{13}\Gamma^3_{41} - \Gamma^2_{14}\Gamma^4_{21} - \Gamma^3_{14}\Gamma^4_{31} + \Gamma^2_{11}\Gamma^t_{2\underline{t}}.$

~~co~~ coeff of $\cot^2\theta$ is $-\frac{2p}{\beta}C - \frac{f^2}{\beta^2}C^2 + C^2 + C^2 - \frac{f^2}{\beta^2}C^2 + \frac{2p}{\beta}C$

$= \cancel{2}\, 2C^2\left(1 - \frac{f^2}{\beta^2}\right)$

This means that p must be zero ~~or c~~ unless $\beta^2 - f^2$ vanishes in which case we shall have to work the Γ^k_{ij} again

But if $\beta^2 - f^2$ is zero ~~then (β has to vanish~~
then the equation determining Γ_{23}^{3} requires
~~$f(r)$ +~~ $f(r,t) + \frac{\beta}{2f}\cot\theta = 0$ so that $\beta = 0$.

~~You~~ do There are no solutions
with spherical symmetry

Coming to Dirac's Electrodynamics
$(F^{\mu\nu})_{\nu} = \lambda v^{\mu}$ gives

~~$\frac{\partial}{\partial r}(F^{21} r^2 \sin\theta) + \frac{\partial}{\partial \phi}(F^{23} r^2 \sin\theta) + \frac{\partial}{\partial t}(F^{24} \frac{}{r^2}$~~

$$\frac{\partial}{\partial\theta}(F^{12} r^2 \sin\theta) + \frac{\partial}{\partial\phi}(F^{13} r^2 \sin\theta) + \frac{\partial}{\partial t}(F^{14} r^2 \sin\theta) = \lambda v^1 r^2 \sin\theta$$

$$\frac{\partial}{\partial r}(F^{21} r^2 \sin\theta) + \frac{\partial}{\partial\phi}(F^{23} r^2 \sin\theta) + \frac{\partial}{\partial t}(F^{24} r^2 \sin\theta) = \lambda v^2 r^2 \sin\theta$$

~~The~~ With the sph. sym (and not rotationally symmetric) forms of F^{12}, F^{24}, F^{13}, F^{34}, F^{14} and F^{23} ~~it is~~ as well as v^{μ}, it is clear that from λ must contain r, t as well as θ. but from (2) λ must contain only (r,t). This can be reconciled by taking $v^2 = 0$ But since we have not yet chosen the polarization of axis $v^2 = 0$ must correspondingly mean $v^3 = 0$

We must have $v^2 = v^3 = 0$ in Dirac's scheme. Hence in $A_{\mu}A^{\mu} = k^2$ scheme $A_2 = A_3 = 0$

so if F_{12} is to be present, A_1 must contain θ, A_4 must contain the same function of θ and $A_1 A^1 + A_4 A^4$ will not give k^2 ~~inde~~ independent of θ.

Hence $A_\mu A^\mu = k^2$ is not possible

$$A_\mu = k v_\mu + \eta \frac{\partial \xi}{\partial x^\mu}$$ Then ~~A~~ again

$$A_1 = k v_1 + \eta \frac{\partial \xi}{\partial x^1}$$

$$A_4 = k v_4 = \eta \frac{\partial \xi}{\partial x^4}$$

$$\therefore \frac{\partial \xi}{\partial x^1} v^1 + \frac{\partial \xi}{\partial x^4} v^4 = 0$$

$v^2 = v^3$ by zero

$$\therefore A_1 v^1 + A_4 u^4 = k$$

Hence A_1 and A_4 still cannot contain θ.

