

  
**MATHEMATICAL GREETINGS FROM PUERTO RICO!**  
 Wishing you a Merry Christmas and a Happy New Year 1952.  


Here is something pretty (and perhaps new):  
 Find digits  $x, y, a, b, c, d, e, f$  such that  
 $xy \cdot abcdef = (100 - yx) \cdot fedcba$ .

Solution:

Let us make

$a = 10 - y.$	$d = y - 1.$
$b = 9 - x - y.$	$e = x + y.$
$c = 9 - x.$	$f = x.$

Then

$$\begin{aligned}
 & (10x + y) [10^5 (10 - y) + 10^4 (9 - x - y) + 10^3 (9 - x) + 10^2 (y - 1) + 10 (x + y) + x] \\
 = & (100 - 10y - x) [10^5 x + 10^4 (x + y) + 10^3 (y - 1) + 10^2 (9 - x) + 10 (9 - x - y) + 10 - y]
 \end{aligned}$$

is an easily proved identity.

So that each letter remain a digit (less than 10) we must have  
 $x + y < 10.$      $x < 9.$      $y < 10.$      $xy < 82.$      $yx < 91.$

There are 45 solutions with

- |          |                                  |
|----------|----------------------------------|
| $x = 0:$ | $y = 1, 2, 3, 4, 5, 6, 7, 8, 9.$ |
| $x = 1:$ | $y = 1, 2, 3, 4, 5, 6, 7, 8.$    |
| $x = 2:$ | $y = 1, 2, 3, 4, 5, 6, 7.$       |
| $x = 3:$ | $y = 1, 2, 3, 4, 5, 6.$          |
| $x = 4:$ | $y = 1, 2, 3, 4, 5.$             |
| $x = 5:$ | $y = 1, 2, 3, 4.$                |
| $x = 6:$ | $y = 1, 2, 3.$                   |
| $x = 7:$ | $y = 1, 2.$                      |
| $x = 8:$ | $y = 1.$                         |

The following table contains all the solutions. Note that every multiplier less than 91 is present.

01. 989010=90.010989	11. 978021=89.120879	41. 945054=86.450549
02. 879120=80.021978	12. 868131=79.131868	42. 835164=76.461538
03. 769230=70.032967	13. 758241=69.142857	43. 725274=66.472527
04. 659340=60.043956	14. 648351=59.153846	44. 615384=56.483516
05. 549450=50.054945	15. 538461=49.164835	45. 505494=46.494505
06. 439560=40.065934	16. 428571=39.175824	51. 934065=85.560439
07. 329670=30.076923	17. 318681=29.186813	52. 824175=75.571428
08. 219780=20.087912	18. 208791=19.197802	53. 714285=65.582417
09. 109890=10.098901	21. 967032=88.230769	54. 604395=55.593406
31. 956043=87.340659	22. 857142=78.241758	61. 923076=84.670329
32. 846153=77.351648	23. 747252=68.252747	62. 813186=74.681318
33. 736263=67.362637	24. 637362=58.263736	63. 703296=64.692307
34. 626373=57.373626	25. 527472=48.274725	71. 912087=83.780219
35. 516483=47.384615	26. 417582=38.285417	72. 802197=73.791208
36. 406593=37.395604	27. 307692=28.296703	81. 901098=82.890109

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