Hence if F_{12} is to survive A_2 must be present

Hence A_μ must be

$A_1(r, t)$, $A_2(r, t)$, $A_4(r, t)$

Now work out the scheme further

$$F_{21} = A_2', \quad F_{24} = \dot{A}_2 \quad A_2 = \eta \frac{\partial \xi}{\partial x^2}$$

$$\therefore F_{21} v^1 + F_{24} v^4 = \left(\eta' \frac{\partial \xi}{\partial x^2} + \eta \frac{\partial^2 \xi}{\partial x^2 \partial x^1}\right) v^1$$

$$+ \left(\dot{\eta} \frac{\partial \xi}{\partial x^2} + \eta \frac{\partial^2 \xi}{\partial x^2 x^4}\right) v^4 = \frac{\partial \xi}{\partial x^2} \left(\eta' v^1 + \dot{\eta} v^4\right)$$

$$+ \eta \frac{\partial}{\partial x^2} \left(\frac{\partial \xi}{\partial x^1} v^1 + \frac{\partial \xi}{\partial x^2} v^4\right) = 0$$

$\therefore F_{21} v^1 + F_{24} v^4 = 0$ $\qquad \therefore \dfrac{v^1}{v^4} = -\dfrac{F_{24}}{F_{21}}$

Now apply the analysis of pages 43, 44 and came to the conclusion that ~~$F_{\mu\nu}$~~ $- J_\mu J^\mu = 0$

Hence Dirac's system does not allow solutions of this type.

Dr. Gatta has sent the following non-linear Diff Eqn.

$$\left[\frac{d^2}{d\xi^2} - x^2\right] u = \eta^2 \xi - \frac{u^{3/2}}{\xi^{1/2}}$$ and

wants to know how to tackle it. We first put in a form where the coefficients are rational functions of u. ∴ $u^{3/2}$ must be removed

Put $u^n = v$ or rather $u = v^n$

$$\therefore \frac{du}{d\xi} = n v^{n-1} \frac{dv}{d\xi} ; \quad \frac{d^2u}{d\xi^2} = n v^{n-1} \frac{d^2v}{d\xi^2} + n(n-1) v^{n-2} \left(\frac{dv}{d\xi}\right)$$

∴ The equation becomes

$$n v^{n-1} \frac{d^2v}{d\xi^2} + n(n-1) v^{n-2} \left(\frac{dv}{d\xi}\right)^2 - x^2 v^n = \eta^2 \xi - \frac{v^{\frac{3n}{2}}}{\xi^{1/2}}$$

$$\therefore \frac{d^2v}{d\xi^2} = -\frac{(n-1)}{v}\left(\frac{dv}{d\xi}\right)^2 + \frac{x^2}{n} v + \frac{\eta^2 \xi}{n} v^{1-n}$$
$$- \frac{1}{n} \frac{v^{\frac{n}{2}+1}}{\xi^{1/2}}$$

In order to get polynomials (or rather rational f^{ns}) ~~or~~ in v it is necessary that n be ~~even and~~ an even integer +ve or −ve.

Now in order to have fixed ~~or~~ singularities it is necessary that the highest power be v^3 and the lowest pow be v^{-1}. This actually restricts the case to $n = 2$ or $n = -2$

With $n = 2$ the equation is

$$\frac{d^2 v}{d\xi^2} = -\frac{1}{v}\left(\frac{dv}{d\xi}\right)^2 + \frac{1}{2}x^2 v + \frac{\eta\xi^2}{2}\cdot\frac{1}{v} - \frac{1}{2}\frac{v^2}{\xi^{1/2}}$$

With $n = -2$, the equation is

$$\frac{d^2 v}{d\xi^2} = \frac{3}{v}\left(\frac{dv}{d\xi}\right)^2 - \frac{1}{2}x^2 v - \frac{\eta^2\xi}{2}v^3 + \frac{1}{2}\frac{1}{\xi^{1/2}}$$

For fixed singularities in the first equation it is necessary that coeff of v^2 be zero (which means that the equ be linear) in the second case it is necessary that the coeff of ~~Const~~ v^0 may vanish which again amounts to the same thing [Ince: Ordinary Diff Equ^ns pp. 335-336 337 . . .]

If however $\eta = 0$ the 1st equation has no term in $\frac{1}{v}$ and so it will have fixed singularity if in that equation the constant term is absent which is really the case

Now $\frac{d^2u}{d\xi^2} - x^2u = \eta^2\xi - \frac{u^{3/2}}{\xi^{1/2}}$ has an obvious sol^n $u = m\xi$ m a constant. Then m satisfies $-x^2m = \eta^2 - m^{3/2}$

Let us put $u = v\xi$

$$\frac{du}{d\xi} = v + \xi\frac{dv}{d\xi} \qquad \frac{d^2u}{d\xi^2} = \cancel{d}\, 2\frac{dv}{d\xi} + \xi\frac{d^2v}{d\xi^2}$$

$$\xi\frac{d^2v}{d\xi^2} + 2\frac{dv}{d\xi} - x^2v\xi = \eta^2\xi - v^{3/2}\xi$$

$$\therefore \frac{d^2v}{d\xi^2} + \frac{2}{\xi}\frac{dv}{d\xi} - x^2v = \eta^2 - v^{3/2}$$

~~$p' + \frac{2}{\xi}p \qquad \frac{p'}{p} + \frac{2}{\xi} = (p\xi^2)'$~~

~~$\frac{dv}{d\xi} = \frac{1}{\frac{d\xi}{dv}} \quad \therefore \frac{d^2v}{d\xi^2} = -\frac{1}{\left(\frac{d\xi}{dv}\right)^2}\frac{d^2\xi}{dv^2}\frac{dv}{d\xi}$~~

$$v = w^n \qquad \frac{dv}{d\xi} = nw^{n-1}\frac{dw}{d\xi} \qquad \frac{d^2v}{d\xi^2}$$

$$nw^{n-1}\frac{d^2w}{d\xi^2} + n(n-1)w^{n-2}\left(\frac{dw}{d\xi}\right)^2 + \frac{2}{\xi}nw^{n-1}\frac{dw}{d\xi} - x^2w^n$$
$$= \eta^2 - w^{\frac{3n}{2}}$$

$$\therefore \frac{d^2w}{d\xi^2} + \frac{(n-1)}{w}\left(\frac{dw}{d\xi}\right)^2 + \frac{2}{\xi}\frac{dw}{d\xi} - \frac{x^2}{n}\cdot w = \frac{\eta^2}{n}w^{1-n} - \frac{w^{\frac{n}{2}+1}}{n}$$

Let us put $n = -2$

$$\frac{d^2w}{d\xi^2} - \frac{3}{w}\left(\frac{dw}{d\xi}\right)^2 + \frac{2}{\xi}\left(\frac{dw}{d\xi}\right) + \frac{x^2}{2}w = -\frac{\eta^2}{2}w^3 + \frac{1}{2}$$

$$\frac{d^2w}{d\xi^2} - \frac{3}{w}\left(\frac{dw}{d\xi}\right)^2 + \frac{2}{\xi}\left(\frac{dw}{d\xi}\right) + \frac{x^2}{2}w = -\frac{\eta^2}{2}w^3 + \frac{1}{2}.$$

$$\xi = \frac{1}{x} \qquad \frac{dw}{d\xi} = \frac{dw}{dx}\cdot\left(-\frac{1}{\xi^2}\right)$$

$$\frac{d^2w}{d\xi^2} = \frac{d^2w}{dx^2}\frac{1}{\xi^4} + \frac{2}{\xi^3}\frac{dw}{dx}.$$

$$\therefore \frac{d^2w}{dx^2}x^4 + 2x^3\frac{dw}{dx} - \frac{3}{w}x^4\left(\frac{dw}{dx}\right)^2$$

$$\cancel{+2} - 2x^3\frac{dw}{dx} + \frac{\alpha^2}{2}w = -\frac{\eta^2}{2}w^3 + \frac{1}{2}$$

$$\therefore \frac{d^2w}{dx^2}\cancel{x^4} - \frac{3}{w}\left(\frac{dw}{dx}\right)^2 = \frac{1}{x^4}\left[-\frac{\alpha^2}{2}w - \frac{\eta^2}{2}w^3 + \frac{1}{2}\right]$$

$$\therefore w\frac{d^2w}{dx^2} - 3\left(\frac{dw}{dx}\right)^2 = \frac{1}{x^4}\left(-\frac{\alpha^2}{2}w^2 - \frac{\eta^2}{2}w^4 + \frac{1}{2}w\right]$$

$$w = x^s(a_0 + a_1x + a_2x^2 + \cdots)$$

$$w = \sum a_r x^{r+s}$$

$$\sum(a_r x^{r+s})\sum a_r(r+s)(r+s-1)x^{r+s-2} - 3\left(\sum a_r(r+s)x^{r+s-1}\right)^2$$

$$= \cancel{x^{-4}} - \frac{\alpha^2}{2}x^{-4}\left[\sum a_r x^{r+s}\right]^2 - \frac{\eta^2}{2}x^{-4}\left(\sum a_r x^{r+s}\right)^4$$

$$+ \frac{1}{2}x^{-4}\sum a_r x^{r+s}$$

$4s-4 \qquad s-4$

$2s-2 \qquad 2s-4$

$2s-2$

$s = 0$

$$\sum a_r x^r \sum a_r r(r-1)x^{r-2} - 3\left(\sum a_r \cancel{x} r x^{r-1}\right)^2$$

$$= -\frac{\alpha^2}{2}x^{-4}\left(\sum a_r x^{r}\right)^2 - \frac{\eta^2}{2}x^{-4}\left(\sum a_r x^r\right]^4$$

$$+ \frac{1}{2}x^{-4}\sum a_r x^{r-4}$$

$$-\frac{\alpha^2}{2}a_0^2 - \frac{\eta^2}{2}a_0^4 + \frac{1}{2}a_0 = 0$$

Coeff of x^2

At large distances w acts like a const. a_0

$w = a_0$ if $a_0 = 0$

$$x^4(\overset{a_2x^2}{\cancel{a_0 + a_1x +}}\cdots)(2a_2 + 3.2\,a_3x + \cdots) - 3x^4(a_1 + 2a_2x + \cdots)^2$$

$$= -\frac{\alpha^2}{2}(a_0 + a_1x + \cdots)^2 - \frac{\eta^2}{2}(a_0 + a_1x + \cdots)^4$$

$$+ \tfrac{1}{2}(a_0 + a_1x + a_2x^2 + \cdots)$$

$$-\frac{\alpha^2}{2}a_0^2 - \frac{\eta^2}{2}a_0^4 + \frac{1}{2}a_0 = 0$$

$$-\frac{\alpha^2}{2}2a_0a_1 - 4\frac{\eta^2}{2}a_0^3a_1 + \frac{1}{2}a_1 = 0$$

Take $a_0 = 0$, $a_1 = 0$ They you will get $a_2 = 0$, $a_3 = 0$ etc

$$\cancel{-\frac{\alpha^2}{2}\frac{a_2^2}{2} - \frac{\eta^2}{2}a_2^4 + \frac{1}{2}a_2 = 0}$$

So a_0 cannot be taken equal to zero.

Hence $\left(\frac{\partial u}{\partial \xi}\right) \not\rightarrow 0$ as $\xi \rightarrow \infty$

Spherical Symmetry of Riemannian Spaces.

Consider a Riemannian space R_n. Take a hypersurface (a sphere?) H_{n-1} of constant curvature in R_n ~~Give an~~ Let the coordinates be so chosen in R_n that ~~the~~ ~~hyp~~ H_{n-1} becomes $x_1 = \text{constant}$. Give an infinitesimal rotation ξ^α $(\alpha = 2 \cdots \overset{n}{\cancel{n-1}})$ to H_{n-1}. As a consequence at ~~P~~ point $P(x^i)$ changes to a pt $P'(x'^i)$. If the g_{ik} of R_n are such that g_{ik} are the same functions of x^i as the g'_{ik} are of x'^i then we say that, R_n is spherically symmetric.

The Hypersurface H_{n-1} can be regarded as embedded in ~~S~~ a flat space S_n and then the coordinates can be chosen to give the ~~equ~~ fundamental form of ~~f~~ H_{n-1} in a simple way. ~~corr~~ With the g_{ik} of this form equation of Killing can be solved to get $\xi^2, \xi^3 \cdots \xi^n$. Add to it $\xi^1 = 0$ and then get the relation satisfied by g_{ik} ~~are~~ in terms of these ξ^k and solve these equations.

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ફ્લુ વાંદરાના [illegible] [illegible] માટે ~~[illegible]~~ લઈ
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તા. ~~૩~~ -૬-૪૩

તે જમાને જનમ્યો ત્યારે બધાં તેના ઓરડામાં ધીમે પગલે ~~[illegible]~~ આવતાં હતાં. તે જમાને પરણ્યો ત્યારે વરઘોડામાં બધાં દોડી દોડી આવતાં હતાં. તે મરણ પામ્યો ત્યારે ડાઘુઓ તેને લઈને સ્મશાન તરફ ઝડપ ભેર દોડ્યા જતા હતા.

તા. ૨૨-૬-૪૩

[illegible] વિદ્યાર્થી પરિષદ્ — સામાન્ય માણસો માટે ની વૈદ્યવિદ્યાની ચર્ચા સભા — વિજ્ઞાનના શિક્ષણના ભાગ રૂપે — [illegible] અને [illegible] [illegible] ? — ચર્ચાનો વિષય — ચર્ચા — પ્રમુખનો ઉપસંહાર — ફરી વાર આ ગોઠવણનું ચાલુ [illegible] એ પરિણામ ની તારવણી.

તા. ૬-૭-૪૩

એક યાદ આવે છે . .

એ ફાજુ ચોરાનું થીએટર. જૂની ફિલ્મના જમાનાનાં ભાવનગરમાં સ્ટાર સીનેમા નામનું થીએટર ઠીક ઠીક ~~જૂ~~ જાણીતું હતું. સામાન્ય રીતે તેના માલિક ના નામ ઉપરથી તે ફાજુ ચોરા નાં થીએટર તરીકે ~~ઓળખ~~ ઓળખાતું. ત્યારે બપોરના ખેલ માં (Matinee Show)માં વિદ્યાર્થીઓ ને રાહત ફી લઈ શો બતાવતા. રાહત ફી ફક્ત ૦-૧-૦. સિનેમા જુની હોય તો માલીક, ~~[illegible]~~ બહાર [illegible] ~~ક~~ સિનેમા ના [illegible] [illegible] [illegible] [illegible] [illegible] જુદી સિનેમા ના પુરા પાટીયા ઉઠીને અને સાથે લઈને બુધવાર અને શનિવાર [illegible] જેવા દિવસ વિદ્યાર્થીઓ ફાજુ ચોરા ના મહેમાન બનતા. શો નો સમય બપોરે [illegible] 3-30 વગાડ્યા હોય પણ જ્યાં સુધી રાહતમાં ટીકીટ [illegible] ભરાય નહિ ત્યાં સુધી ફાજુ ચોરો શરૂ શાનો કરે! અને પણ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] ~~[illegible]~~ [illegible] [illegible] ~~[illegible]~~ [illegible] [illegible] [illegible]. [illegible] [illegible] વગાડવા [illegible] [illegible] [illegible] ~~[illegible]~~ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] "સ્વદેશી" [illegible] [illegible] ન [illegible] તો [illegible] [illegible] "વિદેશી" [illegible] [illegible] [illegible]